Diquarks and Exotic Spectroscopy

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More recently the SU(3)-flavor $\bar{T}_{10}f$ representation has emerged as an interesting feature of chiral soliton (Skyrme) models of baryons. Soon after the study of SU(2) chiral solitons by Adkins, Nappi, and Witten [5], three flavor generalizations led to predictions of a low lying $J = \frac{1}{2}$ $\bar{T}_{10}f$. As early as 1987 Praszalowicz predicted that the $J = 2$ isosinglet member of the $J = \frac{1}{2} \bar{T}_{10}f$ would lie near 1540 MeV [6]. In 1997 Diakonov, Petrov, and Polyakov (DPP) not only predicted a $\Theta$ at about 1540 MeV, but also estimated its width at less than 15 MeV [7].

The discovery of a manifestly exotic baryon provides an opportunity to refine our understanding of quark dynamics at low energy, where it is not perturbative. In this Letter we propose that the $\Theta^+$ can be usefully considered as a bound state of an antiquark with two highly correlated spin-zero ud-diquarks. This interpretation has some desirable features and makes predictions that distinguish it clearly both from the traditional uncorrelated quark model and from the chiral soliton model.

Let us first discuss some general features of the dynamics of a $K^+n$ resonance. The $\Theta^+$ appears to be remarkably narrow. It lies about 100 MeV above $K^+n$ threshold at a center of mass momentum of $k = 270$ MeV. Taking a typical range of hadronic interactions $R$ of 1 fermi, the characteristic parameter $kR$ is about 1.4. Assuming isospin zero, the possible quantum numbers of the $\Theta^+$ (in $KN$ scattering notation, $L_{1(2J)}$) are $S_0$, $P_0$, $P_03$, $D_03$, $D_05$, ... . With $kR \approx 1.4$ only the $S$ or $P$ wave is likely, and there is some indication that the spin of the $\Theta^+$ is $\frac{1}{2} \bar{T}_{10}$. At center of mass momentum 270 MeV the kaon and neutron are not very relativistic. There are no other two-body hadronic channels coupling to $K^+$ below $K\Delta$ threshold at 1725 MeV. Since there are no quark-antiquark annihilation graphs in $K^+n$ scattering (that is what it means to be exotic) one expects, and observes, that low energy $K^+n$ scattering is dominated by ordinary exchange forces. This contrasts with non-exotic channels, for example $\pi N$, which are dominated by confined states that couple by $q\bar{q}$ annihilation, for example the $\Delta$. QCD allows $K^+n$ to couple to $q^4\bar{q}$ configurations with wavefunctions that differ markedly in space, color, and spin from the $K^+n$ scattering state. Without explicit introduction of such effects, one has no choice but to describe $K^+n$ scattering in the region of the $\Theta^+$ in terms of (non-relativistic) potential scattering. Let us see where that leads. Attractive forces in the $s$-wave do not generate resonances. If they are attractive enough they produce bound states like the deuteron (or virtual states like the dineutron), if not, they modulate the phase of the scattering wavefunction in a smooth fashion. In higher partial waves attractive forces typically generate resonances through interplay between the attraction and the repulsive angular momentum barrier. The mass and width of the resonance are related through the range and...
depth of the potential. For a simple attractive potential (we used a square well) of range 1 fermi, the width of a $P$-wave resonance 100 MeV above threshold is above 175 MeV. To produce a width of order 10 MeV requires a range of about 0.05 fermi, which brings in a high energy scale. Thus the observed width of the $\Theta^+$ suggests that its coupling to the $K^+n$ channel is suppressed by some additional dynamics beyond non-relativistic potential scattering.

Now let us discuss the predictions and reliability of the chiral soliton and the traditional (uncorrelated) quark models. In the chiral soliton model the ($\chi$-soliton and the traditional (uncorrelated) quark model are especially pertinent – in the three-flavor case, heavy resonant mesons and baryons by and large fall neatly into the $SU(3)$-flavor multiplets predicted by the quark model. In particular, the ground state multiplets of the $q\bar{q}$ and $q^2$ have the quantum numbers, including parity, expected of a collection of quarks (and antiquarks) in the same eigenstate of some mean field. Thus the lightest mesons – the pseudoscalar and vector nonets – have negative parity; and the lightest baryons (the $(8,\frac{1}{2})$ and the $(10,\frac{3}{2})$) have positive parity.

The nonet of light scalar $(J^{PC}=0^++)$ mesons with masses below 1000 MeV ($f_0(600)$, $f_0(980)$, $a_0(980)$, $\kappa (800)?)$), however, does not seem to fit a $qq$ description. Instead, a heavier scalar nonet above 1000 MeV seems to be largely $\bar{q}q$. The light scalars seem at least to have a significant component of $\bar{q}qqq$ dominated by a configuration where the pairs of quarks (and antiquarks) are separately correlated in a flavor, color, and spin antisymmetric state. This diquark is a boson with color $3_f$, flavor $3_f$, and spin zero. A diquark and antidiquark neatly couple together to give a nonet of color singlet scalar mesons. The apparent fact that the $\bar{q}qqq$ scalars are lighter than the $\bar{q}q$ scalars testifies to the strength of the diquark correlation. There is also evidence for strong diquark correlations in the baryon spectrum.

Attraction between quarks in the color $3_c$ channel has profound roots in microscopic QCD. Indeed, by bringing quarks together in this channel one halves the magnitude of their effective charge, and thus largely cancels the associated field energy. This idea is the basis color superconductivity in dense quark matter, a context in which it can be made quite rigorous.

With these considerations in mind let us now consider the $q^2\bar{q}$ system. In an uncorrelated quark model, in which all the quarks are in the ground state of a mean field, the ground state of $q^2\bar{q}$ has negative parity. This is a striking difference from the chiral soliton model, and from the correlated quark model we are advocating. The full $SU(3)_f \times SU(2)_f$ spin content of this multiplet and bag model estimates of masses and decay couplings can be found in Ref. 3. There are very many states in flavor $1_f$, $8_f$, $10_f$, $SU(2)_f$, $27_f$, etc.. The lightest have the quark content $uudd$ and $uudd$ and would be expected to lie below the $\Theta^+$. There is no evidence for a $\frac{1}{2}^−$ nucleon in this well explored region of the non-strange baryon spectrum.

Furthermore, all the light negative parity baryons are well described as orbital excitations of $q^2$. We propose instead the following picture of the $\Theta^+$ and other $q^2\bar{q}$ baryons. The four quarks are bound into two spin zero, color and flavor $3_f$ diquarks. For our purposes we can regard the diquarks as composite bosons, $[ud]$, $[ds]$, and $[su]$. The diquarks obey Bose statistics, but may be expected to experience a repulsive “Pauli blocking” interaction at short distances. In order to combine with the antiquark into a singlet, the two diquarks must combine into a color $3_f$. Since the $3_f$ is the antisymmetric part of $3_f \times 3_f$, the diquark-diquark wavefunction must be antisymmetric with respect to the rest of its labels. For identical diquarks, like $[ud]^2$ only space labels remain, and we conclude that the lightest state must have a wavefunction antisymmetric under space exchange, i.e. with negative space-parity. When combined with the antiquark, the resulting $q^2\bar{q}$ state therefore has positive parity. The three unlike diquark pairs, $[ud][ds]$, $[ds][su]$, and $[su][ud]$, can be antisymmetrized in flavor, and therefore can have symmetric, i.e. positive parity, spatial wavefunctions. However it is not unreasonable to expect that the blocking repulsion raises the energy of the spatially symmetric states. Evidence, albeit indirect, for Pauli blocking in the $s$-wave may also come from the apparent absence of a tightly bound $H$-dibaryon ($[ud][ds][su]$). We hypothesize that this effect is strong enough to elevate the resulting negative parity nonet of $q^2\bar{q}$ states to masses high enough that their effects on the meson-baryon $S$-wave, where they lie, are weak. In any case all these negative parity states have conventional quantum numbers $(8_f$ and $1_f$) and contain at least one strange quark. Thus the $Y = 1, I = \frac{1}{2}$ “nucleon” has the quark content $uuds\bar{s}$ and would be heavy and weakly coupled to $\pi N$. We put these states aside.

The flavor symmetric, and therefore spatially antisym-
metric, two diquark states form an $SU(3)_f \otimes SU(3)_f$: $[ud]^2$, $[ud][us]+[us]^2$, $[us][ds]+[ds]^2$, and $[ds][ud]$. When combined with the antiquark the result is a degenerate $SU(3)_f 8_f \oplus \overline{10}_f$. The strange quark mass breaks $S(3)_f$ symmetry. In addition to the mass itself, the $[ud]$ diquark is more tightly bound than the $[us]$ or $[ds]$ diquarks, an effect well-known from baryon spectroscopy [13]. This effect can be related to the $\Sigma$–$\Lambda$ mass difference. It leads to a contribution $\alpha = \frac{1}{2}(M_\Lambda - M_\Sigma) \approx 60$ MeV for every strange quark. A schematic Hamiltonian including $SU(3)$ violation is $H_s = M_0 + (n_s + n_\bar{s})m_s + n_s$. It will ideally mix the $\overline{10}_f$ and $8_f$ according to their strange quark content. Of course a more sophisticated treatment would allow for non-ideal mixing and perturb our mass estimates. The quark content of representative states in the $\overline{10}_f \oplus 8_f$ is given in Fig. (1). The mass hierarchy of the states is summarized in Fig. (2) and compared with the chiral soliton model. The spatially antisymmetric wavefunction of the pairs of diquarks would likely correspond to angular momentum one. Thus the total angular momentum of the correlated $(q^4 \bar{q})$ system would be either $\frac{1}{2}$ or $\frac{3}{2}$. We assume that the $\frac{3}{2}^+ 8_f + \overline{10}_f$ is elevated to somewhat higher energies, where the states fall apart quickly into mesons and baryons and do not produce prominent resonances. Of course, if our picture proves correct, a dynamical explanation for this $LS$ splitting must be found.

The $8_f + \overline{10}_f$ has many interesting features:

**$\Theta^+([ud]^2\bar{s})$:** We identify this state with the $\Theta^+$. This fixes $M_0 + m_s \approx 1540$ MeV. It may be possible to explain the narrowness of the $\Theta^+$ by the relatively weak coupling of the $K^+\eta$ continuum to the $[ud]^2\bar{s}$ state from which it differs in color, spin and spatial wavefunctions.

**$N([ud]^2\bar{d})$:** This is the lightest particle in the $8_f + \overline{10}_f$, with mass $M_0$. It has the quantum number of the nucleon. It is tempting to identify this state with the otherwise perplexing Roper resonance, the $N(1440)\, P_{11}$. The assignment would explain the existence of the Roper and fixes $M_0 \approx 1440$ MeV. The $N(1440)$ is much broader than the $\Theta^+$. The greater phase space for $N(1440) \rightarrow N\pi$ compared to $\Theta^+ \rightarrow K^+\eta$, $[p(\Theta^+)/p(N(1440))]^3 \approx 3$ helps, but does not nearly account for the width. Of course, in our picture the internal structure and group-theoretic properties of $\Theta^+$ and $N(1440)$ are quite different.

**$\Sigma((ud)[su]+\bar{s})$ and $\Lambda((ud)[ds]-[su][ud])$:** These $1^+$ hyperon resonances should have masses, $M(\Sigma,\Lambda) \approx M_0 + m_s + \alpha \approx 1600$ MeV, well above threshold for decay to $\Lambda(1116)\pi$ and $\Sigma(1192)\pi$. We expect them to be broader than the $\Theta^+$. The PDG identify $\Sigma$ “bumps” at 1480 MeV and 1560 MeV. There is a well established $\Lambda(1600)\, P_{01}$ and $\Sigma(1660)\, P_{11}$ both with $J^P = \frac{1}{2}^+$. $N_s([ud][su]+\bar{s})$: Although it has the same quantum numbers as the nucleon, this state has hidden strangeness (like the $\phi(1020)$) and is therefore heavier and should couple anomalously strongly to strange particles. We estimate $M(N_s) \approx M_0 + 2m_s + \alpha \approx 1700$ MeV. There is a candidate, the $N(1710)\, P_{11}$. If this assignment is correct, then the $N(1710)$ should couple more strongly to $N\eta, K\Lambda$, and $K\Sigma$ than currently suspected.

**$N_s([us]^2\bar{s})$:** Like the $N_s$, this state has hidden strangeness. It is predicted to be heavy, $M(N_s) \approx M_0 + 3m_s + 2\alpha \approx 1850$ MeV, and to couple predominantly to $\Sigma\eta$ and $\Xi\pi$. Since it couples weakly to $\overline{10}_f N$, it would have been difficult to see. $\Xi(\sqrt{2}[us]^2\bar{u} - [su][ds]\bar{d})$: This $\frac{1}{2}^+$ cascade resonance should have mass $M(\Xi) \approx M_0 + 2m_s + 2\alpha \approx 1750$ MeV. Some $\Xi$s are known in this region, but none have established spin-parity assignments. $\Xi_s(\sqrt{2}[us]^2\bar{u})$: This isospin 3/2 multiplet contains two $\Xi$s with ordinary charge assignments, $\Xi^{+}_s$ and $\Xi^{-}_s$. In addition it includes the exotic $\Xi^{+}_s$ and $\Xi^{-}_s$. These exotics should have mass near 1750 MeV. There is a clean $SU(3)$ relation between the matrix elements for $\Theta^+ \rightarrow pK^0/nK^+$ and $\Xi^{--} \rightarrow \Sigma^- K^-/\Xi^-\pi^-$. Adjusting for differences in phase space, the width of the $\Xi^{--}$ at a mass of 1750 MeV should be about 50% greater than the width of the $\Theta^+$.

The spectrum of the $8_f + \overline{10}_f$ correlated quark picture differs in several dramatic ways from the spectrum predicted by the chiral soliton model. The differences are highlighted in Fig. (3). The chiral soliton model predicts only a $\overline{10}_f$. First order perturbation theory in $m_s$ predicts equal spacing between isomultiplets of decreasing hypercharge. None of the phenomena related to ideal mixing of the $8_f + \overline{10}_f$ occur. In particular in the chiral soliton model: a) There is no $N$ lighter than the $\Theta^+$, therefore no candidate for the Roper resonance; b) The $N$ that is heavier than the $\Theta$ couples both to strange and non-strange channels. The $N_s$ in the correlated
FIG. 2: Relative masses of states in the ideally mixed $(q^4q)_{10} \oplus 8_{1}$, compared with the mass hierarchy in the chiral soliton model.

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<tr>
<th>Mass</th>
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<td>$\Xi_{3/2}$</td>
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The quark picture should couple predominantly with hidden strangeness (like the $\phi(1020)$ or $f_0(980)$); c) The $\Xi$ states are heaviest – above 2 GeV. In the correlated quark picture they are close to the $N_s$ because they both contain two $s$-quarks. The prediction of light charge-exotic $\Xi$'s is the most distinctive signature of our model; d) There is no second multiplet of $\Sigma$’s with hidden strangeness and no $\Lambda$. The striking departure of predicted mass splittings in the chiral soliton model from an approximate additive quark mass formula arises from that model’s powerful embodiment of chiral flavor $SU(3) \times SU(3)$.

Exotic channels ought to be accessible to numerical investigation using lattice gauge theory. The predicted resonances are above threshold, but not by much, so they should be visible in long-range correlators. By comparing different spatial and flavor trial wave functions, one could isolate the effect of the suggested (diquark) configurations. One could also bring the states closer to threshold, or perhaps even below it, by increasing quark masses, as in an analogous study of the scalar mesons.

Our picture suggests that the charm and bottom analogs of the $\Theta^+$ may be stable against strong decays. The $\Theta^+_c = [ud]^2 \bar{c}$ and $\Theta^+_b = [ud]^2 \bar{b}$ differ from the $\Theta^+$ by the replacement $\bar{s} \rightarrow \bar{c}$ or $\bar{b}$. The cost of this can be estimated by comparing the masses of the $\Lambda(1116)$, $\Lambda_c(2285)$, and $\Lambda_b(5624 \pm 9)$. The $[ud]$ diquark in the $\Lambda$ is coupled to color 3 and spin zero and so provides an environment for the $s$ quark nearly identical to the environment of the $\bar{s}$ quark in the $\Theta^+$ (plausible in the limit of tight diquark binding). Therefore we estimate $M(\Theta^+_c) - M(\Theta^+) = M(\Lambda_c) - M(\Lambda) = 1170 \text{ MeV}$. This yields $M(\Theta^+_c) = 2710 \text{ MeV}$, which is 100 MeV below threshold for the strong decay $\Theta^+_c \rightarrow pD^-$. Likewise, we estimate $M(\Theta^+_b) - M(\Theta^+) = M(\Lambda_b) - M(\Lambda) = 4510 \pm 10 \text{ MeV}$, and therefore $M(\Theta^+_b) = 6050 \pm 10 \text{ MeV}$, which is 165 ± 10 MeV below threshold for the strong decays $\Theta^+_b \rightarrow pB^0$ and $nB^+$. The sequence of binding energies, $-100 \text{ MeV}$, $100 \text{ MeV}$, and $165 \text{ MeV}$, for the $\Theta^+, \Theta^0,$ and $\Theta^+_b$ reflects the non-linear dependence of the pseudoscalar meson $(K, D, \text{and } B)$ masses on the quark masses. The $\Theta^+_c$ could be produced, for example, in $\pi^- p \rightarrow \Theta^+_cD^0$ or $\Theta^+_bD^+\pi^-$ or even in non-leptonic $B$-decays, such as $B^0 \rightarrow \Theta^+_bD^+\pi^-$.

We have presented a description of $q^4\bar{q}$ baryons based on the strong color-spin correlation force that drives color superconductivity and may be responsible for the peculiarities of the light scalar mesons. A light positive parity $K^+n$ exotic is natural in this picture. So is an even lighter $\frac{1}{2}^+$ nucleon, which we tentatively identify with the Roper resonance. Our description differs in important ways from the chiral soliton description of the $\Theta^+$, and our prediction of relatively light charge-exotic cascades should be testable in both numerical and accelerator experiments.

After submission of this Letter, evidence for an exotic $\Xi^{-}\Xi^+$ was reported by the NA49 Collaboration [21], at a mass about 100 MeV above our rough estimate.

Acknowledgments

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[20] For a different analysis, which predicts instability of the $\Theta^0$ and $\Theta^+$, see M. Karliner and H. J. Lipkin, arXiv:hep-ph/0307343.