The Super-little Higgs

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with

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Outline

• The fine tuning of SUSY and little Higgs theories – why and how to combine them?
• The simplest little Higgs and SUSY
• A beautiful (old) SU(6) GUT model and a super-little Higgs
• Higgs potential and quartic coupling
• Phenomenology
  • Gauge coupling unification
  • Extra fermions
  • LHC

• Conclusions
The Fine Tuning of SUSY

• SUSY solves hierarchy – no quadratic divergences
• But log divergences are present
• General Higgs potential of MSSM:

\[ V(H_1, H_2) = (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 \]
\[ -B\mu(H_1 H_2 + \text{h.c.}) + \frac{g^2}{2}(H_1^\dagger \tau H_1 + H_2^\dagger \tau H_2)^2 + \]
\[ \frac{g'2}{2}(H_1^\dagger H_1 - H_2^\dagger H_2)^2 \]

• EWSB can happen only due to soft SUSY breaking terms

\[ M_Z^2 = 2 \left( \frac{m_{H_1}^2 - m_{H_2}^2}{\tan^2 \beta} \tan^2 \beta - 1 - \mu^2 \right) \]

• For large tan \( \beta \) (needed for Higgs mass) and neglecting \( \mu \)

\[ M_Z^2 \sim -2m_{H_2}^2 \]
• Leading expression

\[ M_Z^2 = \frac{3y_t^2}{2\pi^2} m_{\tilde{t}}^2 \ln \frac{f}{m_{\tilde{t}}} \]

• One loop suppressed vs. stop mass, but usually \( f \gg m_{\tilde{t}} \)

• Log compensates loop suppression, need fine tuning to ensure \( f \sim \text{TeV} \)

Goal here: find a rationale why \( f \sim \text{TeV} \), and thus avoid fine tuning of SUSY

• Simplest possibility: Higgs a pseudo-Goldstone boson of symmetry broken at scale \( f \)
• This idea already used in SUSY GUTs to solve D-T splitting
Fine-tuning of Little Higgs

• Little Higgs: realistic model for Higgs as PGB
• Aim: to raise cutoff of SM to \( \sim 10 \) TeV to solve little hierarchy
• But: Higgs does **NOT** look like generic PGB!

\[
V(h) = 0 \cdot |h|^2 + 0 \cdot |h|^4 + f^4 \cos^n\left(\frac{|h|}{f}\right)
\]

- Tree-level vanishes
- Due to PGB nature
- Generic PGB pot.

• Both mass and quartic generated at one loop: \( \langle h \rangle \sim f \)
• Does not raise cutoff \( \Lambda = 4\pi f \)
• Little higgs introduces **Collective symmetry breaking**

\[
m^2 = 0 \cdot \Lambda^2 + \frac{\mathcal{O}(g^2, \lambda_t^2)}{(4\pi)^2} f^2 \quad \lambda_h = \mathcal{O}(g^2, \lambda_t^2)
\]

• Higgs VEV now \( \langle h \rangle \sim f/4\pi \)
Fine-tuning of Little Higgs

• But: many new states at the f~TeV scale
• Generically large corrections to EWPO’s

• In the end usually need f~4-5 TeV to avoid conflict

• Possible way out: T-parity (Cheng & Low) – will not use here
MSSM: \[ m^2 \sim \frac{\mathcal{O}(g^2, \lambda_t^2)}{(4\pi)^2} m_{\text{soft}}^2 \ln \frac{\Lambda}{m_{\text{soft}}} \]

Problem: large log, EWPT not a problem due to R-parity

LH: \[ m^2 \sim \frac{\mathcal{O}(g^2, \lambda_t^2)}{(4\pi)^2} f^2 \ln \frac{f}{m_{\text{soft}}} \]

Problem: EWPT

Problems complementary

Super-little Higgs:

\[ m^2 \sim \frac{\mathcal{O}(g^2, \lambda_t^2)}{(4\pi)^2} m_{\text{soft}}^2 \ln \frac{f}{m_{\text{soft}}} \]

If we take \( m_{\text{soft}} \sim \) few 100 GeV (usual SUSY bound)
\[ f \sim 4-5 \text{ TeV} \] (EWP bound on LH, cuts off log)

\[ \langle h \rangle \sim \frac{m_{\text{soft}}}{4\pi} \left[ \log(\frac{f}{m_{\text{soft}}}) \right]^{\frac{1}{2}} \sim \mathcal{O}(100 \text{GeV}) \]

Higgs VEV super-little!
The simplest little Higgs

- Extend $SU(2) \times U(1)$ to $SU(3) \times U(1)$
- Use two sets of triplets $H_1, H_2$ to break $SU(3) \times U(1) \rightarrow SU(2) \times U(1)$
- If no $H_1 \dot{H}_2$ -type terms, global symmetry breaking pattern
  \[
  SU(3)_{H_1} \times SU(3)_{H_2} \\
  \langle H_1 \rangle \quad \langle H_2 \rangle \\
  SU(2) \quad SU(2)
  \]

- Two sets of Goldstones, one set eaten, one set remains as physical pseudo-Goldstone boson (PGB)

  \[
  \Pi = \begin{pmatrix}
  h_1 & h_1^* \\
  h_2 & h_2^*
  \end{pmatrix} \\
  H_1 = e^{i\Pi/f} \begin{pmatrix} f \end{pmatrix} \\
  H_2 = e^{-i\Pi/f} \begin{pmatrix} f \end{pmatrix}
  \]
• Gauging of diagonal SU(3) explicitly breaks global sym.
• Symmetry breaking terms:

\[ |gA_\mu H_1|^2 + |gA_\mu H_2|^2 \]

• If either coupling turned off: larger global symmetry intact
• Any diagram contributing to Higgs mass has to involve both
• Lowest vertex:
SUSY and little Higgs: a difficult marriage

• Make it supersymmetric: \( H_1 \rightarrow H_1, \bar{H}_1 \)
• Two sets of chiral SF’s \( H_2 \rightarrow H_2, \bar{H}_2 \)
• Generic VEVs and parameterization:
  \[
  H_1 = e^{i\frac{F_2}{F_1}F} (0, 0, f_1/\sqrt{2}), \quad \bar{H}_1 = (0, 0, \bar{f}_1/\sqrt{2}) e^{-i\frac{F_2}{F_1}F}
  \]
  \[
  H_2 = e^{i\frac{F_1}{F_2}F} (0, 0, f_2/\sqrt{2}), \quad \bar{H}_2 = (0, 0, \bar{f}_2/\sqrt{2}) e^{-i\frac{F_1}{F_2}F}
  \]

• But D-terms necessarily break global symmetry at tree-level:
  \[
  V_D \in \frac{g^2}{8} \left( |H_1^\dagger \cdot H_2|^2 - |\bar{H}_1 \cdot H_2|^2 - |\bar{H}_2 \cdot H_1|^2 + |\bar{H}_2^\dagger \cdot \bar{H}_1|^2 \right) =
  \]
  \[
  \frac{g^2}{8} (f_1^2 - \bar{f}_1^2)(f_2^2 - \bar{f}_2^2) \cos^2 \left[ \frac{\sqrt{G^\dagger G}}{F} \left( \frac{F_1}{F_2} - \frac{F_2}{F_1} \right) \right]
  \]
  • Tree-level Goldstone mass if \( f_1 \neq \bar{f}_1 \) or \( f_2 \neq \bar{f}_2 \)
  • VEVs need to be supersymmetric, how to ensure?

Early attempt global symmetry only: Birkedal, Chacko, Gaillard; Pokorski et al.
• Possibility #1:

Add a $Z_2$ symmetry in one of the H sectors

Berezhiani, Chankowski, Falkowski, Pokorski; Roy, Schmaltz

• Possibility #2:

Choose a gauge representation that ensures SUSY VEV

$$H_2, \bar{H}_2 \rightarrow \Sigma = \begin{pmatrix}
w & h_1 \\
w & h_2 \\
h_1^* & h_2^* & -2w
\end{pmatrix}$$

• D-term issue automatically resolved

• Global sym. breaking pattern:

$$SU(3)_\Sigma \times SU(3)_H \times U(1)_X$$

$$SU(2) \times U(1) \quad SU(2) \times U(1)$$
A beautiful old model

- SU(6) GUT theory, with Higgs sector $\Sigma$, $H$
- SU(6)$\times$SU(6) global symmetry

\[
\langle \Sigma \rangle = w \begin{pmatrix}
4 & 4 \\
-2 & -2 \\
-2 & -2
\end{pmatrix}
\]

\[
\langle H \rangle = \langle \bar{H} \rangle = (0, 0, 0, 0, 0, f)
\]

SU(6) $\rightarrow$ SU(4)$\times$SU(2)$\times$U(1)
A beautiful old model

- SU(6) GUT theory, with Higgs sector $\Sigma$, $H$
- SU(6)$\times$SU(6) global symmetry

$$\langle \Sigma \rangle = w \begin{pmatrix} 4 & 4 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 \end{pmatrix}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (0, 0, 0, 0, 0, f)$$

SU(6) $\rightarrow$ SU(5)
A beautiful old model

- SU(6) GUT theory, with Higgs sector $\Sigma$, $H$
- SU(6)$\times$SU(6) global symmetry

$$\langle \Sigma \rangle = w \begin{bmatrix} 4 & 4 \\ -2 & -2 \\ -2 & -2 \end{bmatrix}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (0, 0, 0, 0, 0, f)$$

Uneaten Goldstone boson: one complex doublet
Fermion sector of the SU(6) model

Barbieri, Dvali, Strumia, Berezhiani, Hall

•SU(5): $3 \times (10 + \bar{5})$
•SU(6): $3 \times (15 + \bar{6} + \bar{6}')$ need to extend, more chiral fields?

•But Yukawa coupling:
  \[ 15^{ab}_{i} \bar{H}_{a} \bar{6}_{b}^{j} \]

•After VEV gives mass to 3x(5+5)
•Chiral matter content that of SU(5) MSSM

To get natural top Yukawa coupling

•Unusual representation in SU(6): 20, three-index antisym.
•Self-adjoint (anomaly free), but no mass term: $20^{abc}20^{def}\epsilon_{abcdef} = 0$
•Under SU(5): 20 → 10 + 1\bar{0}$
• Renormalizable Yukawa couplings involving 20:

\[
\lambda_1 20^{abc} H^d 15^{ef} + \lambda_2 20^{abc} \sum_d 20^{efg} \epsilon_{abcf}g
\]

Exchanges a 10 from 15 with a 10 from 20

Produces an order one top Yukawa coupling 10 10 H

• Automatically has the collective breaking pattern:
need both couplings to generate top mass
The **matter content of the super-little model**

- Decompose SU(6) to SU(3)×SU(3)×U(1)
- Automatically anomaly free, flavors universal
- Top Yukawa via collective breaking
- One set of PGB doublet

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The Higgs sector

- The superpotential (with $\lambda'< 0.01$ to ensure global sym.)

$$W_{\text{Higgs}} = \frac{M}{2} \text{Tr} \Sigma^2 + \frac{\lambda}{3} \text{Tr} \Sigma^3 + S(\lambda'' H \bar{H} - M'^2) + \lambda' \bar{H} \Sigma H$$

- VEVs

$$\langle \Sigma \rangle = \text{diag} (w/2, w/2, -w), \quad \langle H \rangle = (0, 0, f/\sqrt{2}), \quad \langle \bar{H} \rangle = (0, 0, \bar{f}/\sqrt{2})$$

- Goldstones: \( F^2 = (f^2 + \bar{f}^2)/2, \quad V^2 = F^2 + 9w^2 \)

$$\begin{align*}
H & = \exp \left( i \Pi \frac{3w}{FV} \right) \langle H \rangle, \\
\bar{H} & = \langle \bar{H} \rangle \exp \left( -i \Pi \frac{3w}{FV} \right) \\
\Sigma & = \exp \left( -i \Pi \frac{F}{3wV} \right) \langle \Sigma \rangle \exp \left( i \Pi \frac{F}{3wV} \right)
\end{align*}$$
• Pion matrix:

\[ \Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 2 \\ \frac{H^t_d}{H_u} & 0 \end{pmatrix} \]

• Goldstone vs. sGoldstone

\[ G \equiv (H_u + H_d^\dagger) / \sqrt{2} \quad \tilde{G} \equiv (H_u - H_d^\dagger) / \sqrt{2} \]

• Need to make sure VEV is along Goldstone direction
  (sGoldstone NOT protected by global symmetry)
The top sector

• To ensure that matter content is that of MSSM

\[ W_{\text{matter}} = \alpha_{ij} Q_i \bar{H} D_j + \beta_{ij} E_i \bar{H} L_j \]

• Will use trick from SU(6) model to get O(1) top Yukawa:

\[ 20 \rightarrow (\bar{3}, 3)_{-\frac{1}{3}} + (3, \bar{3})_{\frac{1}{3}} + \text{singlets} \]

\[ \uparrow \quad Q' \quad \bar{Q}' \]

• Superpotential for top Yukawa:

\[ W_{\text{top}} = \lambda_1 \bar{Q}' \Sigma Q' + \lambda_2 \bar{Q}' HU + \lambda_3 Q H Q' \]

• Has collective form: need all three couplings to generate top Yukawa
Heavy top partners and top Yukawa:

\[
M_{T_1}^2 = \lambda_1^2 w^2 + \frac{1}{2} \lambda_2^2 f^2, \quad M_{T_{2,3}}^2 = \frac{1}{4} \left( \lambda_1^2 w^2 + 2 \lambda_3^2 f^2 \right)
\]

\[
y_t = \frac{f^2 \sqrt{F^2 + 9w^2} \lambda_1 \lambda_2 \lambda_3}{F \sqrt{2(w^2 \lambda_1^2 + f^2 \lambda_2^2)(w^2 \lambda_1^2 + 2f^2 \lambda_3^2)}}
\]
Electroweak precision constraints

• Little Higgs models usually tightly constrained, need T-parity
• SUSY models usually have R-parity (or matter parity)

Which one?

• T-parity does not commute with SU(3)\times U(1): Z’ T-even
• Constraint from Z’ exchange: F>3 TeV
• If w too small: SU(2) breaking VEV partly in triplet: w>0.5 TeV

Assume F>3 TeV, w>0.5 TeV, and impose usual R-parity
Higgs potential

• D-terms do not give significant contribution to mass or quartic

\[ \frac{(m_H^2 - m_{\bar{H}}^2)(f^2 - \bar{f}^2)}{(f^2 + \bar{f}^2)^2} w^2 G^2 \quad \text{for} \quad f \gg w, m_H, m_{\bar{H}} \]

• But this is NOT enough: need to make sure VEV is actually along the Goldstone direction (NOT sGoldstone)

• Soft breaking terms + D-terms introduce mass and mixing

\[ V_{soft} = m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + m_\Sigma^2 \text{Tr} \Sigma \Sigma^+ \]

• Mixing matrix:

\[
\begin{pmatrix}
0 & m_{\bar{G}\bar{G}}^2 \\
 m_{\bar{G}\bar{G}}^2 & m_{\bar{G}G}^2
\end{pmatrix}
\]

• The mixing \( m_{\bar{G}\bar{G}}^2 \sim m_H^2 - m_{\bar{H}}^2 \) can be suppressed by \( m_\Sigma \sim \text{TeV} \gg m_H, m_{\bar{H}} \)
The Higgs quartic

- Now we achieved the Higgs VEV along Goldstone $\tan \beta \sim 1$
- No tree-level quartic from D-terms along this direction
- Top induced potential:

$$m^2 \sim -\frac{3}{8\pi^2}y_t^2(m_t^2 - m_{\tilde{t}}^2)(2 \ln \frac{m_{T2}}{m_{\tilde{t}}} + 1 + 2c \ln \frac{m_{T1}}{m_{T2}})$$

Mass term as expected super-little

Quartic too small to exceed 115 GeV Higgs mass

$$\lambda \sim \frac{3y_t^4}{8\pi^2}(\ln \frac{m_{\tilde{t}}}{m_t} + \frac{A_t^2}{2m_{\tilde{t}}^2}(1 - \frac{A_t^2}{12m_{\tilde{t}}^2}))$$

Need a tree-level quartic!
A tree-level quartic

• Need operator that gives quartic but no mass to Goldstone
• Notoriously difficult in simplest little Higgs already
• Could try to get an NMSSM-type superpot. term $SG^2$

First try: 

$$S \bar{H} \Sigma H$$

• No good because $\bar{H} \Sigma H = -\frac{f^2 w}{2} + \frac{V^2}{12w}|G|^2$
  contains both a mass and a quartic
• Need to absorb VEV of operator: “sliding singlet”

Second try: 

$$S (S' + \bar{H} \Sigma H)$$

• VEV absorbed since $\langle S' \rangle = -\langle \bar{H} \Sigma H \rangle$ but also FULL Goldstone dependence absorbed
Third try:

\[ \frac{1}{\lambda} (SH + \frac{1}{\lambda} S'\Sigma H)(\bar{S}\bar{H} + \Sigma\bar{H}) \]

Similar to Roy and Schmaltz

- "collective sliding singlet"
- VEV’s will cancel: no mass
- Goldstone dependence remains: O(1) tree-level quartic
- Operator could be generated via exchange of heavy triplets

EOM:

\[ \bar{S}\bar{H}H + \bar{H}\Sigma H = 0, \]
\[ \bar{S}\bar{H}\Sigma H + \bar{H}\Sigma^2 H = 0 \]

Goldstone expansion:

\[ \bar{S}f^2 + (-wf^2 + V^2|G|^2/6w) = 0 \]
\[ \bar{S}(-wf^2 + V^2|G|^2/6w) + (f^2w^2 - V^2|G|^2/6) = 0 \]
Other possibilities for generating the quartic

• Use MSSM quartic, but may still be too small (as MSSM has generically hard time getting a heavy higgs)
  (Berezhiani, Chankowski, Falkowski, Pokorski)

• Can be implemented here as well, need extra triplets in $\Sigma$ sector, and $w\Box f, \bar{f}$

• Most recently: supersymmetric version of twin Higgs idea
  (Chacko, Goh, Harnik)

• Falkowski, Pokorski, Schmaltz: VEV still along $\tan \beta = 1$
  • Still need 4 singlets, though superpotential less complicated

• Chang, Hall, Weiner: use supersoft D-term breaking
  • In the end Higgs mass suppression comes down to controlling soft masses to a scalar and a triplet
The good news:

The beta functions are such that unification would happen at a high scale (~Planck) with the minimal fermion matter content.
The bad news:

• Adding the matter needed for generating the top Yukawa will introduce Landau pole before unification
• Seiberg duality, duality cascade a la Klebanov, Strassler?
• Unifies into string theory on warped throat? Meaning of betas?
New particles at LHC

• At low energies model = MSSM with \( m \) ~ few 100 GeV
• R-parity conservation: traditional SUSY searches apply
• Around \( f \sim \text{TeV} \): lot of new states: little partners + their superpartners

Gauge bosons

• \( Z' \): from EWP \( m > 1.7 \) TeV. Should be cleanly visible to multi-TeV range (can be singly produced)
• \( W' \): \( m > 1.5 \) TeV, but couplings for single production v/f suppressed

Heavy top partners

• Expected in 2-3 TeV range (if \( f \) close to lower bound)
• LHC reach \( \sim 2-2.5 \) TeV
Other additional fermions

• Main distinction from usual 3-3-1 charge assignment:

  Anomaly free
  Generation independent

• Singlet leptons extended into SU(3) triplets (rather than singlets)

• Vectorlike SU(2) singlet quarks from Q,D’: O(f) mass
• Vectorlike SU(2) doublet leptons from E,L’: O(f) mass
• Two light SU(2) singlet “sterile neutrino” from L,L’

  • Light (no renormalizable mass)
  • Not completely sterile (SU(3) interactions)
  • Can add full SU(6) states to give them O(f) mass
Conclusions

• SUSY models could still be natural if Higgs super-little:

\[ m_h^2 \sim \frac{g^2}{16\pi^2} m_{\text{soft}}^2 \ln \frac{f}{m_{\text{soft}}} \]

• Generic SUSY little Higgs models will **NOT** have this property:
  • D-terms induce tree-level mass
  • Non-Goldstones can be dominant for EWSB

• Simple model based on SU(6) SUSY GUT → SU(3) × SU(3) × U(1)
• Higgs sector: \( \Sigma + H, \bar{H} \)
  • Anomaly free, generation independent charges
  • No Goldstone mass from D-terms
  • Collective top Yukawa
  • Need acrobatics for quartic
  • Unification?

• New particles at LHC:
  • MSSM few 100 GeV
  • \( W', Z', T, \) extra fermions: few TeV
  • Sterile neutrinos