Quark-Gluon Plasma and Relativistic Heavy Ion Collisions

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OUTLINE

- Introduction
- QCD thermodynamics
- Heavy ion collisions and hadroproduction
- Heavy ion collisions and hydrodynamics
- Jet production
- Conclusions

ICTP 2012-
Workshop on Recente Developments in Astronuclear and Astroparticle Physics
Matter under extreme conditions...

Fermi Notes on Thermodynamics

[Diagram of matter under unusual conditions with labels such as 'Electron-proton gas', 'Non-deg. electron gas', 'Atomic gas', 'Condened state', 'Degenerate electron gas', 'Relativ. degenerate electron gas', and 'Nucleon gas'.]

[Hot QGP]
Sketch of the phase diagram today

- quark-gluon plasma (QGP)
- hadron gas
- nucleon gas
- Colour superconductor
- neutron stars
- nuclei

Temperature axis

Critical point?
Phase Transition: from the hadronic side

- Hagedorn: density of hadronic states \( \rho(m) \) grows exponentially

\[
\rho(m) = C m^\alpha e^{m/T_0}
\]

\( T_0 = 160 \text{ MeV} \)

\( T_0 \) is called Hagedorn limiting temperature (1965) for an hadronic system (no quarks at that time)

Partition function for a gas of hadrons, with \( m > T \)

\[
\log Z(T,V) \propto \int_{m_0}^{\infty} dm \, m^{3/2} \rho(m) e^{m/T} \propto \int_{m_0}^{\infty} dm \, m^{\alpha+3/2} e^{-m\left(\frac{1}{T} - \frac{1}{T_0}\right)}
\]

Integral diverges for \( T \to T_0 \): hadronic matter cannot have a \( T > T_0 \)

Cabibbo-Parisi, PLB59(1975) - one year after Gross -Wilczek paper:

Divergency of the partition function has to be associated with a phase transition of hadronic matter to quark-gluon matter + asymptotic freedom at large \( T \) -> weakly quark gluon gas
Theory of Strong Interaction: QCD

\[ L_{QCD} = \sum_{i=1}^{n_f} \bar{\psi}_i \gamma_\mu \left( i \partial_\mu - g A_\mu^a \frac{\lambda_a}{2} \right) \psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} \sum_a F_{a \mu \nu} F^{a \mu \nu} \]

\[ F_{a \mu \nu} = \partial_\mu A^\nu_a - \partial_\nu A^\mu_a + i f_{abc} A^\mu_b A^\nu_c \]

Similar to QED, but gluons self-interact!

- Asymptotic freedom
- Confinement

\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log \left( \frac{Q^2}{\Lambda^2} \right)} \]

\[ \Lambda \sim 200 \text{ MeV} \approx 1 \text{ fm}^{-1} \approx (\text{hadron size})^{-1} \]

Two regimes:
- \( Q \gg \gg \Lambda_{QCD} \) one can use perturbative QCD (pQCD)
- \( Q \sim \Lambda_{QCD} \), \( Q > \Lambda_{QCD} \) non-perturbative methods:
  - lattice QCD (lQCD) and effective lagrangian approach
Lattice QCD: a huge computational effort

Solving QCD on a grid of points in space and time with size \((Ns)^3 \times Nt\)

**Partition function**

\[
Z = \int DA_\mu^a(x)D\bar{\psi}(x)D\psi(x)e^{\frac{i}{\beta} \int d\tau d^3x [A,\bar{\psi},\psi]}
\]

\(i\tau \rightarrow 1/T = \beta\)

**Dynamics -> Statistics**

Physical size: \(L = NS \ a\),  Time -> Temperature: \(T = 1/(Nt \ a)\)

\[
U(n,n + \hat{\mu}) = \exp(igt^a A_\mu^a(n))
\]

- Only thermodynamical observables (EoS, susceptibility, ...) or correlators
  No formulation for dynamical processes!
- Cannot be directly used at finite baryon density (no neutron star aargh!!)
- Only very recently, calculations for physical quark masses became feasible!

It is a fundamental guidance, but it is not sufficient!
Degrees of freedom in the Universe

\[ g(T) = \frac{\epsilon}{3T^4} \]

BBN
\[ e^+e^- \text{ annihilation} \]
QCD transition
EW transition

MSSM
SM

\[ g(T) = 2\gamma + 4e + 4\nu + 3\mu + 3\pi + \ldots + 16g + 31.5\mu ds + 21cb + \ldots \]

Order Parameters of the Phase Transition

Polyakov Loop - Confinement

- Crossover nature of the transition
  - no real order parameter
  - smooth behavior with temperature
  - still exhibit a rapid change in the vicinity of the phase transition

\[ L = \text{tr}(\exp[i g \int_0^\beta A_0(x, \tau) d\tau]) \approx e^{-H_{int}/T} \]

Order parameter for infinite quark mass

\[ H_{int} \to \infty \Rightarrow L = 0 \]

WB collaboration, JHEP (2010)

Reference a gas of non-interacting massless particle ... Stefan-Boltzmann

\[ \frac{\varepsilon_{SB}}{T^4} = \frac{\pi^2}{30} \left[ \frac{7}{8} d_{q+\bar{q}} + d_g \right] \]

Stefan-Boltzmann limit not reached by 20%: QGP as a weak interacting gas?

\[ \varepsilon_c \approx 0.7 \text{ GeV} / \text{fm}^3 \]
\[ T_c \approx 160 \text{ MeV} \]

T > Tc not a hadron gas but not a massless quark-gluon gas

No interaction means also

\[ I = \varepsilon - 3p = 0 \]
(for a massless gas)

Wuppertal-Budapest collaboration, QM2012 talk
How to produce a matter with $\varepsilon \gg 1$ GeV/fm$^3$ lasting for $\tau > 1$ fm/c in a volume much larger than a hadron?

<table>
<thead>
<tr>
<th>Accelerator</th>
<th>Lab</th>
<th>$\sqrt{s_{NN}}$</th>
<th>Nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPS (90's)</td>
<td>CERN</td>
<td>6-18</td>
<td>Pb-Pb</td>
</tr>
<tr>
<td>RHIC (00-..)</td>
<td>RHIC</td>
<td>7.7-200</td>
<td>Au-Au</td>
</tr>
<tr>
<td>LHC (09-..)</td>
<td>CERN</td>
<td>2750</td>
<td>Pb-Pb</td>
</tr>
</tbody>
</table>
Sketch of the nuclear collision process

- Pre-equil. phase
- Freeze-Out
- Hadron Gas
- QGP
- $\tau_{eq}$
- $T_c$
- Beam
Probing the Quark Gluon Plasma formation

The plasma is a transient and rapidly decays. This situation is dramatically different from the idealized situation of lattice QCD calculations.

Several possible probes at our disposal, that can be clustered in three groups

- Hadron radiation
- Electromagnetic radiation
- Hard probes: heavy quarks and quarkonia, jets, hard photons etc.
  Common feature: early production, “calculable” in perturbative QCD, easily comparable to pp and pA
Some typical definitions

Transverse view

$y_z = \tanh^{-1} \beta = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \approx v_z$

In terms of $p_T$ and $y$

$p^\mu = (E, p_x, p_y, p_z)$

$\rho = \mu$

Centrality of the collisions

$p^\mu = (m_T \cosh y, p_T, m_T \sinh y)$

$m_T = \sqrt{p_T^2 + m^2}$
Hadron radiation

Provides direct information about the hadronization stage of the plasma.
Allows to determine the thermodynamical state at the stage when hadrons cease interactions and decouple

Enhanced production of strange particles was predicted to be a signature of QGP formation
The temperature at the chemical freeze-out at RHIC is that of a hadronic black-body and it is the largest ever measured on Earth (~ $2 \times 10^{12}$ K).
Interesting the relative location between the freeze-out in HIC and critical line of the phase diagram
Increasing the energy...

Baryon stopping decreases, but a larger energy density is achieved (hotter, denser, longer)
Have we overcome the critical T?

Same observation as in elementary collisions. Statistical equilibrium as an Intrinsic feature of QCD at the soft scale, i.e. hadronization (F.B., U. Heinz, R. Stock, H. Satz et al)

In order to show that Tc has been overcome, i.e. that a QGP has been produced, we need to go to other observables.
Strangeness enhancement: Wroblewski ratio

\[ \lambda_S = 2 \frac{\langle s \bar{s} \rangle}{\langle u \bar{u} + d \bar{d} \rangle} \]
Relativistic hydrodynamical model

Early assumptions

- Local thermodynamical equilibrium at chemical decoupling (Cooper-Frye distribution)
- Lattice QCD Equation of State
- Ideal fluid (now viscous)
- Bjorken longitudinal solution
  \[ v_z = \frac{z}{t} \ (2+1) \]
  (now only at initial local time)
“Measuring” the initial temperature of the plasma with hydrodynamical model

(Huovinen, Kolb, Heinz, Hirano, Romatschke, Teaney,..)

The idea is to determine the initial conditions from the observed final spectra (in the transverse plane)

\[
\frac{dN}{dp_T} \approx e^{-m_T/T_{eff}}
\]

\[
T_{eff} \approx T_{dec} + \frac{1}{2} m \langle u_T^2 \rangle
\]

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>SPS</th>
<th>RHIC 1</th>
<th>RHIC 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{eq}$ (fm$^{-3}$)</td>
<td>43</td>
<td>95</td>
<td>110</td>
</tr>
<tr>
<td>$T_{eq}$ (MeV)</td>
<td>257</td>
<td>340</td>
<td>360</td>
</tr>
<tr>
<td>$\tau_{eq}$ (fm/c)</td>
<td>0.8</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Fitted parameters. The initial $T$ is correlated to the initial equilibration time.
Elliptic flow in peripheral collisions

Anisotropic pressure gradient: if there is collective flow, particles get a larger momentum on the reaction plane!

\[
\frac{dN_i}{dy p_{\perp} dp_{\perp} d\varphi_p}(b) = \frac{1}{2\pi} \frac{dN_i}{dy p_{\perp} dp_{\perp}}(b) \left( 1 + 2 v_2(p_{\perp}, b) \cos(2\varphi_p) + \ldots \right). 
\]
Spacial anisotropy gets converted into momentum anisotropy provided that:

- Thermalization occurs while the system is still almond-shaped
- This requires short times (~ 1 fm/c) when the system is still in the QGP phase
Elliptic flow coefficient $v_2$

\[ \frac{dN_i}{dy p_{\perp} dp_{\perp} d\varphi_p}(b) = \frac{1}{2\pi} \frac{dN_i}{dy p_{\perp} dp_{\perp}}(b) \left( 1 + 2 v_2^i(p_{\perp}, b) \cos(2\varphi_p) + \ldots \right). \]

STAR, PRL 90 032301 (2003)
$v_2$ in good agreement with quasi-ideal fluid

\[ T^{\mu \nu} = (\rho + p)u^\mu u^\nu - pg^{\mu \nu} \]

- Local equilibrium distribution

\[ \varepsilon \frac{d\sigma}{d^3p} = \int d\Sigma \mu p^\mu \frac{1}{e^{\beta \cdot p} \pm 1} \]

- Lattice QCD EoS

- Initial energy density $\sim 25$ GeV/fm$^3$

**Mass ordering** at low pT also in agreement with hydro predictions
RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a liquid.

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain fundamental properties of the universe using 10 dimensions instead of the usual three spatial dimensions plus time.
Viscous hydrodynamics

Relativistic Navier-Stokes

\[ T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \eta (\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \partial^\alpha u_\alpha) \]

but it violates causality,
2nd order expansion needed -> Israel-Stewart

\[ \pi^{\mu\nu} = \eta \nabla \langle u^\mu u^\nu \rangle + \tau_\pi \left[ \Delta^\mu_\alpha \Delta^\nu_\beta D\pi^{\alpha\beta} \ldots \right] \]

two more parameters appears +
\[ \delta f \sim f_{\text{eq}} \] reduce the pT validity range
QGP: an almost ideal fluid

Similar phenomenon observed in atomic physics:
Cold atom clouds (strongly coupled) expanding after switching off an optical trap (Lithium)
Shear viscosity in strongly interacting conformal quantum field theory (from AdS/CFT correspondance)

Kovtun, Son, Starinets PRL 94, 111601 (2005)

\[ \frac{\eta}{s} = \frac{\hbar}{4\pi} \]

**Conjecture:** it is a universal quantum bound which is reached for the maximal coupling

\[ \eta \sim \rho v \ell, \quad s \sim n = \frac{\rho}{m} \]

\[ \frac{\eta}{s} \sim m v \ell \sim \hbar \quad \text{mean free path} \]

\[ \text{de Broglie wavelength} \]

Quasiparticles: de Broglie wavelength \( \lesssim \) mean free path

Therefore \( \eta/s \gtrsim \hbar \)
Universal bound?

\[
\eta/s \quad \text{(limit)} = \frac{1}{4\pi}
\]

\[
\eta/s \quad \text{(water)} > 10
\]

\[
4\pi \cdot \eta/s
\]

Physics Today, May 2005

- Helium 0.1 MPa
- Nitrogen 10 MPa
- Water 100 MPa

\[
\eta/s \quad \text{(water)} > 10
\]

\[
\eta/s \quad \text{(limit)} = \frac{1}{4\pi}
\]
Recent development: from averages to Event-by-event hydrodynamics

$\varepsilon_n$ for $n>2$ of similar size

When including fluctuations, all moments appear:

$n = 2 \quad n = 3 \quad n = 4 \quad n = 5 \quad n = 6$

also $v_1$ and $n > 6$

Compute $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$
Viscosity smoothens fluctuations and affect more higher harmonics

\[ \eta/s = 0 \]

\[ \eta/s = 0.16 \]

\[ t = 3.7 \text{ fm/c} \]
High harmonics fluctuations reminds the CMB fluctuations...

\[ \frac{\Delta T}{T} \approx 10^{-5} \]
\[ l_{\text{maximum}} \approx 210 \]
\[ \delta \phi \approx 2\pi/l_{\text{maximum}} \approx 1^\circ \]

Freeze-out \( \tau \approx 380,000 \) y's (QGP \( \sim 10^{-22} \) s)
Sound horizon \( R \approx \text{Mps} \) (QGP \( \sim 6 \) fm)

Of course \( n=200 \) is not possible to be seen for a hadron system with \( R \sim 10 \) fm
A recent ALICE measurement

None of the models reproduce the correct shapes:
- No peak at \( n=3 \)
- Too large for \( n>6 \)

A promising new challenge -> new findings and knowledge

A first schematic calculation

Hydrodynamics, \( e^4=3p \)
Renormalized to \( m=3 \)

Staig & Shuryak, PRC (2011)
Jets as a probe of the plasma

Nucleus-nucleus collisions

- hard initial scattering
- scattered partons probe traversed hot and dense medium
- ‘jet tomography’

- Initial parton-parton scattering with large momentum transfer
  - Unlike in vacuum, quarks and gluons lose energy (brehmsstrahlung and collisions) before hadronizing

Theoretical task: to calculate energy distribution of hard partons traversing a length L within the medium
Suppression of high-\(p_T\) hadrons
(not so for \(\gamma\))

- Due to energy loss of partons, there is a strong suppression of high \(p_T\) hadrons
- Even larger at LHC than at RHIC
- Nuclear modification factor \(R_{AA}(p_T)\) for charged particles produced in 0-5% centrality range
  - minimum (~ 0.14) for \(p_T\) ~ 6-7 GeV/c
  - then slow increase at high \(p_T\)

\[ R_{AA}(p_T) = \frac{\text{Yield}_{AA}(p_T)}{\langle N\text{bin} \rangle_{AA} \text{Yield}_{pp}(p_T)} \]

\(R_{AA} = 1\) for hard QCD processes in absence of nuclear modifications

Including CDF data

\[ 0.9 \text{ TeV} \times \text{NLO (2.76 TeV)/NLO(0.9 TeV)} \]

PLB 696 (2011) 30-39
Di-jet imbalance

- Pb-Pb events with large di-jet imbalance observed at the LHC

→ recoiling jet strongly quenched!
The largest temperature ever achieved on Earth (> $2 \times 10^{12}$ K)

- $\sim 10^3$ K
- $\sim 10^7$ K
- $\sim 10^8$ K
- $\sim 5 \times 10^{12}$ K!
Conclusions and outlook

- There is little doubt that QGP has been produced in relativistic heavy ion collisions

- Many observations point to a system which is hotter than $T_c$ and strongly interacting (low viscosity), in accordance with QCD. The study of its properties is ongoing at LHC

- Search of the QCD critical point and the onset of deconfinement at lower energies

- Relativistic heavy ion collisions have been a very effective laboratory for advanced theoretical subjects: not only QCD, but also relativistic statistical mechanics, relativistic hydrodynamics and even (maybe) string theory

- Hopefully, there will be useful results for relativistic astrophysics and cosmology
**Soft and Hard probes**

**SOFT** ($p_T \sim \Lambda_{QCD,T}$) driven by *non perturbative QCD*
- Hadron yields, **collective modes of the bulk**, strangeness enhancement, fluctuations, thermal radiation, dilepton enhancement

**HARD** ($p_T \gg \Lambda_{QCD}$)
- Early production, pQCD applicable, Baseline pp, pA
- Quenching, **heavy quarks**, quarkonia, hard photons (W,Z)

Nuclear modification factor

$$R_{AA}(p_T) = \frac{\frac{d^2 N_{AA}^{s}}{d p_T}}{N_{coll} \frac{d^2 N_N^{NN}}{d p_T}} = \frac{\text{medium}}{\text{vacuum}}$$