Color
Superconductivity in High Density QCD

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Motivations for the study of high-density QCD:

- Understanding the interior of CSO’s
- Study of the QCD phase diagram at $T\sim 0$ and high $\mu$

Asymptotic region in $\mu$ fairly well understood: existence of a CS phase. Real question: does this type of phase persists at relevant densities ($\sim 5-6 \rho_0$)?
Summary

- Mini review of CFL and 2SC phases
- Pairing of fermions with different Fermi momenta
- The gapless phases g2SC and gCFL
- The LOFF phase and its phonons
Study of CS back to 1977 (Barrois 1977, Frautschi 1978, Bailin and Love 1984) based on Cooper instability:

At \( T \sim 0 \) a degenerate fermion gas is unstable

*Any weak attractive interaction leads to Cooper pair formation*

- Hard for electrons (Coulomb vs. phonons)
- Easy in QCD for di-quark formation (attractive channel \( 3 \))
  \[ 3 \otimes 3 = 3 \oplus 6 \]
In QCD, CS easy for large $\mu$ due to asymptotic freedom

At high $\mu$, $m_s, m_d, m_u \sim 0$, 3 colors and 3 flavors

Possible pairings: $\langle 0 | \psi_{i\alpha} \psi_{j\beta} | 0 \rangle$

- Antisymmetry in color ($\alpha, \beta$) for attraction
- Antisymmetry in spin (a,b) for better use of the Fermi surface
- Antisymmetry in flavor (i, j) for Pauli principle
Favorite state **CFL** (color-flavor locking)  
(Alford, Rajagopal & Wilczek 1999)

\[
\langle 0 | \psi^\alpha_{aL} \psi^\beta_{bL} | 0 \rangle = -\langle 0 | \psi^\alpha_{aR} \psi^\beta_{bR} | 0 \rangle = \Delta \varepsilon^{\alpha\beta\epsilon} \varepsilon_{ab\epsilon}
\]

**Symmetry breaking pattern**

\[SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \Rightarrow SU(3)_{c+L+R}\]
What happens going down with $\mu$? If $\mu \ll m_s$ we get 3 colors and 2 flavors (2SC)

$$\langle 0 | \psi^\alpha_{aL} \psi^\beta_{bL} | 0 \rangle = \Delta \varepsilon^{\alpha\beta 3} \varepsilon_{ab}$$

$\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \Rightarrow \text{SU}(2)_c \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R$

But what happens in real world?
- $M_s$ not zero
- Neutrality with respect to em and color
- Weak equilibrium

(No free energy cost in neutral $\rightarrow$ singlet, Amore et al. 2003)

All these effects make Fermi momenta of different fermions unequal causing problems to the BCS pairing mechanism
Consider 2 fermions with $m_1 = M$, $m_2 = 0$ at the same chemical potential $\mu$. The Fermi momenta are

$$p_{F1} = \sqrt{\mu^2 - M^2} \quad \quad \quad \quad p_{F2} = \mu$$

Effective chemical potential for the massive quark

$$\mu_{eff} = \sqrt{\mu^2 - M^2} \approx \mu - \frac{M^2}{2\mu}$$

Mismatch:

$$\delta \mu \approx \frac{M^2}{2\mu}$$
If electrons are present, weak equilibrium makes chemical potentials of quarks of different charges unequal:

\[ d \rightarrow u e \bar{\nu} \quad \Rightarrow \quad \mu_d - \mu_u = \mu_e \]

In general we have the relation: \((\mu_i = \mu + Q \mu_Q)\)

\[ \mu_e = -\mu_Q \]

N.B. \( \mu_e \) is not a free parameter
Neutrality requires:
\[
\frac{\partial V}{\partial \mu_e} = -Q = 0
\]

Example 2SC: normal BCS pairing when

\[
\mu_u = \mu_d \Rightarrow n_u = n_d
\]

But neutral matter for

\[
n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u \neq 0
\]

Mismatch:
\[
\delta \mu = \frac{p_F^d - p_F^u}{2} = \frac{\mu_d - \mu_u}{2} = \frac{\mu_e}{2} \approx \frac{\mu_u}{8} \neq 0
\]
Also color neutrality requires

$$\frac{\partial V}{\partial \mu_3} = T_3 = 0, \quad \frac{\partial V}{\partial \mu_8} = T_8 = 0$$

As long as $\delta \mu$ is small no effects on BCS pairing, but when increased the BCS pairing is lost and two possibilities arise:

- The system goes back to the normal phase
- Other phases can be formed
In a simple model with two fermions at chemical potentials $\mu + \delta\mu$, $\mu - \delta\mu$ the system becomes normal at the Chandrasekhar-Clogston point. Another unstable phase exists.

$$\delta\mu = \Delta_{\text{BCS}}$$

$$\delta\mu_1 = \frac{\Delta_{\text{BCS}}}{\sqrt{2}}$$
The point $|\delta\mu| = \Delta$ is special. In the presence of a mismatch new features are present. The spectrum of quasiparticles is

$$E(p) = \left| \delta\mu \pm \sqrt{(p - \mu)^2 + \Delta^2} \right|$$

For $|\delta\mu| < \Delta$, the gaps are $\Delta - \delta\mu$ and $\Delta + \delta\mu$. For $|\delta\mu| = \Delta$, an unpairing (blocking) region opens up and gapless modes are present.

Energy cost for pairing $2\delta\mu$ begins to unpair $2\delta\mu > 2\Delta$

Energy gained in pairing $2\Delta$
Same structure of condensates as in 2SC

(Huang & Shovkovy, 2003)

4x3 fermions:
- 2 quarks **ungapped** $q_{ub}$, $q_{db}$
- 4 quarks **gapped** $q_{ur}$, $q_{ug}$, $q_{dr}$, $q_{dg}$

General strategy (NJL model):

- Write the free energy:

\[
V(\mu, \mu_3, \mu_8, \mu_e, \Delta)
\]

- Solve:

\[
\frac{\partial V}{\partial \mu_e} = \frac{\partial V}{\partial \mu_3} = \frac{\partial V}{\partial \mu_8} = 0
\]

**Neutrality**

\[
\frac{\partial V}{\partial \Delta} = 0
\]

**Gap equation**
• For $|\delta \mu| > \Delta$ ($\delta \mu = \mu_e / 2$) 2 gapped quarks become gapless. The gapless quarks begin to unpair destroying the BCS solution. But a new stable phase exists, the gapless 2SC (g2SC) phase.

• It is the unstable phase which becomes stable in this case (and CFL, see later) when charge neutrality is required.
solutions of the gap equation

\[ \mu_e = \text{const.} \]

Normal state

\[ Q = 0 \]

neutrality line

\[ \frac{\mu_e}{2} = \Delta \]

gap equation

\[ \mu_e = 148 \text{ MeV} \]

\[ V(\text{GeV/fm}^3) \]

\[ \Delta(\text{MeV}) \]

\[ V(\text{GeV/fm}^3) \]

\[ \Delta(\text{MeV}) \]
• But evaluation of the gluon masses (5 out of 8 become massive) shows an instability of the g2SC phase. Some of the gluon masses are imaginary (Huang and Shovkovy 2004).

• Possible solutions are: gluon condensation, or another phase takes place as a crystalline phase (see later), or this phase is unstable against possible mixed phases.

• Potential problem also in gCFL (calculation not yet done).
Generalization to 3 flavors

\[
\langle 0 | \psi_\alpha^a \psi_\beta^b | 0 \rangle = \Delta_1 \varepsilon_{\alpha\beta}^{ab1} \varepsilon_{ab1} + \Delta_2 \varepsilon_{\alpha\beta}^{ab2} \varepsilon_{ab2} + \Delta_3 \varepsilon_{\alpha\beta}^{ab3} \varepsilon_{ab3}
\]

Different phases are characterized by different values for the gaps. For instance (but many other possibilities exist)

**CFL**: \( \Delta_1 = \Delta_2 = \Delta_3 = \Delta \)

**g2SC**: \( \Delta_3 \neq 0, \Delta_1 = \Delta_2 = 0 \)

**gCFL**: \( \Delta_3 > \Delta_2 > \Delta_1 \)
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<th>-1</th>
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**Gaps in gCFL**
Strange quark mass effects:

- **Shift of the chemical potential for the strange quarks:**
  \[ \mu_{\alpha s} \Rightarrow \mu_{\alpha s} - \frac{M_s^2}{2\mu} \]

- **Color and electric neutrality in CFL requires**
  \[ \mu_8 = -\frac{M_s^2}{2\mu}, \quad \mu_3 = \mu_e = 0 \]

- **gs-bd unpairing catalyzes CFL to gCFL**
  \[ \delta \mu_{bd-gs} = \frac{1}{2} (\mu_{bd} - \mu_{gs}) = -\mu_8 = \frac{M_s^2}{2\mu} \]

  \[ \delta \mu_{rd-gu} = \mu_e, \quad \delta \mu_{rs-bu} = \mu_e - \frac{M_s^2}{2\mu} \]
It follows:

\[
\frac{M^2}{\mu} \quad \text{Energy cost for pairing} \quad \text{begins to unpair} \quad \frac{M^2}{\mu} > 2\Delta \\
2\Delta \quad \text{Energy gained in pairing}
\]

Again, by using NJL model (modelled on one-gluon exchange):

- Write the free energy: \( V(\mu, \mu_3, \mu_8, \mu_e, M_s, \Delta_i) \)
- Solve:

Neutrality \( \frac{\partial V}{\partial \mu_e} = \frac{\partial V}{\partial \mu_3} = \frac{\partial V}{\partial \mu_8} = 0 \)

Gap equations \( \frac{\partial V}{\partial \Delta_i} = 0 \)
• CFL ↔ gCFL 2nd order transition at $M_s^2/\mu \sim 2\Delta$, when the pairing gs-bd starts breaking

• gCFL has gapless quasiparticles. Interesting transport properties
gCFL has $\mu_e$ not zero, with charge cancelled by unpaired $u$ quarks
● LOFF (Larkin, Ovchinnikov, Fulde & Ferrel, 1964): ferromagnetic alloy with paramagnetic impurities.

● The impurities produce a constant exchange field acting upon the electron spins giving rise to an effective difference in the chemical potentials of the opposite spins producing a mismatch of the Fermi momenta.
According to LOFF, close to first order point (CC point), possible condensation with \textbf{non zero total momentum}:

\[
\vec{p}_1 = \vec{k} + \vec{q} \quad \vec{p}_2 = -\vec{k} + \vec{q} \quad \rightarrow \quad \langle \psi(x)\psi(x) \rangle = \Delta e^{2i\vec{q} \cdot \vec{x}}
\]

More generally:

\[
\langle \psi(x)\psi(x) \rangle = \sum_m \Delta_m c_m e^{2i\vec{q}_m \cdot \vec{x}}
\]

\[
\vec{p}_1 + \vec{p}_2 = 2\vec{q}
\]

\[
|\vec{q}| \quad \text{fixed variationally}
\]

\[
\frac{\vec{q}}{|\vec{q}|} \quad \text{chosen spontaneously}
\]
Single plane wave:

\[ E(\vec{p}) - \mu \rightarrow E(\pm \vec{p} + \vec{q}) - \mu \mp \delta\mu \approx \sqrt{(p - \mu)^2 + \Delta^2} \mp \bar{\mu} \]

\[ \bar{\mu} = \delta\mu - \vec{v}_F \cdot \vec{q} \]

Also in this case, for \[ |\bar{\mu}| = \delta\mu - \vec{v}_F \cdot \vec{q} < \Delta \]
a unpairing (blocking) region opens up and gapless modes are present.

Possibility of a crystalline structure (Larkin & Ovchinnikov 1964, Bowers & Rajagopal 2002)

\[ \langle \psi(x)\psi(x) \rangle = \Delta \sum_{|\vec{q}_i| = 1.2\delta\mu} e^{2i\vec{q}_i \cdot \vec{x}} \]

The \( q_i \)'s define the crystal pointing at its vertices.
Crystalline structures in LOFF
The LOFF phase is studied via a Ginzburg-Landau expansion of the grand potential

$$\Omega = \alpha \Delta^2 + \frac{\beta}{2} \Delta^4 + \frac{\gamma}{3} \Delta^6 + \ldots$$

(for regular crystalline structures all the $\Delta_q$ are equal)

The coefficients can be determined microscopically for the different structures (Bowers and Rajagopal (2002))
**Gap equation**

**Propagator expansion**

**Insert in the gap equation**
We get the equation

\[ \alpha \Delta + \beta \Delta^3 + \gamma \Delta^5 + \cdots = 0 \]

Which is the same as

\[ \frac{\partial \Omega}{\partial \Delta} = 0 \]

with

\[ \alpha \Delta = \quad \beta \Delta^3 = \quad \gamma \Delta^5 = \]

The first coefficient has universal structure, independent on the crystal. From its analysis one draws the following results.
\[ \Omega_{\text{BCS}} - \Omega_{\text{normal}} = -\frac{\rho}{4}(\Delta_{\text{BCS}}^2 - 2\delta\mu^2) \]

\[ \Omega_{\text{LOFF}} - \Omega_{\text{normal}} = -0.44\rho(\delta\mu - \delta\mu_2)^2 \]

\[ \Delta_{\text{LOFF}} \approx 1.15(\delta\mu_2 - \delta\mu) \]

\[ \delta\mu_1 = \Delta_{\text{BCS}} / \sqrt{2} \]

\[ \delta\mu_2 \approx 0.754\Delta_{\text{BCS}} \]

Small window. Opens up in QCD?

(Leibovich, Rajagopal & Shuster 2001; Giannakis, Liu & Ren 2002)
Single plane wave

Critical line from

\[ \frac{\partial \Omega}{\partial \Delta} = 0, \quad \frac{\partial \Omega}{\partial q} = 0 \]

Along the critical line

(at T = 0, q = 1.2\delta \mu_2)
<table>
<thead>
<tr>
<th>Structure</th>
<th>P</th>
<th>$\mathcal{G}$(Föppl)</th>
<th>$\bar{\beta}$</th>
<th>$\bar{\eta}$</th>
<th>$\bar{\Omega}_{\text{min}}$</th>
<th>$\delta \mu_\kappa / \Delta_\theta$</th>
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<td>0.569</td>
<td>1.637</td>
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Preferred structure: face-centered cube

General analysis (Bowers and Rajagopal (2002))
Effective gap equation for the LOFF phase

For the single plane wave ($P = 1$) the pairing region is defined by

$$\Delta_{\text{eff}} = \Delta \theta(E_u) \theta(E_d) = \begin{cases} \Delta & \text{for} \ ((p, \vec{v}_F)) \in \text{PR} \\ 0 & \text{elsewhere} \end{cases}$$

$$E_{\text{u,d}} = \pm (\delta \mu - \vec{v}_F \cdot \vec{q}) + \sqrt{\xi^2 + \Delta^2}, \quad \xi = p - \mu$$

$$\Delta = \frac{g \rho}{2} \int \frac{d \vec{v}}{4\pi} \int_{\delta} d\xi \frac{\Delta_{\text{eff}}}{\sqrt{\xi^2 + \Delta_{\text{eff}}^2}}$$

$$\rho = 4 \frac{\mu^2}{\pi^2}$$
How to obtain this result starting from an effective theory for fermions close to the Fermi surface? Problem:

\[ \mathcal{L} \sim \Delta e^{2i\vec{q} \cdot \vec{r}} \psi^T \mathcal{C} \psi \]

where in the Fermi fields the large part in the momentum has been extracted

\[ \mathbf{p} = \mu \mathbf{v}_F + \ell \]

Solution: appropriate average procedure over the cell size

\[ \mathcal{L} \rightarrow \Delta_{\text{eff}} \psi^T \mathcal{C} \psi \]
Average by

$$g_R(\vec{r}) = \prod_{k=1}^{3} \frac{\sin(\pi qr_k / R)}{\pi r_k}$$

When $R/\pi \sim 1$ different from zero in a region of the order of the cell size. Condition satisfied if the gap is not too small.
For P plane waves

\[ \langle \psi(x)\psi(x) \rangle = \Delta \sum_{k=1}^{P} e^{2i\vec{q}_k \cdot \vec{x}} \]

an analogous average procedure gives pairing regions and effective gap given by

\[ P_k = \{(p, \vec{v}_F) \mid \Delta_E(p, \vec{v}_F) = k\Delta \} \]

\[ \Delta_E(p, \vec{v}_F) = \sum_{m=1}^{P} \Delta_{\text{eff}}(p, \vec{v}_F \cdot \vec{q}_m) \]
We obtain the following gap equation

\[ P \Delta = \frac{g \rho}{2} \sum_{k=1}^{P} \iint_{p_k} d\vec{v} \ d\xi \frac{\Delta_E}{\sqrt{\xi^2 + \Delta_E^2}} = \]

\[ = \frac{g \rho}{2} \sum_{k=1}^{P} \iint_{p_k} d\vec{v} \ d\xi \frac{k \Delta}{\sqrt{\xi^2 + k^2 \Delta^2}} \]

The result can be interpreted as having \( P \) quasi-particles each of one having a gap \( k \Delta \), \( k = 1, \ldots, P \).
The approximation is better far from a second order transition and it is exact for $P = 1$ (original FF case).
Evaluating the free energy at the CC point we see that the P=6 case (octahedron) is favored. Then the cube takes over at $\delta\mu_2 \sim 0.95 \Delta$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$z_q$</th>
<th>$\Delta/\Delta_0$</th>
<th>$2\Omega \rho\Delta_0^2$</th>
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<td>0.24</td>
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<td>0.9</td>
<td>0.21</td>
<td>-0.09</td>
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<table>
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<tr>
<th>$P$</th>
<th>$\delta\mu_2/\Delta_0$</th>
<th>Order</th>
<th>$z_q$</th>
<th>$\Delta/\Delta_0$</th>
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</thead>
<tbody>
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<td>1</td>
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<td>II</td>
<td>0.83</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.83</td>
<td>I</td>
<td>1.0</td>
<td>0.81</td>
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<tr>
<td>6</td>
<td>1.22</td>
<td>I</td>
<td>0.95</td>
<td>0.43</td>
</tr>
<tr>
<td>8</td>
<td>1.32</td>
<td>I</td>
<td>0.9</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Two phase transitions from the CC point 
($M_s^2/\mu = 4 \Delta_{2SC}$) up to the cube case 
($M_s^2/\mu \sim 7.5 \Delta_{2SC}$). Extrapolating to CFL 
($\Delta_{2SC} \sim 30$ MeV) one gets that LOFF should be favored from about 
$M_s^2/\mu \sim 120$ MeV up $M_s^2/\mu \sim 225$ MeV.

\[
\begin{array}{|c|c|c|c|c|}
\hline
P & \delta \mu_2/\Delta_0 & \text{Order} & z_q & \Delta/\Delta_0 \\
\hline
1 & 0.754 & II & 0.83 & 0 \\
2 & 0.83 & I & 1.0 & 0.81 \\
6 & 1.22 & I & 0.95 & 0.43 \\
8 & 1.32 & I & 0.9 & 0.35 \\
\hline
\end{array}
\]
Conclusions

- Under realistic conditions ($M_s$ not zero, color and electric neutrality) new CS phases might exist.
- In these phases gapless modes are present. This result might be important in relation to the transport properties inside a CSO.