Effective Field Theories of QCD

Effective theories of QCD at high density

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Summary

- Introduction
- Effective theory for CFL phase
- Weak coupling calculations
- Effective theory for LOFF phase
- Weak coupling calculations
- Conclusions

Introduction

Ideas about **Color Superconductivity** (CS) back to B. Barrois, NP **B129** (1977),390; S. Frautschi, Erice 1978; D. Bailin and A. Love, Phys. Report **107** (1984) 325. Only recently it has been realized that CS methods are very powerful to analyze in a rigorous fashion the high density and zero temperature region of QCD phase space (for a complete review see; K. Rajagopal and F. Wilczek, hep-ph/0011333)

- Naive expectation at very high density: asymptotic freedom $\Rrightarrow$ Fermi sphere of almost free quarks

- **BCS proved the instability of the Fermi surface in presence of an attractive arbitrarily weak interaction.** The previous picture changes to a **coherent state of particle-hole pairs**, the **Cooper pairs**

- The dominant interaction in QCD (gluon exchange) is attractive. **A diquark condensation is expected**

Effective Theory for the CFL phase

In 3 massless flavors QCD at high density the diquark condensate has the form

$$\langle q^i_{\alpha L}(\vec{p}) C\, q^j_{\beta L}(-\vec{p})\rangle \approx \epsilon_{ijX}\epsilon^{\alpha\beta X} + \kappa(\delta^\alpha_i \delta^\beta_j + \delta^\beta_i \delta^\alpha_j)$$

$$\langle q^i_{\alpha L}(\vec{p}) C\, q^j_{\beta L}(-\vec{p})\rangle = -\langle q^i_{\alpha R}(\vec{p}) C\, q^j_{\beta R}(-\vec{p})\rangle$$

This produces the symmetry breaking

$$G = SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_V \otimes U(1)_A$$

$$\Downarrow$$

$$H = SU(3)_{c+L+R} \otimes Z_2 \otimes Z_2$$

The $U(1)_A$ symmetry is restored at very high density for $N_f = 3$. The corresponding breaking originates asymptotically a massless Goldstone boson ($U(1)_A \to Z_2$) (R. Rapp, T. Schäfer, E,V, Shuryak, Velkowsky, hep-ph/9904353; T. Schäfer, hep-ph/9909574; D.T. Son and M.A. Stephanov, Phys. Rev. **D61** (2000) 074012, hep-ph/9910941; *ibidem*, Erratum, **D62** (2000) 059902, hep-ph/0004095).

The condensate belongs to the representation $(\bar{3},\bar{3}) \oplus (6,6)$ of $SU(3)_c \otimes SU(3)_{L,R}$, and $(\bar{3},\bar{3})$ for $\kappa = 0$.

To represent the Goldstone fields it is enough to consider one representation, say the $(\bar{3}, \bar{3})$. We then introduce **Goldstone fields as the phases of the following condensates**

$$X_\alpha^i \approx \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \langle q_{\beta L}^j q_{\gamma L}^k \rangle^*$$

$$Y_\alpha^i \approx \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \langle q_{\beta R}^j q_{\gamma R}^k \rangle^*$$

With the notations

$$g_c \in SU(3)_c, \quad g_{L(R)} \in SU(3)_{L(R)}$$

$$\exp(i\alpha) \in U(1)_V, \quad \exp(i\beta) \in U(1)_A$$

we have

$$q_{L(R)} \in (\mathbf{3}, \mathbf{3}) \text{ of } SU(3)_c \otimes SU(3)_{L(R)}$$

$$q_L \to \exp i(\alpha + \beta) q_L \text{ under } U(1)_V \otimes U(1)_A$$

$$q_R \to \exp i(\alpha - \beta) q_R \text{ under } U(1)_V \otimes U(1)_A$$

and under the total symmetry group

$$X \to g_c X g_L^T \exp(-2i\alpha - 2i\beta)$$

$$Y \to g_c Y g_R^T \exp(-2i\alpha + 2i\beta)$$

Being X and Y the phases of the condensates $(\bar{3}, \bar{3})$ we have

$$X, Y \in U(3)$$

The number of fields is

$$\#X + \#Y = (1+8) + (1+8) = 18$$

8 of these fields give mass to the gluons. **The physical NGB are 10** correponding to the breaking of the global symmetry

$$SU(3)_L \otimes SU(3)_R \otimes U(1)_V \otimes U(1)_A$$
$$\Downarrow$$
$$SU(3)_{L+R} \otimes Z_2 \otimes Z_2$$

It is convenient to separate the $U(1)$ factors defining

$$X = \hat{X} \exp(2i\phi + 2i\theta), \quad Y = \hat{Y} \exp(2i\phi - 2i\theta)$$

with $\hat{X}, \hat{Y} \in SU(3)$. We introduce also

$$\det(X) = \exp(6i\phi + 6i\theta)$$

$$\det(Y) = \exp(6i\phi - 6i\theta)$$

The transformation properties are

$$\hat{X} \to g_c \hat{X} g_L^T, \quad \hat{Y} \to g_c \hat{Y} g_R^T$$

$$\phi \to \phi - \alpha, \quad \theta \to \theta - \beta$$

The breaking of the global symmetry can be also described by gauge invariant order parameters given by

$$\Sigma_j^i = (\hat{Y}_\alpha^j)^* \hat{X}_\alpha^i \to \Sigma = \hat{Y}^\dagger \hat{X}$$

$$d_X = \det(X), \qquad d_Y = \det(Y)$$

These are the 8+2 NGB's corresponding to the breaking of the global symmetry. Notice that

$$\Sigma \to g_R^* \Sigma \, g_L^T$$

Σ^{T} transforms as the usual chiral field

The most general invariant lagrangian for the fields Σ, ϕ and θ under G, the space rotation group $O(3)$ and Parity ($X \leftrightarrow Y$, $\phi \leftrightarrow \phi$, $\theta \leftrightarrow -\theta$) is (R. C. and R. Gatto, Phys. Lett. **B464** (1999) 111)

$$\mathcal{L} = \frac{F_T^2}{4}\left(Tr\,[\dot{\Sigma}\dot{\Sigma}^\dagger] - v^2\,Tr[\vec{\nabla}\Sigma\cdot\vec{\nabla}\Sigma^\dagger]\right)$$

$$-\frac{{f_T^V}^2}{2}\left(\dot{\phi}^2 - v_\phi^2|\vec{\nabla}\phi|^2\right)$$

$$-\frac{{f_T^A}^2}{2}\left(\dot{\theta}^2 - v_\theta^2|\vec{\nabla}\theta|^2\right)$$

Notice that the first term coincides with the chiral lagrangian except for the breaking of the Lorentz invariance

Perturbative calculations

Once taken into account the diquark condensation, it is possible to do perturbative calculations at very high density taking advantage of asymptotic freedom. We will follow the following steps

- We go from $\mathcal{L}_{QCD}$ at high density to an effective theory describing gapped fermionic excitations close to the Fermi surface.

- We couple Goldstone and gluons in an invariant way to the fermions at the Fermi surface and evaluate the relevant $n-$point functions. This allows the determination of the couplings appearing in the effective lagrangian for NG bosons and gluons

We start describing the effective theory around the Fermi surface (the physics has been described by J. Polchinski, TASI 1992, hep-th/9210046, see also: D.K. Hong, Phys. Lett. **B473** (2000) 118, hep-ph/9812510 and Nucl. Phys. **B582** (2000) 451, hep-ph/9905523; S.R. Beane, P.F. Bedaque, M.J. Savage, Phys. Lett. **B483** (2000) 131, hep-ph/0002209)

We consider QCD at finite density, with a chemical potential μ ($a = 1, \cdots, 8$)

$$\mathcal{L}_{QCD} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \mu \bar{\psi} \gamma_0 \psi$$

For $\mu \gg \Lambda_{QCD}$ quarks are almost free . We have, ($\vec{\alpha} = \gamma_0 \vec{\gamma}$)

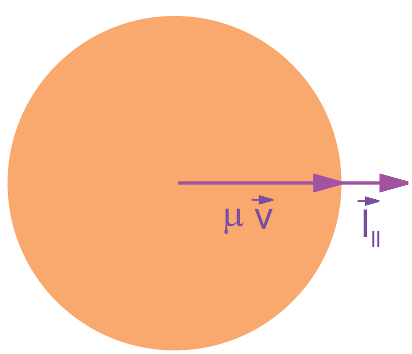
$$(\not{p} + \mu\gamma_0)\psi(p) = 0 \quad \Rightarrow \quad (p^0 + \mu)\psi = \vec{\alpha} \cdot \vec{p}\,\psi$$

The energy eigenvalues are

$$p^0 = E_\pm = -\mu \pm |\vec{p}|$$

with eigenstates $|\pm\rangle$. For momenta close to the Fermi momentum $|\vec{p}| \approx \mu$ only the states $|+\rangle$ close to the Fermi surface ($E_+ \approx 0$) can be excited. The states $|-\rangle$ with $E_- \approx -2\mu$ decouple at large μ. More formally write

$$p^\mu = \mu v^\mu + \ell^\mu, \quad v^\mu = (0, \vec{v}_F), \quad |\vec{v}_F| = 1$$



$\vec{p} = \mu \vec{v} + \vec{l}_\parallel$

The Hamiltonian is ($\vec{\alpha} = \gamma^0 \vec{\gamma}$)

$$H = -\mu + \vec{\alpha}\cdot\vec{p} \;\; \rightarrow \;\; H = -\mu(1 - \vec{\alpha}\cdot\vec{v}_F) + \vec{\alpha}\cdot\vec{\ell}$$

Introducing the projectors

$$P_\pm = \frac{1 \pm \vec{\alpha}\cdot\vec{v}_F}{2}$$

and $|\pm\rangle = P_\pm\,\psi$, we get

$$H\,|+\rangle = \vec{\alpha}\cdot\vec{\ell}\,|+\rangle, \quad H|-\rangle = (-2\mu + \vec{\alpha}\cdot\vec{\ell})\,|-\rangle$$

We decompose the fields with $P_\pm$ and integrate out all the modes with $|\vec{\ell}| > \delta$ ($\Delta < \delta \ll \mu$)

$$\psi(x) = \sum_{\vec{v}_F} e^{-i\mu v\cdot x}\left[\psi_+(x) + \psi_-(x)\right]$$

$$\psi_\pm(x) = e^{i\mu v\cdot x} P_\pm\,\psi(x) = \int_{|\vec{\ell}|<\delta} \frac{d^4\ell}{(2\pi)^4} e^{-i\ell\cdot x}\psi_\pm(\ell)$$

Substituting inside the lagrangian we get (**off-diagonal terms in the velocity are cancelled by the exponential oscillations for $\mu \to \infty$**)

$$\mathcal{L} = \sum_{\vec{v}_F} \Big[\psi_+^\dagger iV \cdot D\,\psi_+ + \psi_-^\dagger (2\mu + i\tilde{V}\cdot D)\psi_-$$

$$+ \, (\bar{\psi}_+ i \not{D}_\perp \psi_- + \text{h.c.}) \Big]$$

$$V^\mu = (1, \vec{v}_F), \qquad \tilde{V}^\mu = (1, -\vec{v}_F)$$

$$\not{D}_\perp = D_\mu \gamma_\perp^\mu, \quad \gamma_\perp^\mu = P_\perp^{\mu\nu}\gamma_\nu$$

$$P_\perp^{\mu\nu} = (2g^{\mu\nu} - V^\mu \tilde{V}^\nu - \tilde{V}^\mu V^\nu)$$

Fields inside $\mathcal{L}$ are evaluated at the same Fermi velocity, or

Fermi velocity selection rule

For large chemical potential the field ψ_- decouples and it can be eliminated through its equation of motion. At the leading order

$$iV \cdot D\,\psi_+ = 0, \quad \psi_- = -\frac{i}{2\mu}\gamma_0 \not{D}_\perp \psi_+$$

For fixed $\vec{v}_F$ only energy and momentum along the Fermi velocity are relevant.

Due to the velocity selection rule we have

infinite copies of 2-d physics

Eliminating the field ψ_-:

$$\mathcal{L} = \sum_{\vec{v}_F} \left[\psi_+^\dagger i V \cdot D \psi_+ - \frac{1}{2\mu + i\tilde{V} \cdot D} \psi_+^\dagger (\not{D}_\perp)^2 \psi_+ \right]$$

The $1/\mu$ term may contribute to one-loop diagrams giving rise to an extra μ factor (see later).

Couplings to Goldstone bosons

We have seen that the NG fields, $\hat{X}$ ($\hat{Y}$), transform under G as $q_L(q_R)$, for instance

$$q_L \to g_c\, q_L g_L^T, \qquad \hat{X} \to g_c \hat{X} g_L^T$$

There are two possible invariant couplings with the NGB's, and similar for $\hat{Y}$ and ψ_R, corresponding to the two channels $(\bar{3}, \bar{3})$ and $(6, 6)$

$$\gamma_1\, Tr\,[q_L^T \hat{X}^\dagger] C Tr[q_L \hat{X}^\dagger] + \gamma_2\, Tr\,[q_L^T C \hat{X}^\dagger q_L \hat{X}^\dagger]$$

$$+\ \text{h.c.}$$

Since in the fundamental state $\langle \hat{X} \rangle = \langle \hat{Y} \rangle = 1$, the two couplings reproduce the correct breaking of the symmetry in the CFL phase. For simplicity we will take

$$\gamma_1 = -\gamma_2 \propto \frac{\Delta}{2}$$

corresponding to a condensate in the representation $(\bar{3}, \bar{3})$

In this case the coupling can be written as

$$-\frac{\Delta}{2}\sum_{I=1,2,3} Tr\,[(q_L\hat{X}^\dagger)^T C\epsilon_I(q_L\hat{X}^\dagger)\epsilon_I]$$

with$(\epsilon_I)_{ab}=\epsilon_{Iab}$. It is convenient to define (λ_a, $a=1,\cdots,8$, are the Gell-Mann matrices, $\lambda_0=\sqrt{2/3}\ 1$, and $\Delta_a=\Delta$, $\Delta_9=-2\Delta$)

$$\hat{X}=1+(\hat{X}-1)\equiv 1+X_1$$

$$\psi_\pm=\frac{1}{\sqrt{2}}\sum_{A=1}^{9}\lambda_A\psi_\pm^A$$

In terms of the velocity decomposition we get the lagrangian (R.C., R. Gatto and G. Nardulli, Phys. Lett. **B498** (2001) 179, hep-ph/0010321)

$$\mathcal{L}=\sum_{\vec{v}_F}\frac{1}{2}\Big[\sum_{A=1}^{9}\Big(\psi_+^{A\dagger}iV\cdot D\psi_+^A+\psi_-^{A\dagger}i\tilde{V}\cdot D\psi_-^A -\Delta_A(\psi_-^{A^T}C\psi_+^A+\text{h.c.})\Big) -\Delta\sum_{I=1,3}\Big(Tr[(\psi_-X_1^\dagger)^T C\epsilon_I(\psi_+X_1^\dagger)\epsilon_I]+\text{h.c.}\Big)\Big]$$

Goldstone and gap terms couple fields with opposite Fermi velocity (Cooper pairs). ψ_- **is obtained from** ψ_+ **sending** $\mathbf{v_F}\to-\mathbf{v_F}$

Formalism neater introducing Nambu-Gorkov fields

$$\chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}$$

We get the quadratic part of the lagrangian (plus analogous term for the right-handed fields)

$$\boxed{\mathcal{L}_0 = \int \frac{d\vec{v}_F}{8\pi} \, \frac{1}{2} \sum_{A=1}^{9} \chi^{A\dagger} \begin{bmatrix} iV \cdot D & \Delta^A \\ \Delta^A & i\tilde{V} \cdot D^* \end{bmatrix} \chi^A}$$

and the propagator

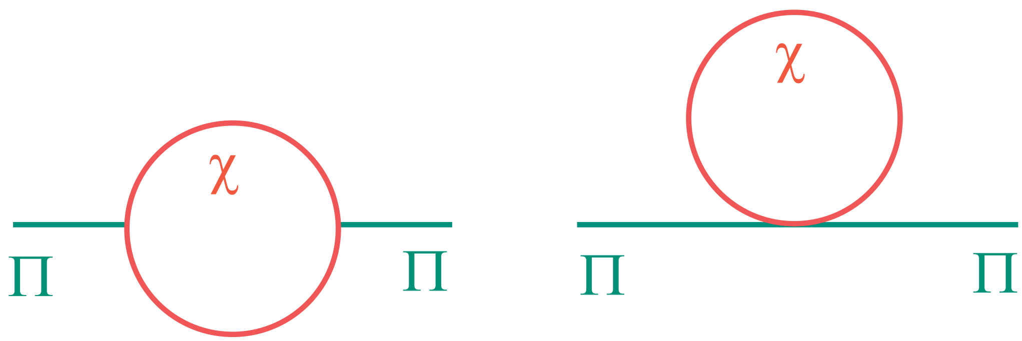
$$S_{AB}(p) = \frac{2\delta_{AB}}{(V \cdot p\,\tilde{V} \cdot p) - \Delta_A^2} \begin{bmatrix} \tilde{V} \cdot p & -\Delta_A \\ -\Delta_A & V \cdot p \end{bmatrix}$$

Expanding the Goldstone fields $\hat{X}$ and $\hat{Y}$ in the gauge $\hat{X} = \hat{Y}^\dagger$

$$\hat{X} = \exp i \left(\frac{\lambda_a \Pi^a}{2F} \right), \qquad a = 1, \cdots, 8$$

we get vertices $\Pi\chi\chi$ and $\Pi\Pi\chi\chi$. The Goldstone self-energy is given by the diagrams

Goldstone self-energy



Expanding to $\mathcal{O}(p^2)$ we get

$$i\mu^2 \frac{21 - 8\ln 2}{72\pi^2 F^2} \int \frac{d\vec{v}_F}{4\pi} \sum_{a=1}^{8} \Pi^a (V \cdot p)(\tilde{V} \cdot p) \Pi^a$$

from which

$$\mathcal{L}_{\text{eff}}^{\text{kin}} = \frac{\mu^2 (21 - 8\ln 2)}{72\pi^2 F^2} \sum_{a=1}^{8} \left(\dot{\Pi}^a \dot{\Pi}^a - \frac{1}{3} |\vec{\nabla} \Pi_a|^2 \right)$$

To get the proper normalization we must have

$$F^2 = \frac{\mu^2(21 - 8 \ln 2)}{36\pi^2}$$

Comparing with the effective lagrangian we see that

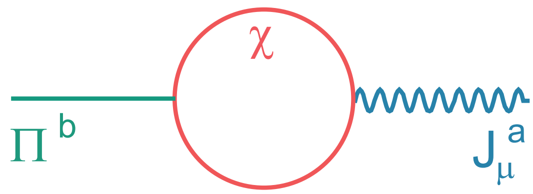
$$F_T = F, \quad v = \frac{1}{\sqrt{3}}$$

Therefore the pions satisfy the dispersion relation

$$(p^0)^2 - \frac{1}{3}|\vec{p}|^2 = 0 \quad \mapsto \quad p^0 = \pm \frac{1}{\sqrt{3}} |\vec{p}|$$

The same result has been obtained through the evaluation of the Debye and Meissner masses of the gluons (D. T. Son and M. A. Stephanov, Phys. Rev. **D61**, 074012 (2000), hep-ph/9910491; erratum, *ibid.* **D62**, 059902 (2000), hep-ph/0004095; M. Rho, A. Wirzba and I. Zahed, Phys. Lett. **B473**, 126 (2000), hep-ph/9910550; D. K. Hong, T. Lee and D. Min, Phys. Lett. **B477**, 137 (2000), hep-ph/9912531; C. Manuel and M. H. Tytgat, Phys. Lett. **B479**, 190 (2000), hep-ph/0001095; M. Rho, E. Shuryak, A. Wirzba and I. Zahed, Nucl. Phys. **A676**, 273 (2000), hep-ph/0001104; S. R. Beane, P. F. Bedaque and M. J. Savage, Phys. Lett. **B483**, 131 (2000), hep-ph/0002209; C. Manuel and M. Tytgat, hep-ph/0010274)

To the same result one can arrive through the evaluation of the diagram (R.C., R. Gatto and G. Nardulli, Phys. Lett. **B498** (2001) 179)



giving

$$\langle 0|J_\mu^a|\Pi^b\rangle = iF\delta_{ab}\tilde{p}_\mu, \quad \tilde{p}^\mu = \left(p^0, \frac{1}{3}\vec{p}\right)$$

This shows the **current is conserved**, due to the dispersion relation for the pions

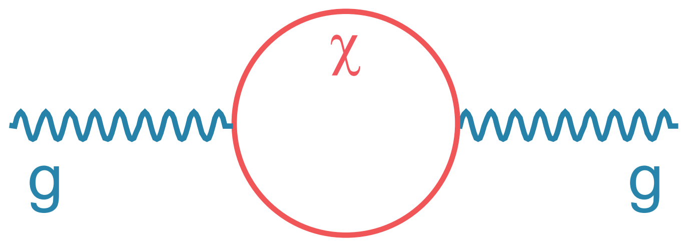
$$p\cdot\tilde{p} = (p^0)^2 - \frac{1}{3}|\vec{p}|^2 = 0$$

Similar calculations can be done for the NG fields ϕ and θ

Couplings to the gluons

By the same techniques we can evaluate the gluon self-energy. The coupling of gluons to fermions is given by the covariant derivative. This gives rise to the diagram

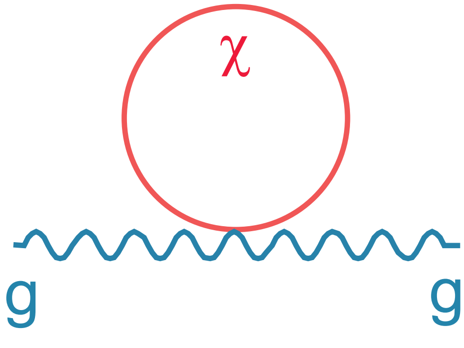
Gluon self-energy



We have also the tadpole contribution arising from the term

$$-\frac{1}{2\mu + i\tilde{V}\cdot D}\psi_+^\dagger (\not{D}_\perp)^2 \psi_+$$

Contribution to the Meissner mass



The loop integration gives an extra μ factor compensating the one in the denominator. From the constant part of the diagrams we get Debye and Meissner masses

$$m_D^2 = g_s^2 F^2 = \frac{\mu^2 g_s^2}{36\pi^2}(21 - 8\log 2)$$

$$m_M^2 = \frac{\mu^2 g_s^2}{108\pi^2}\left(-33 - 8\log 2 + \underbrace{54}_{\text{tadpole}}\right) = \frac{m_D^2}{3}$$

Showing once again $v^2 = 1/3$. The tadpole term is also essential in order to satisfy the Ward identity ($\Pi_{ab}^{\mu\nu}$ is the gluon self-energy)

$$p_\mu \Pi_{ab}^{\mu\nu} \propto \langle 0|J^\nu|\Pi^b\rangle = iF\delta_{ab}\tilde{p}^\nu$$

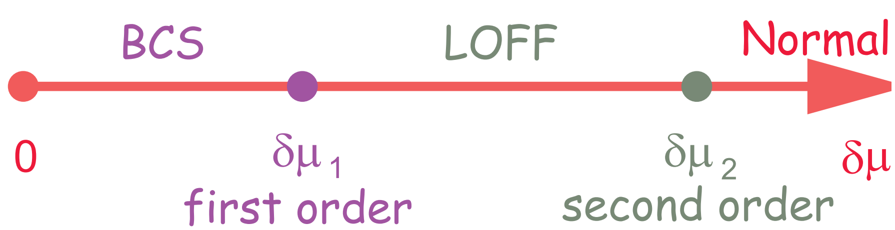
The Meissner and the Debye masses are not the physical masses of the gluons. This comes from the wave-function renormalization proportional to $\mu^2 g_s^2/\Delta^2$ making the **effective square masses proportional to Δ^2** rather than to $g_s^2\mu^2$($m_{\text{gluon}} \lesssim 2\Delta$). This changes also $g_s \to g_s\Delta/(g_s\mu) = \Delta/\mu$

The LOFF phase

BCS condensation happens for pairs of opposite momenum. In presence of mass difference between quarks of different flavor, if the corresponding energy difference exceeds the gap the condensate gets disrupted. Simulated in a simple model (M. Alford, J.A. Bowers and K. Rajagopal, hep-ph/0008208) with two species of quarks, say up and down, with different μ's

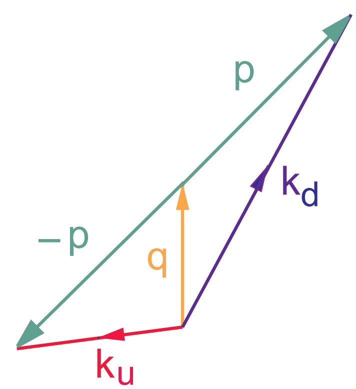
$$\mu_u = \bar{\mu} - \delta\mu, \quad \mu_d = \bar{\mu} + \delta\mu$$

Two critical values of $\delta\mu$:



For $\delta\mu_1 < \delta\mu < \delta\mu_2$ condensation happens between pairs of non-vanishing total momentum $2\vec{q}$

$$\vec{k}_u = \vec{p} + \vec{q}, \quad \vec{k}_d = -\vec{p} + \vec{q}$$



This state (**LOFF state**) (A.I. Larkin and Yu.N. Ovchinnikov, JETP **20** (1965) 762; P. Fulde and R.A. Farrell, Phys. Rev. **135** (1964) A550) is somewhat favored since quarks can stay close to their own Fermi surface. In the LOFF phase two condensates, the scalar

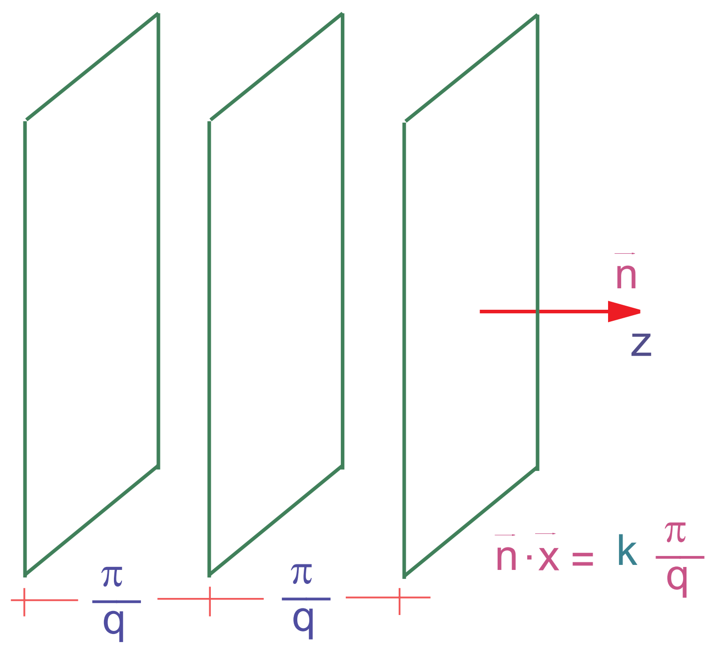
$$\langle \epsilon^{\alpha\beta 3}\epsilon_{ij}\psi^T_{i\alpha}(x)C\psi_{i\beta}(x)\rangle = \Gamma^{(s)}\, e^{2i\vec{q}\cdot\vec{x}}$$

and the vector ($\vec{n} = \vec{q}/|\vec{q}\,|$)

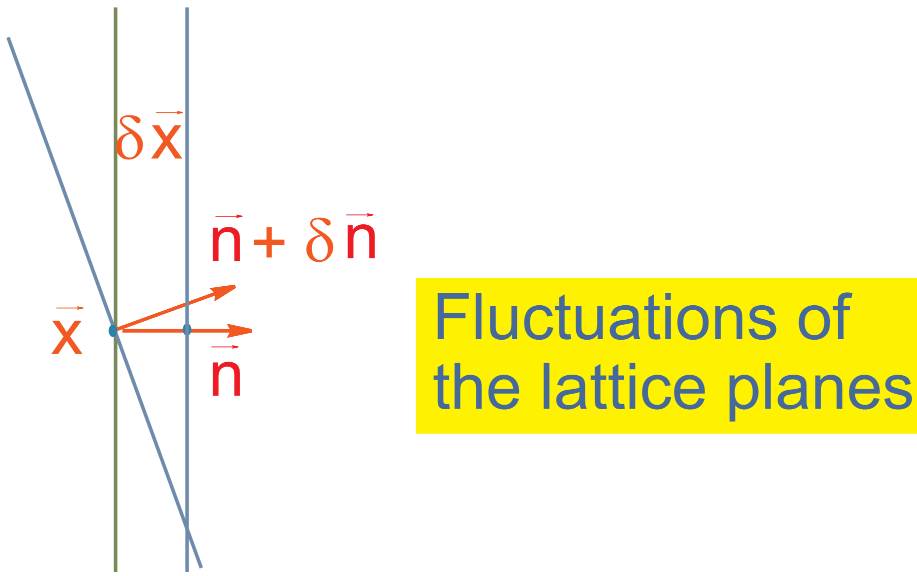
$$\langle \epsilon^{\alpha\beta 3}\psi_{i\alpha}(x)\sigma^1_{ij}C(\vec{\alpha}\cdot\vec{n})\psi_{j\beta}(x)\rangle = \Gamma^{(v)}\, e^{2i\vec{q}\cdot\vec{x}}$$

Both terms break space symmetries.

How to count NG bosons? The condensates induce a lattice structure ($\vec{n} = \vec{q}/|\vec{q}|$)



The planes define maxima of the Majorana masses which oscillate along $\vec{n}$. The NG bosons are related to the fluctuations of the planes due to translations and rotations



We describe the total fluctuation through a field $\phi(x)$

$$\vec{n}\cdot\vec{x} \to \vec{n}\cdot\vec{x} + \frac{1}{2fq}\phi(x) \equiv \frac{1}{2q}\Phi(x), \quad \langle\phi(x)\rangle_0 = 0$$

The two fluctuations can be described by fields

$$T(x) = \vec{n}\cdot\delta\vec{x}, \quad \vec{R}(x) = \vec{n} + \delta\vec{n}$$

satisfying

$$\langle T(x)\rangle_0 = 0, \quad |\vec{R}(x)|^2 = 1, \quad \langle\vec{R}(x)\rangle_0 = \vec{n}$$

The field $\vec{R}(x)$ can be defined as

$$\vec{R}(x) = \left[e^{i(\xi_1 L_1 + \xi_2 L_2)}\right]_{i3}, \quad (L_i)_{jk} = -i\epsilon_{ijk}$$

We now get

$$\frac{\phi}{2fq} = \delta(\vec{n}\cdot\vec{x}) = (\vec{R} - \vec{n})\cdot\vec{x} + T$$

In order the field ϕ describes small fluctuations we must have the fields T and $\vec{R}$ functionally related, meaning that $\vec{R}(x)$ can be expressed in terms of $\Phi(x)$

$$\vec{R}(x) = \frac{\vec{\nabla}\Phi(x)}{|\vec{\nabla}\Phi(x)|}$$

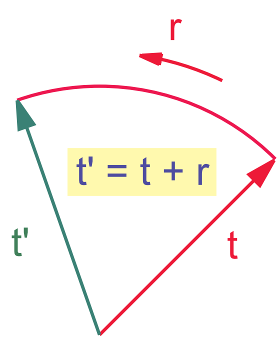
In terms of the ξ_i fields

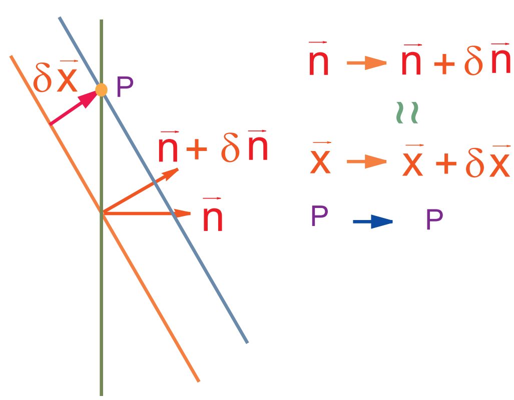
$$R_i \approx n_i + \epsilon_{ijk} n_j \xi_k \approx n_i + \frac{1}{2fq}(\partial_i - n_i(\vec{n}\cdot\vec{\nabla}))\phi$$

that is $\delta\vec{n} = (-\xi_2, \xi_1, 0)$, and

$$\frac{1}{2fq}\partial_1\phi = -\xi_2, \quad \frac{1}{2fq}\partial_2\phi = \xi_1$$

The physical origin for this is that the NG fields are nothing but the local parameters of the broken symmetries and locally there is no difference between translations and rotations





In the LOFF phase only one NGB is present: the phonon

$\phi(x)$ is the physical field, but the invariant lagrangian is more easily obtained in terms of Φ

$$\mathcal{L} = \frac{f^2}{2}\left[\dot{\Phi}^2 - \sum_{n=1}^{\infty} c_n(|\vec{\nabla}\Phi|^2)^n\right]$$

Gradient expansion in Φ is not valid since

$$\langle\vec{\nabla}\Phi\rangle = 2\vec{q}$$

of order Δ. However the expansion is feasible for ϕ. Noticing that

$$|\vec{\nabla}\Phi|^2 = 4q^2 + \frac{4q}{f}\vec{n}\cdot\vec{\nabla}\phi + \frac{1}{f^2}|\vec{\nabla}\Phi|^2$$

at two spatial derivatives order we get

$$\mathcal{L} = \frac{1}{2}\left[\dot{\phi}^2 - v_{\|}^2(|\vec{\nabla}_{\|}\phi|^2) - v^2(4qf\vec{\nabla}_{\|}\phi + |\vec{\nabla}\phi|^2)\right]$$

with

$$\vec{\nabla}_{\|}\phi = \vec{n}\cdot\vec{\nabla}\phi$$

The phonon satisfies an anisotropic dispersion relation. The lack of rotational invariance in $\mathcal{L}(\phi)$ follows from the gradient expansion. Similar to chiral case.

Conclusions

- In high density QCD **color superconducting phases** are formed with different features according to m_s

  - $m_s = 0 \Rightarrow$ **CFL**

  - $m_s = \infty \Rightarrow$ **2SC**

- An **effective lagrangian** description for the CFL phase has been proposed (for 2SC see R.C., Z. Duan , F. Sannino, Phys. Rev. **D62** (2000) 094004)

- Asymptotic freedom allows for weak coupling calculations of the parameters of $\mathcal{L}_{\text{eff}}$. Use is made of a formalism describing **excitations close to the Fermi surface** which simplifies calculations a lot. Equivalent to

  ∞ **copies of 2-dim physics**

- The effective description of a **superconductive crystalline state** has been also considered. This state **spontaneously breaks space symmetries**. The low lying excitation is a **phonon** with **anisotropic dispersion relation**.