The QCD phase diagram

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The QCD phase diagram

- We will discuss the QCD phase diagram in the following variables:
  - Temperature, T
  - Chemical potential, μ
  - Strange quark mass, m_s

- To set the stage we start considering various limiting cases:
  - $T = \mu = m_s = 0$. Hadronic **confined** phase with spontaneous breaking
  $$SU(3)_L \otimes SU(3)_R \otimes U(1)_V \rightarrowtail SU(3)_V \otimes U(1)_V$$
  - $T = \mu = 0$, $m_s \neq 0$. Hadronic **confined** phase with spontaneous breaking
  $$SU(2)_L \otimes SU(2)_R \otimes U(1)_V \rightarrowtail SU(2)_V \otimes U(1)_V$$

- $T \to \infty$, $\mu = 0$. Using asymptotic freedom we expect an **unbroken, deconfined** phase with symmetry $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$ ($m_s = 0$), or $SU(2)_L \otimes SU(2)_R \otimes U(1)_V$ ($m_s \neq 0$)

- Asymptotic freedom suggests that at high density and low temperature matter will consist of a Fermi sphere of almost free quarks, but this is **too naive!!**

The Fermi surface

- The naive picture suggested by asymptotic freedom does not work. In fact Bardeen, Cooper and Shrieffer (BCS) proved the instability of the Fermi surface in presence of an attractive interaction.

- An attractive quark-quark interaction changes the Fermi surface to a coherent state of particle-hole pairs the **Cooper pairs**. This is because it costs no free energy to make a pair of particles and the interaction makes this favorable.

An arbitrarily weak interaction breaks the fermion number symmetry spontaneously

- In the case of normal matter, only for some crystals the attractive phonon interaction can overcome the repulsive em interaction. This leads to superconductivity (SC). However, thermal fluctuations can easily destroy this state making SC survive only at low temperature.

- In QCD the dominant interaction due to gluon-exchange is attractive. Therefore a condensate of quark pairs is expected.

- The relevant degrees of freedom are the ones close to the Fermi surface. Also the qq interaction has two channels available the antisymmetric $\bar{3}$ (the attractive and dominant one) and the symmetric 6.

- The diquark condensate cannot be a color singlet and therefore the gauge symmetry $SU(3)_c$ is broken. We speak of **color superconductivity**. The strange quark mass plays a peculiar role in determining different phases

Finite temperature and zero density

- This part of the phase diagram can be studied in lattice simulations starting from first principles. In heavy ion collisions at higher and higher energies the phase diagram is explored closer and closer to the surface $\mu = 0$

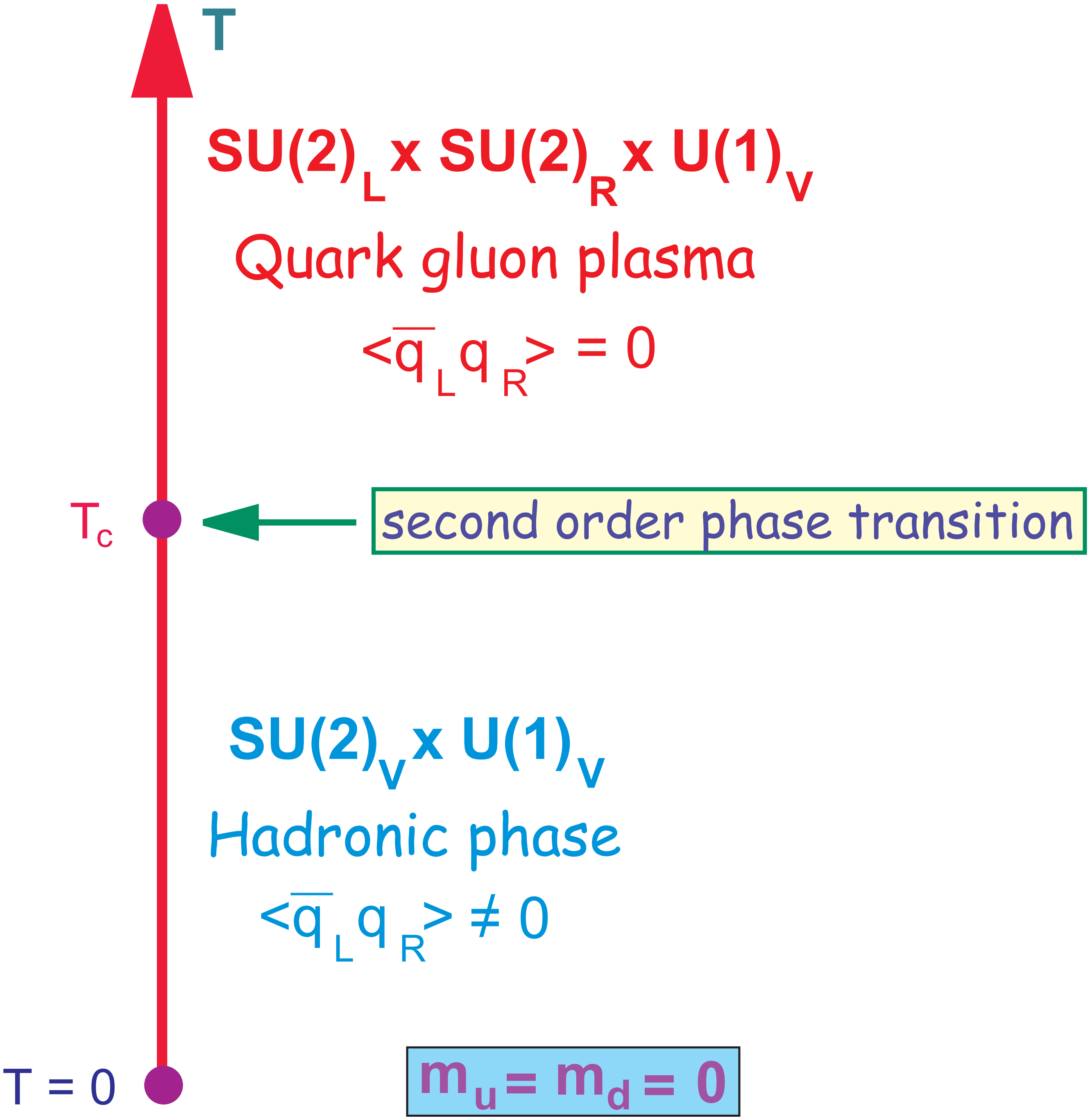
- Let us start with $m_u = m_d = 0$, $m_s \to \infty$. The phase structure is characterized by the chiral condensate

$$\langle \bar{\psi}_L \psi_R \rangle$$

  At low T the non vanishing of the condensate locks together the L and R symmetries breaking the global symmetry to $SU(2)_V \otimes U(1)_V$.

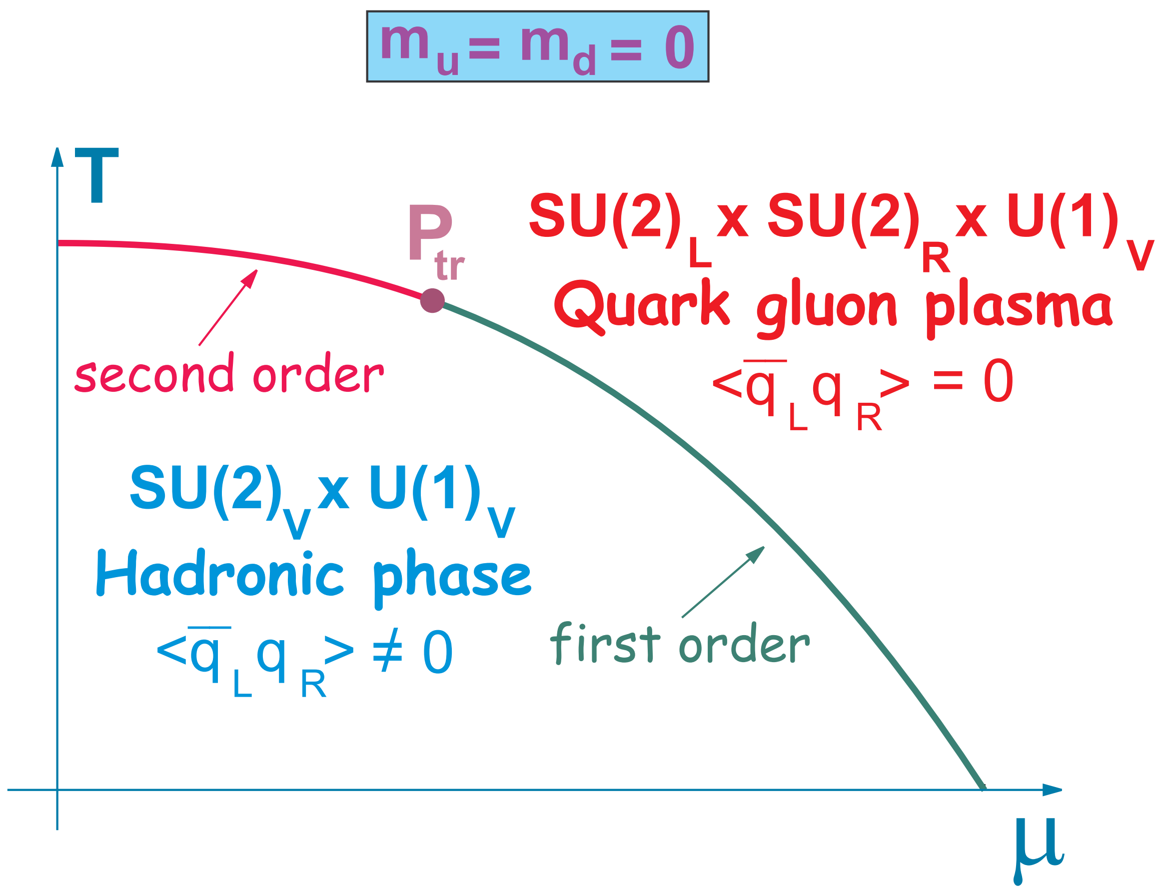
- The excitations of the theory are the hadrons and the low-lying spectrum is characterized by 3 massless Goldstone bosons. This phase can be described by means of the chiral effective lagrangian. Heating the system we meet the **critical temperature, $\mathbf{T_c}$**, where $\langle\bar{\psi}_L\psi_R\rangle \to 0$. After that we go to the **quark-gluon plasma phase**. The transition is likely to be **second-order** (Pisarski and Wilczek, 1984; Wilczek 1992, Rajagopal and Wilczek, 1993)

- Since $m_u, m_d \neq 0$ the second order phase transition goes into an analytical crossover. This agrees with lattice simulations (reviews by Ukawa, 1997; Laermann, 1998; Karsch, 1999) suggesting $T_c \approx 140 \div 190\ MeV$

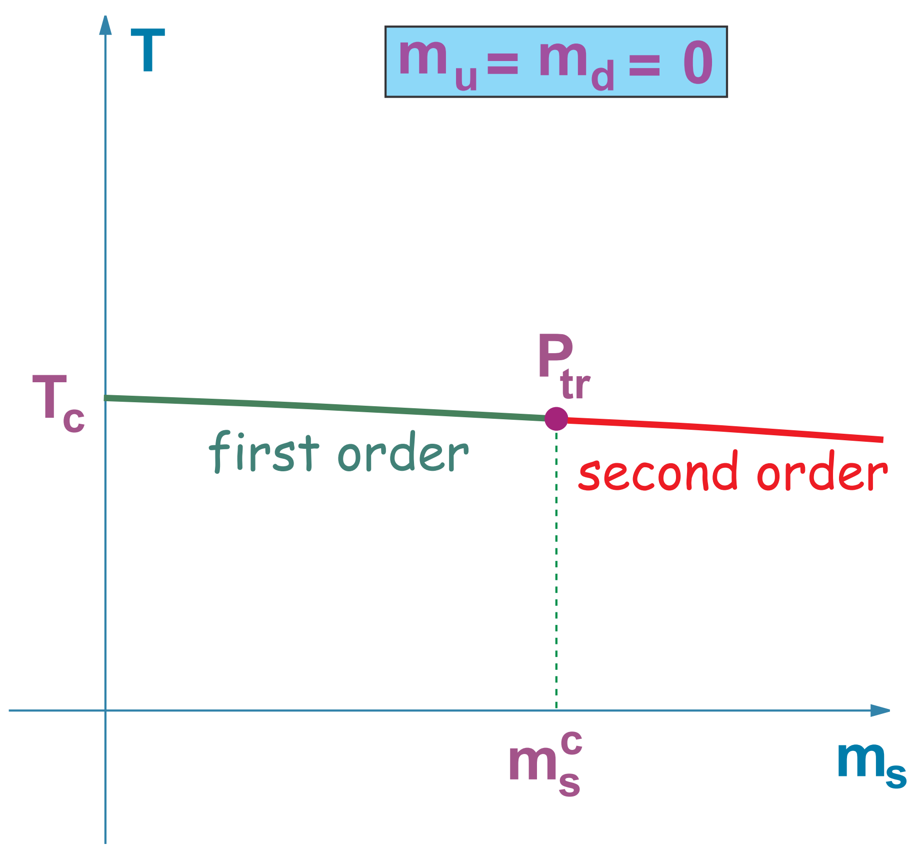

T
$SU(2)_L \times SU(2)_R \times U(1)_V$
Quark gluon plasma
$\langle \bar{q}_L q_R \rangle = 0$
T_c
second order phase transition
$SU(2)_V \times U(1)_V$
Hadronic phase
$\langle \bar{q}_L q_R \rangle \neq 0$
T = 0
$m_u = m_d = 0$

Finite temperature and density

- Many indications (mainly from model calculations: Barducci, Casalbuoni, De Curtis, Gatto, Pettini, 1989 and 1990; Klevansky, 1992; Alford, Rajagopal, Wilczek, 1998; Berges, Rajagopal, 1999) suggest that chiral symmetry at finite μ is restored through a **first order phase transition** leading to the idea of the presence of a **tricritical point,** $\mathrm{T_{tr}}$, in the QCD phase diagram



- Moving m_s down to zero, universality arguments and lattice calculations suggest that the transition at $\mu = 0$ would be first order, rather than second one or a cross over (for massive quarks). Therefore, dialing m_s one expects a critical value, $\mathbf{m_s^c}$, such that at $\mu = 0$ starting from large values of m_s the phase transition in temperature goes from second order to first order. Lattice simulations suggest $m_s^c \approx m_s/2$. Therefore a **tricritical point** should exist in the real QCD phase diagram.



Finite density and zero temperature

- At high density we expect the formation of diquark condensates and color superconductivity. **How to prove this statement?** There are various possibilities:

  - Construct a trial wavefunctional and use a many-body variational approach

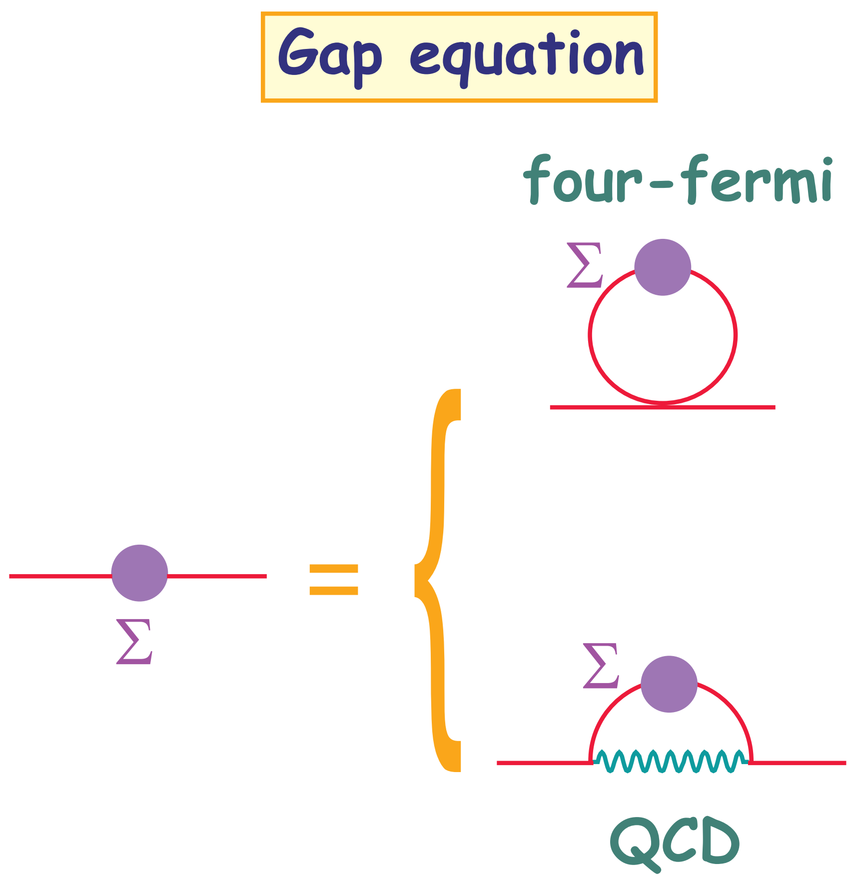
  - Proceed in a field-theoretical way considering the quark self-energy. The poles of this function give the energies of the fermionic excitations close to the Fermi surface (quasi-particles). This is done through a self-consistency condition, **the gap equation**, assuming the self-energy with the structure of the condensate and then solving it to find the actual self-energy

At asymptotically high density one can do this calculation from first principles since QCD is weakly coupled (Son, 1999; Schäfer and Wilczek, 1999; Pisarski and Rischke, 2000; Hong, 2000; Hong, Miransky, Shovkovy and Wijewardhana, 2000; Brown, Liu and Ren, 2000; Hsu and Schwetz, 2000; Schäfer, 2000; Shovkovy and Wijewardhana, 2000). However the physically interesting density regime for neutron star or heavy ions is up to a few times the nuclear density, $\mu \lesssim 500\ MeV$, whereas the actual calculations are unlikely to be extrapolated below $10^8\ MeV$. Alternatively one can use phenomenological interactions known to capture the essential features of QCD in the regime of interest. For two flavors one can use the instanton vertex producing a four-fermi interactions, or with more flavors to work in the one-gluon exchange approximation.

The corresponding gap equations are illustrated in figure. The results agree within a factor of about two. The gap equation has the form

$$\Sigma(k) = -\frac{1}{(2\pi)^4}\int d^4q D^{-1}(q)V(k-q)$$

with $D^{-1}(q)$ the full propagator and $V(p)$ the vertex function, momentum independent in the four-fermi case, whereas for QCD includes the gluon propagator



Since one expects a diquark condensate it is convenient to make use of the Nambu-Gorkov formalism introducing

$$\Psi = (\psi, \bar{\psi}^T)$$

The wave operator is then postulated of the form

$$D(q) = D(q)_{\rm free} + \Sigma = \begin{pmatrix} \not{q} + \mu\gamma_0 & \gamma_0\Delta\gamma_0 \\ \Delta & (\not{q} - \mu\gamma_0)^T \end{pmatrix}$$

equivalent to introduce in the lagrangian a mass gap term of the type $\psi^T C \Delta \psi$. For a four-fermi interaction Δ is a matrix in color-flavor-spin space. In the gluon-exchange case there is also a momentum dependence. The typical gap equation for a four-fermi interaction (neglecting antiparticle contribution) is

$$\Delta = 4G \int_0^\Lambda \frac{d^4p}{(2\pi)^4} \left(\frac{\Delta}{p_0^2 + (|\vec{p}| - \mu)^2 + \Delta^2} \right)$$

When the interaction strenght, $G \to 0$, and $\Delta \ll \mu,\ \Lambda$

$$\Delta \propto \Delta \mu^2 G \log(\mu/\Delta) \to \Delta \propto \mu \exp(-c/(G\mu^2))$$

Replacing the one-gluon exchange by an effective four-fermi interaction one expects

$$\Delta \propto \mu \exp(-c/g^2)$$

But taking into account the gluon propagator the gap equation is rather of the type

$$\Delta \propto g^2 \int d\epsilon \frac{\Delta}{\sqrt{\epsilon^2 + \Delta^2}} d\theta \frac{\mu^2}{\theta\mu^2 + \Delta^2}$$

θ comes from the gluon propagator (θ is the integration angle between the internal and external momentum) which, due to the spontaneous breaking of color, picks up a **Meissner mass** $\propto \Delta$ (Casalbuoni, Gatto, Nardulli, 2001). This gives a double-log contribution (Son, 1999; Rajagopal and Wilczek, 2001)

$$\Delta \propto \Delta g^2 (\log(\mu/\Delta))^2 \to \boxed{\Delta \propto \mu \exp(-c/g)}$$

The gap at large μ is much larger in QCD than in the case of a point-like interaction

If the coupling g is evaluated at μ and assume $1/g^2 \approx \log \mu$ the exponential gives a very weak suppression and for $\mu \to \infty$ we have $\Delta \to \infty$ and $\Delta/\mu \to 0$.

Color superconductivity is bounded to dominate physics at high density

Two massless quark flavors

We consider the case $m_u = m_d = 0$, $m_s \to \infty$. The gap equation has been solved with different interactions, including QCD at weak coupling. All the indications are that the quarks pair in the color $\bar{\mathbf{3}}$ and flavor singlet channel

2SC phase:

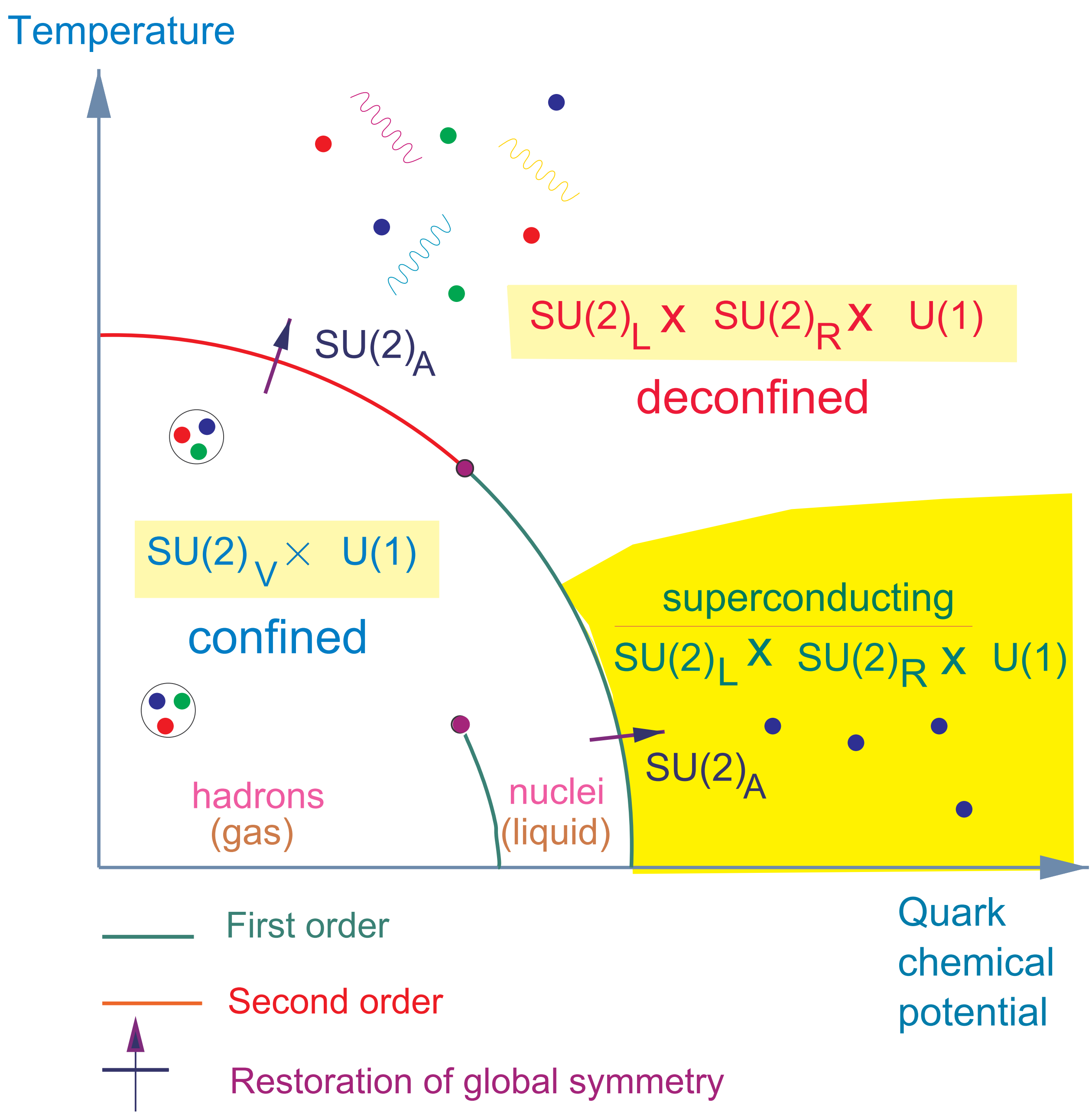
$$\Delta^{\alpha\beta}_{ijL(R)} = \langle q^{\alpha}_{iL(R)} q^{\beta}_{jL(R)} \rangle \propto C\gamma_5 \epsilon_{ij} \epsilon^{\alpha\beta 3}$$

with $\alpha, \beta = 1, 2, 3$ are color indices and $i, j = 1, 2$ are flavor indices. The calculation gives a gap of order $100\ MeV$ for $\mu \approx 500\ MeV$.
The condensate breaks color but does not break flavor

$$[SU(3)_c] \otimes [U(1)_Q] \otimes SU(2)_L \otimes SU(2)_R$$

$$\downarrow$$

$$[SU(2)_c] \otimes [U(1)_{\tilde{Q}}] \otimes SU(2)_L \otimes SU(2)_R$$



Temperature
SU(2)_A
SU(2)_L x SU(2)_R x U(1)
deconfined
SU(2)_V × U(1)
confined
superconducting
SU(2)_L x SU(2)_R x U(1)
SU(2)_A
hadrons
(gas)
nuclei
(liquid)
Quark
chemical
potential
First order
Second order
Restoration of global symmetry

The main features are

- $SU(3)_c$ is broken down to $SU(2)_c$. Five gluons become massive

- No global symmetries are broken. The 2SC phase has the same global symmetries as the quark-gluon plasma QGP. Therefore one cannot construct a gauge invariant order parameter such to discriminate between the two phases. **There is no need of a phase transitions between 2SC and QGP phases**. Notice that the baryon number, B is broken, but the following linear combination of B and of the color charge, T_8 is conserved

$$\tilde{B} = B - \frac{2\sqrt{3}}{3} T_8 = (0, 0, 1)$$

meaning that the quarks of color 1 and 2 taking part in the condensate have $\tilde{B} = 0$, whereas the other two quarks of color 3 q_i^3 have $\tilde{B} = 1$. Notice that the colors 1 and 2 are confined, whereas the color 3 is deconfined.

- The electric charge is broken, but again the linear combination

$$\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8$$

is not broken. This gives the following charges to the quarks

$$\tilde{Q}_u = \frac{2}{3} - \frac{1}{6}(1, 1, -2) = (\frac{1}{2}, \frac{1}{2}, 1)$$

$$\tilde{Q}_d = -\frac{1}{3} - \frac{1}{6}(1, 1, -2) = (-\frac{1}{2}, -\frac{1}{2}, 0)$$

- Quarks of color 1 and 2 acquire a mass gap Δ (mass of the physical excitation around the Fermi surface). Quarks of color 3 remain ungapped. The 't Hooft anomaly matching could prevent any condensation (Sannino, 2000; Hsu, Sannino, Schwetz, 2000; Casalbuoni, Duan, Sannino, 2000)

- No Goldstone bosons in the spectrum, since no global symmetry is broken

- The low energy degrees of freedom are 3 gluons and the almost free quarks of color 3. The symmetries determining the effective lagrangian are: the gauge symmetry $SU(2)_c$ and rotation invariance (Lorentz is broken being at finite density). For the gluons one gets (Rischke, Son, Stephanov, 2000)

$$\mathcal{L}_{\text{eff}} = \frac{\epsilon}{2}\vec{E}^a \cdot \vec{E}^a - \frac{1}{2\lambda}\vec{B}^a \cdot \vec{B}^a$$

with a propagation velocity for the gluons given by

$$v = \frac{1}{\sqrt{\epsilon\lambda}}$$

One can evaluate the constants integrating out the quark degrees of freedom

$$\epsilon = 1 + \frac{g^2\mu^2}{18\pi^2\Delta^2} \approx \frac{g^2\mu^2}{18\pi^2\Delta^2}, \quad \lambda = 1$$

The strong coupling constant gets modified

$$\alpha_s \;\to\; \alpha_s' = \frac{g^2_{\text{eff}}}{4\pi v} = \frac{g^2}{4\pi\sqrt{\epsilon}} = \frac{3}{2\sqrt{2}}\frac{g\Delta}{\mu}$$

since the Coulomb force is modified

$$g^2/r \to g^2/(\epsilon r) \;\Rightarrow\; g^2 \to g^2_{\text{eff}} = g^2/\epsilon$$

Three massless quark flavors

Both for left and right condensates one finds

CFL phase:

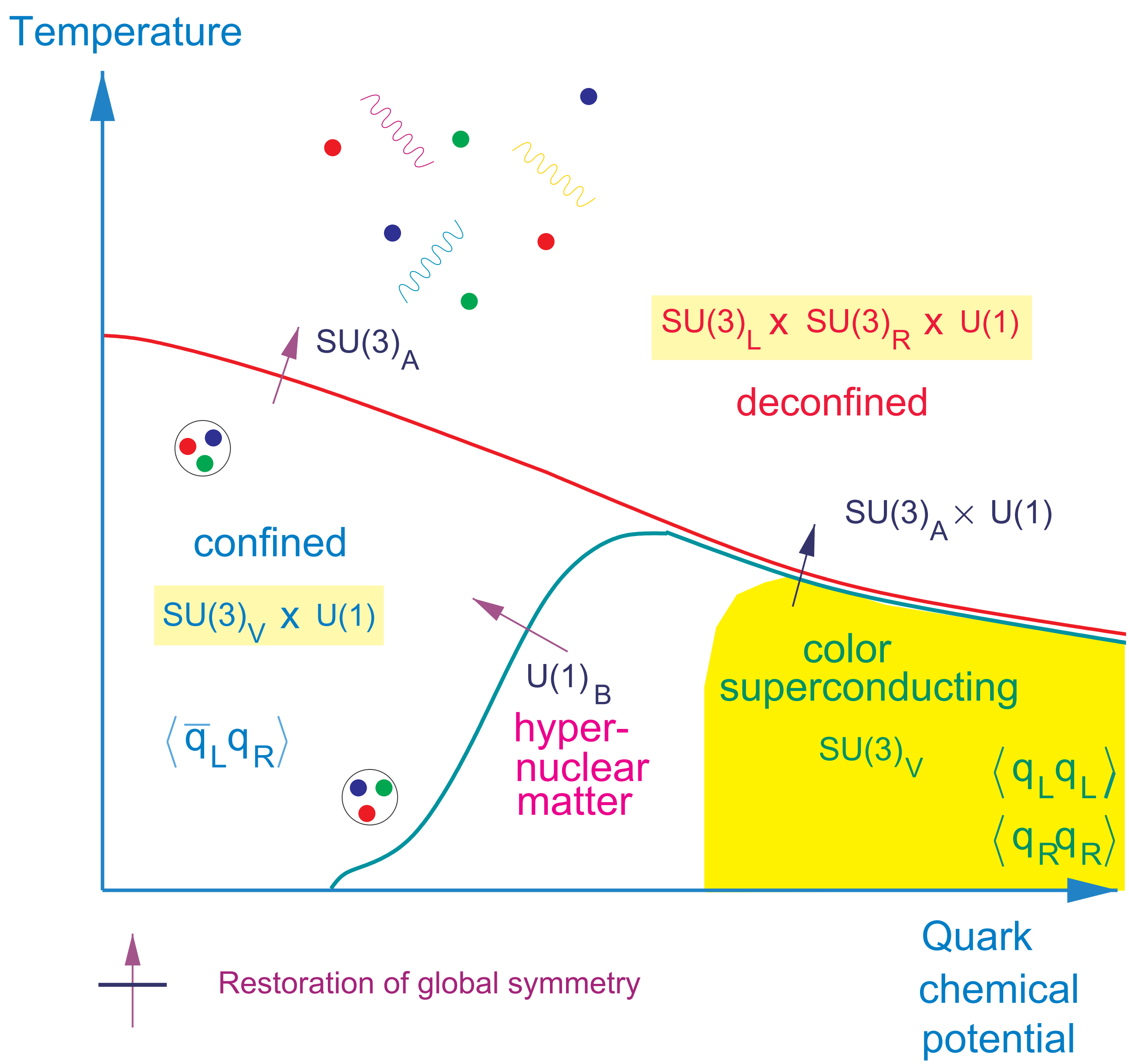
$$\Delta^{\alpha\beta}_{ijL(R)} = \langle q^{\alpha}_{iL(R)} q^{\beta}_{jL(R)} \rangle$$
$$\propto C\gamma_5 \left[\epsilon_{ijX}\epsilon^{\alpha\beta X} + \kappa(\delta^{\alpha}_i \delta^{\beta}_j + \delta^{\beta}_i \delta^{\alpha}_j) \right]$$
$$\propto C\gamma_5 \left[(\kappa + 1)\delta^{\alpha}_i \delta^{\beta}_j + (\kappa - 1)\delta^{\beta}_i \delta^{\alpha}_j \right]$$

For $\kappa = 0$ the pairing is in the color and flavor channel $(\mathbf{\bar{3}}, \mathbf{\bar{3}})$. The term proportional to κ corresponds to the channel $(\mathbf{6}, \mathbf{6})$. Since this term breaks the same symmetries as the former, it is not zero, but it is generally small. In weak coupling calculations one finds $\kappa = g\sqrt{2}\log(2)/(36\pi)$ (Schäfer, 2000). The gap locks together left, right and color symmetries

$$[SU(3)_c] \otimes \underbrace{SU(3)_L \otimes SU(3)_R}_{\supset [U(1)_Q]} \otimes U(1)_B$$

$$\downarrow$$

$$\underbrace{SU(3)_{c+L+R}}_{\supset [U(1)_{\tilde{Q}}]} \otimes Z_2$$


Temperature
SU(3)$_A$
SU(3)$_L$ x SU(3)$_R$ x U(1)
deconfined
SU(3)$_A$ × U(1)
confined
SU(3)$_V$ x U(1)
$\langle \bar{q}_L q_R \rangle$
U(1)$_B$
hyper-
nuclear
matter
color
superconducting
SU(3)$_V$
$\langle q_L q_L \rangle$
$\langle q_R q_R \rangle$
Quark
chemical
potential
Restoration of global symmetry

The main features are

- The color group is completely broken, all the gluons become massive

- Both chiral symmetry and baryon number are broken. One can introduce the (non-gauge invariant) order parameters

$$X_i^\alpha \approx \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}\langle q_{jL}^\beta q_{kL}^\gamma\rangle^*$$

$$Y_i^\alpha \approx \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}\langle q_{jR}^\beta q_{kR}^\gamma\rangle^*$$

  Starting from these one can construct two gauge invariant order parameters, one breaking the chiral symmetry $X_i^\alpha(Y_j^\alpha)^*$ and the other, $\epsilon^{ijk}\epsilon_{\alpha\beta\gamma}X_i^\alpha X_j^\beta X_k^\gamma$, **breaks B making the CFL phase a superfluid**

- Also in this case the electric charge is broken spontaneously, but the combination

$$\tilde{Q} = Q - \frac{2}{\sqrt{3}}T_8$$

  with eigenvalues $0, \pm 1$ as in the old Han-Nambu model, is conserved

- All the nine quarks are gapped. With respect to the unbroken $SU(3)_{c+L+R}$ they behave as $\mathbf{8} \oplus \mathbf{1}$. The singlet gap is higher than the octet one

- The symmetries in the CFL phase are the same as the ones in the hypernuclear matter (Schäfer, Wilczek, 1999) where one expects the existence of a pairing interaction in the flavor singlet dibaryon channel of the type ($\langle\Lambda^0\Lambda^0\rangle, \langle\Sigma^0\Sigma^0\rangle, \langle n\Xi^0\rangle$), breaking the baryon number and making this phase superfluid. A chiral condensate would break chiral symmetry. Therefore it would be possible to connect the hypernuclear matter to the CFL phase

pions $\Leftrightarrow$ **Goldstones**

vector mesons $\Leftrightarrow$ **massive gluons**

baryons $\Leftrightarrow$ **gapped quarks**

This suggests that there is no need of phase transitions between these two phases

Quark-hadron continuity

ψ^i_α = quark field

ψ^i_α	u	d	s
R	2/3	-1/3	-1/3
B	2/3	-1/3	-1/3
W	2/3	-1/3	-1/3

$D^\gamma_k = \epsilon_{ijk}\epsilon^{\alpha\beta\gamma}\psi^i_\alpha\psi^j_\beta$ = diquark field

D^γ_k	R	B	W
u	2/3	2/3	2/3
d	-1/3	-1/3	-1/3
s	-1/3	-1/3	-1/3

$B^i_k = \psi^i_\gamma D^\gamma_k = \psi^i_\gamma \left(\epsilon_{rsk}\epsilon^{\alpha\beta\gamma}\psi^r_\alpha\psi^s_\beta\right)$ = baryon field

B^i_k	u	d	s
u	0	-1	-1
d	1	0	0
s	1	0	0

$G^i_k = (D^*)^i_\alpha g^\alpha_\beta D^\beta_k$ = vector meson field

G^i_k	u	d	s
u	0	-1	-1
d	1	0	0
s	1	0	0

In the CFL phase the charge $\tilde{Q}$ of diquarks is zero whereas for quarks, ψ^i_α, and gluons, $g_\alpha\beta$, coincides with the charge Q of baryons, B^i_k, and of vector mesons, G^i_k. For instance, remember

$$\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^- & \Xi^- \\ \Sigma^+ & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Xi^0 \\ p & n & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

The low energy degrees of freedom are the nine Goldstone bosons arising from the global symmetry breaking of chiral and baryon symmetry. They are described by the field $\Sigma = Y^\dagger X$ ($\Sigma \to g_L \Sigma g_R^\dagger$) and by the Goldstone ϕ appearing in the $U(1)$ part of X. The effective lagrangian for Σ is the same as the chiral lagrangian, except for the breaking of Lorentz invariance (Casalbuoni, Gatto, 1999)

$$\mathcal{L} = \frac{F_T^2}{4}\left(Tr\,[\dot\Sigma\dot\Sigma^\dagger] - v^2\,Tr[\vec\nabla\Sigma\cdot\vec\nabla\Sigma^\dagger]\right)$$

One can also add mass terms for the goldstones (Son, Stephanov, 2000)

$$-c\left(\det M\,\mathrm{Tr}(M^{-1}\Sigma) + \mathrm{h.c.}\right)$$

M is the quark mass matrix, and the reason why $\mathcal{L}$ is of the second order in the quark masses relies in the Z_2 discrete symmetries allowing a change of sign in the left-handed quark fields. The constants have been evaluated by many authors in the weak coupling limit, with the results: $F_T \propto \mu$, $c \propto \Delta^2$, $m_\Sigma^2 \propto m_q^2\Delta^2/\mu^2$, $v^2 = 1/3$.

From the couplings of the Goldstone fields X and Y to gluons one finds that the Meissner mass is given by

$$m_M \propto g_s F_T \approx g_s \mu$$

But one-loop fermion corrections to gluon self-energy give a wave-function renormalization $\propto g\mu/\Delta$, giving a physical mass (or an inverse screening lenght) proportional to Δ (Casalbuoni, Gatto, Nardulli, 2000)

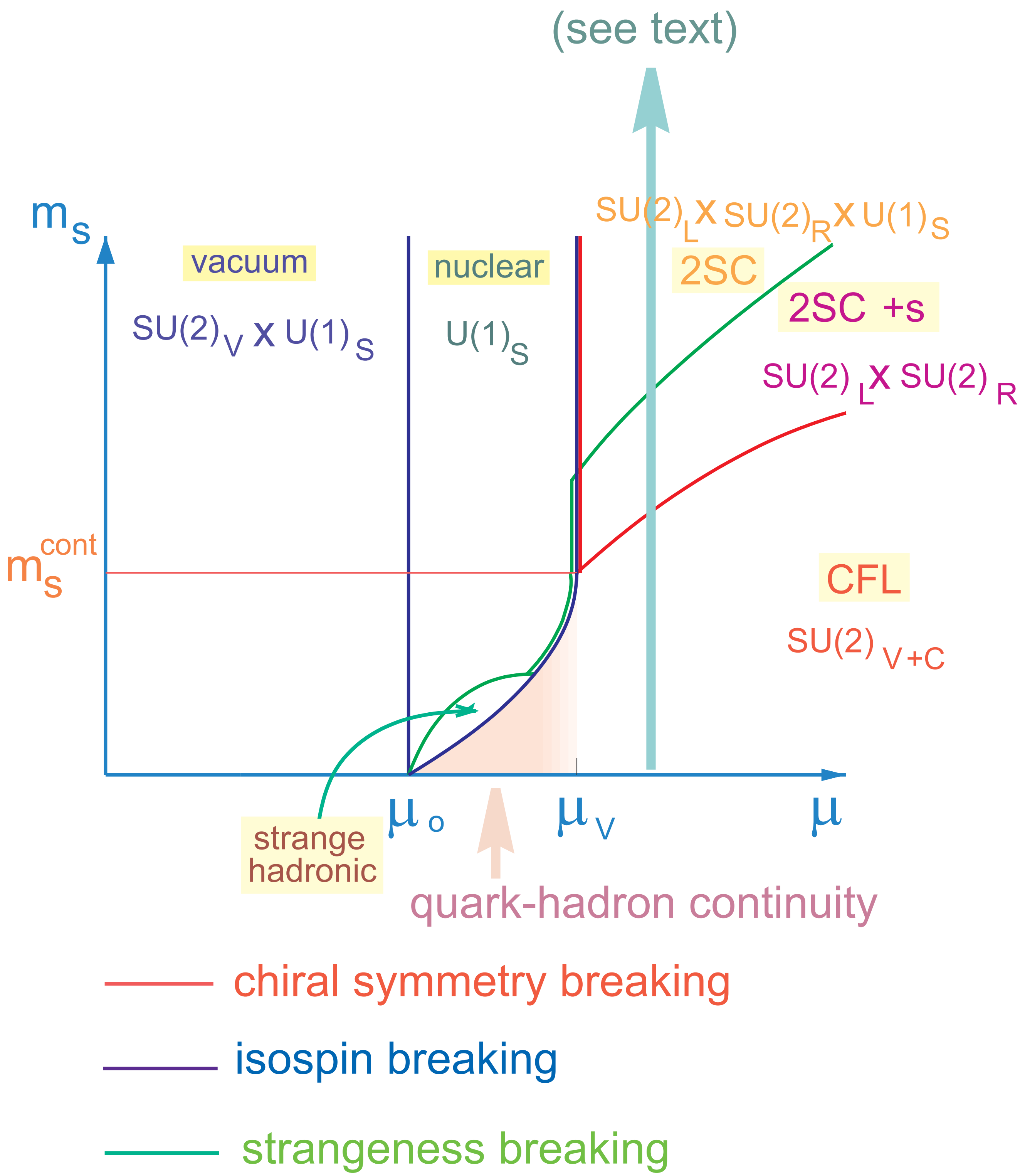
Two massless + one massive quark flavors

- $m_s \neq 0$ breaks explicitly

$$SU(3)_L \otimes SU(3)_R \;\rightarrow SU(2)_L \otimes SU(2)_R$$

If m_s is large, the strange quark decouples and. at large μ, the 2SC phase will occur. If m_s is small we expect a CFL of $SU(2)_c \subset SU(3)_c$ with $SU(2)_L$ and $SU(2)_R$ producing chiral symmetry breaking and leaving unbroken $SU(2)_{c+L+R}$

- **As $\mathrm{m_s}$ goes from 0 to ∞, color and flavor must unlock and the full symmetry $\mathbf{SU(2)_L \otimes SU(2)_R}$ gets restored**. It can be argued that there is a critical value for m_s corresponding to a **first order phase transition**



(see text)
m_s
vacuum
$SU(2)_V \times U(1)_S$
nuclear
$U(1)_S$
$SU(2)_L \times SU(2)_R \times U(1)_S$
2SC
2SC +s
$SU(2)_L \times SU(2)_R$
m_s^{cont}
CFL
$SU(2)_{V+C}$
μ_o
μ_V
μ
strange hadronic
quark-hadron continuity
chiral symmetry breaking
isospin breaking
strangeness breaking

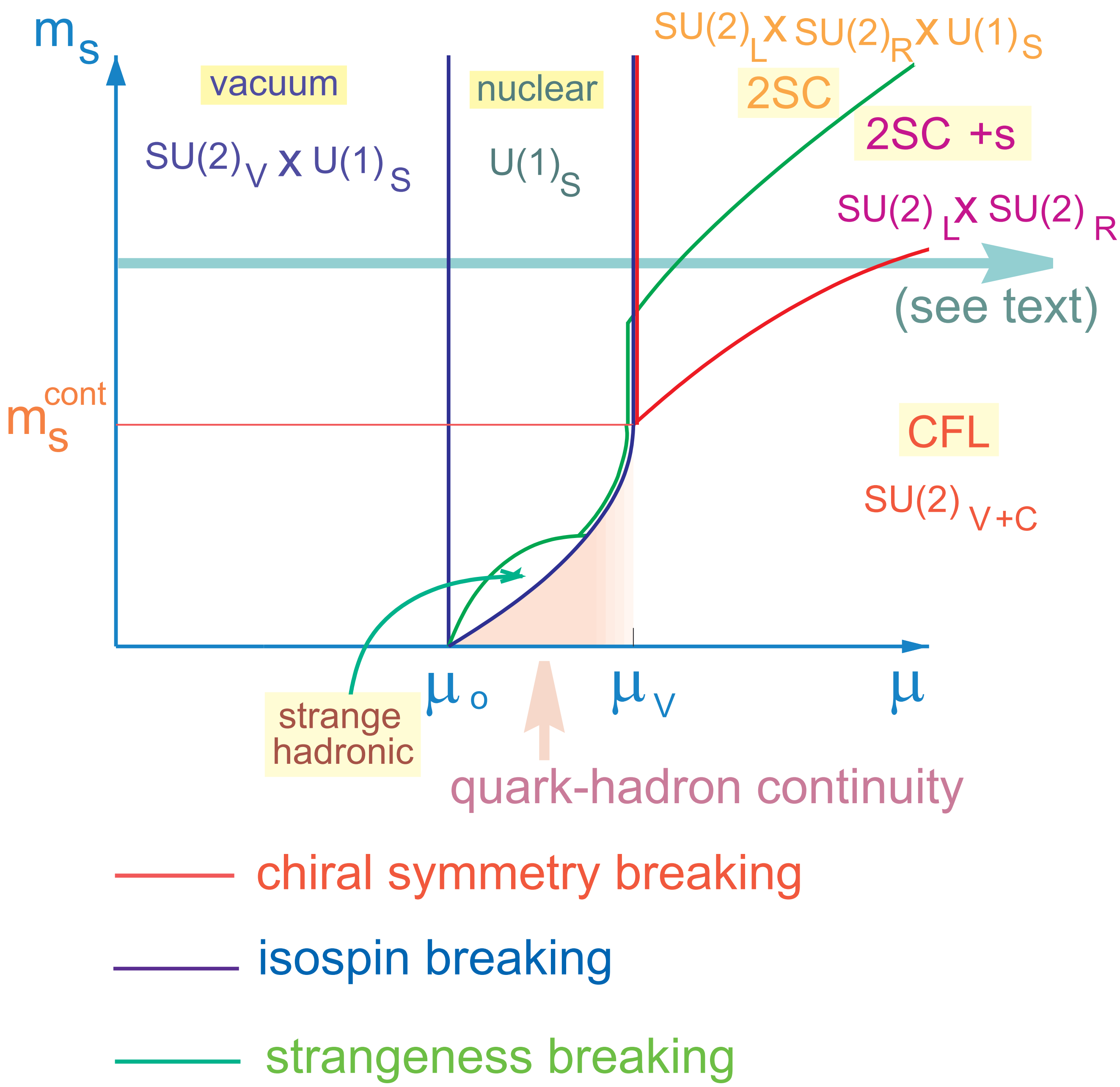
Consider now ($T = 0$), at fixed m_s (not too light), to vary μ from 0 to ∞

- For small μ, chiral symmetry is broken, $U(1)_s$ (strangeness) is not broken.

- After a first order phase transition ($\mu_0 \approx 300\ MeV$) we meet nuclear matter. Here the instability of the nucleon Fermi surface can lead to Cooper pair pp and nn possibly breaking isospin. At this stage $U(1)_s$ survives since no ss pairing is possible.

- Passing through μ_V we meet the $2SC$ phase with a first order phase transition (NJL model).

- When μ exceeds the strange quark mass we may have ss pairing in a color-spin locked phase, $2SC + s$ ($\langle s^\alpha C \gamma^i s^\beta \rangle = \Delta \epsilon^{\alpha\beta i}$). For very high μ, where all the quark masses are negligible we meet again the CFL phase



m_s
vacuum
SU(2)_V x U(1)_S
nuclear
U(1)_S
SU(2)_L x SU(2)_R x U(1)_S
2SC
2SC +s
SU(2)_L x SU(2)_R
(see text)
m_s^cont
CFL
SU(2)_V+C
μ_o
μ_V
μ
strange hadronic
quark-hadron continuity
chiral symmetry breaking
isospin breaking
strangeness breaking

Consider now ($T = 0$), with a fixed light m_s, to vary μ from 0 to ∞

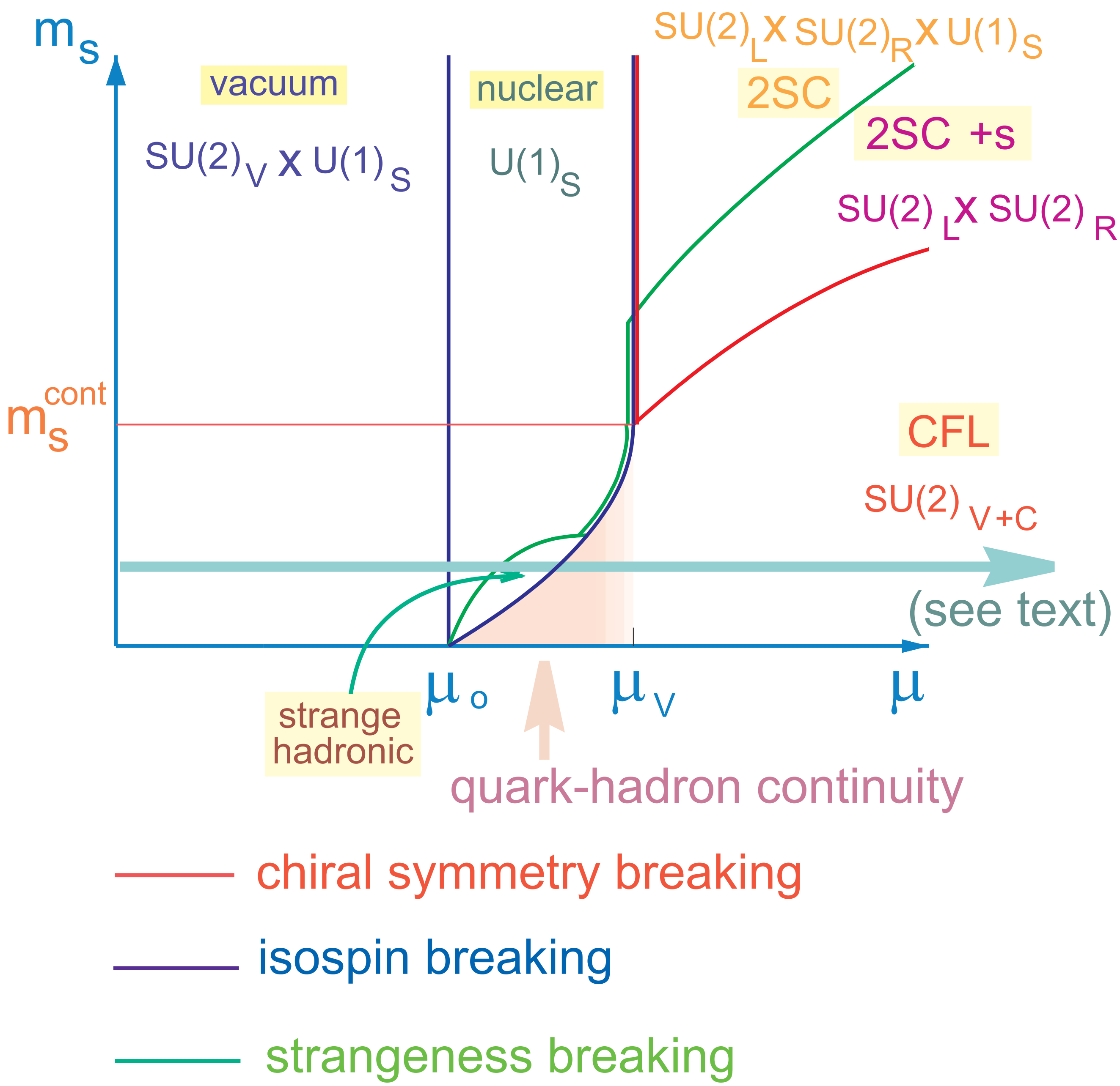
- Up to μ_0 we have the vacuum

- At μ_0, nuclear matter phase with pp and nn pairing breaking isospin

- Increasing μ strangeness is broken through kaon condensation (Kaplan, Nelson, 1986; Brown, Kubodera, Rho, 1987; Schäfer, 2000) or via $\Lambda\Lambda$, $\Sigma\Sigma$, $\Xi\Xi$ pairing (**strange hadronic phase**)

- Leaving the previous phase we have two possibilities:

  - **Deconfinement**, the baryonic Fermi surfaces are replaced by the quark $(u, d, s,)$ ones . Isospin locks to color, $SU(2)_{c+V}$ is restored, χ-symmetry remains broken



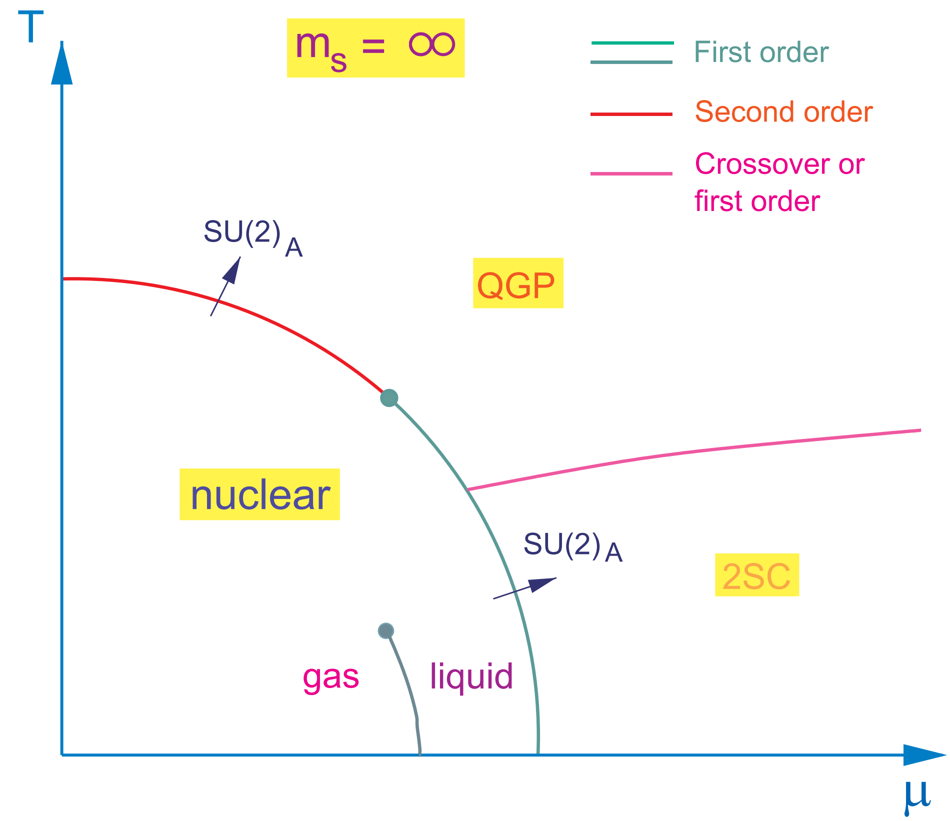
m_s
vacuum
$SU(2)_V \times U(1)_S$
nuclear
$U(1)_S$
$SU(2)_L \times SU(2)_R \times U(1)_S$
2SC
2SC +s
$SU(2)_L \times SU(2)_R$
m_s^{cont}
CFL
$SU(2)_{V+C}$
(see text)
strange hadronic
μ_o
μ_V
μ
quark-hadron continuity
chiral symmetry breaking
isospin breaking
strangeness breaking

- **No deconfinement**. Fermi momenta of the baryon octet are similar and pairing among baryons of different starngeness is possible. $SU(2)_V$ is restore, χ-symmetry and $U(1)_S$ are broken.

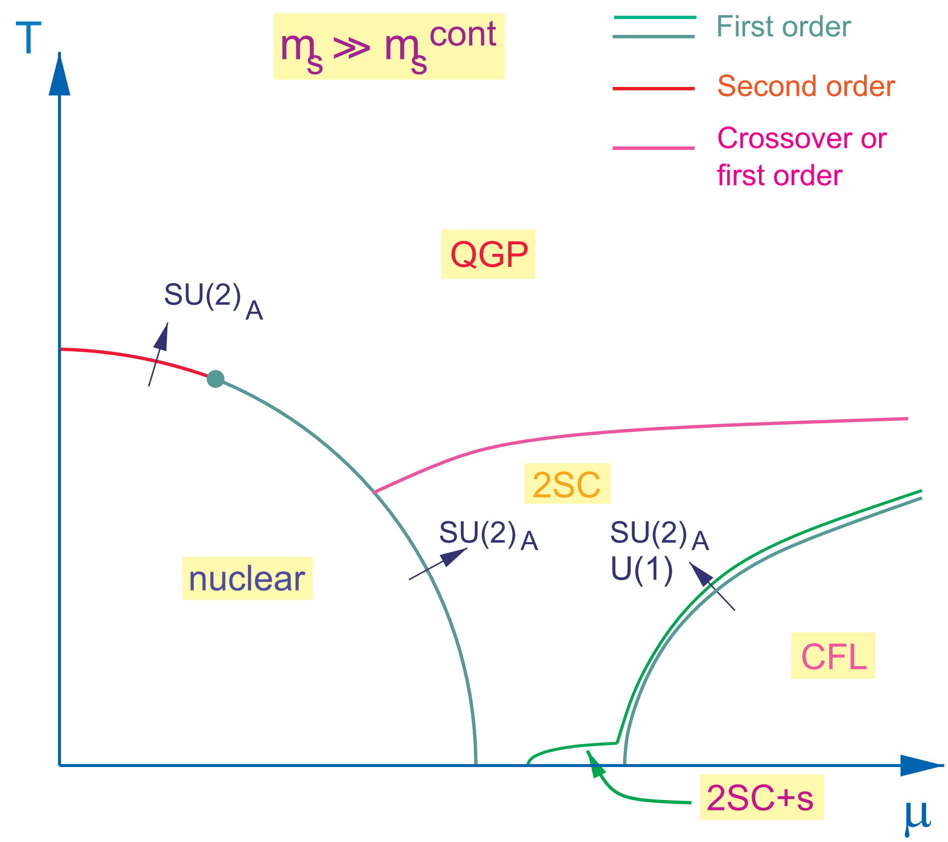
The symmetries are the same in the two scenarios. This is the **quark-hadron continuity** (Schäfer, Wilczek, 1999). At very high μ the CFL phase is present anyway, but a phase transition is not necessary

Particle type	Hyperonic matter	$\Leftrightarrow$	CFL quark matter
Fermions:	8 Baryons	$\Leftrightarrow$	9 Quarks
Chiral (pseudo)Goldstone:	8 pion/kaons	$\Leftrightarrow$	8 pseudoscalars
Baryon number (pseudo)Goldstone:	1	$\Leftrightarrow$	1
Vector Mesons:	9	$\Leftrightarrow$	8 massive gluons

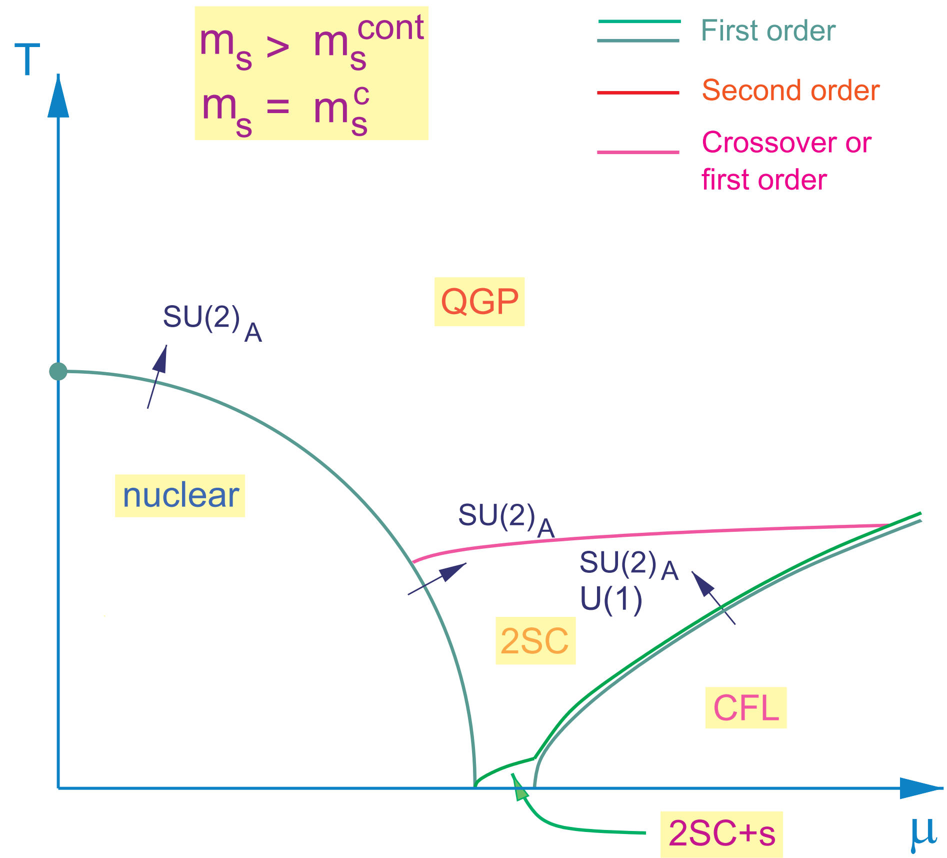
Quark	$SU(2)_{\text{color}+V}$	$\tilde{Q}$	Hadron	$SU(2)_V$	Q
$\begin{pmatrix} bu \\ bd \end{pmatrix}$	$\mathbf{2}$	$\begin{pmatrix} +1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} p \\ n \end{pmatrix}$	$\mathbf{2}$	$\begin{pmatrix} +1 \\ 0 \end{pmatrix}$
$\begin{pmatrix} gs \\ rs \end{pmatrix}$	$\mathbf{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}$	$\mathbf{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$\begin{pmatrix} ru - gd \\ gu \\ rd \end{pmatrix}$	$\mathbf{3}$	$\begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \Sigma^0 \\ \Sigma^+ \\ \Sigma^- \end{pmatrix}$	$\mathbf{3}$	$\begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$
$ru + gd + \xi_- bs$	$\mathbf{1}$	0	Λ	$\mathbf{1}$	0
$ru + gd - \xi_+ bs$	$\mathbf{1}$	0	—		

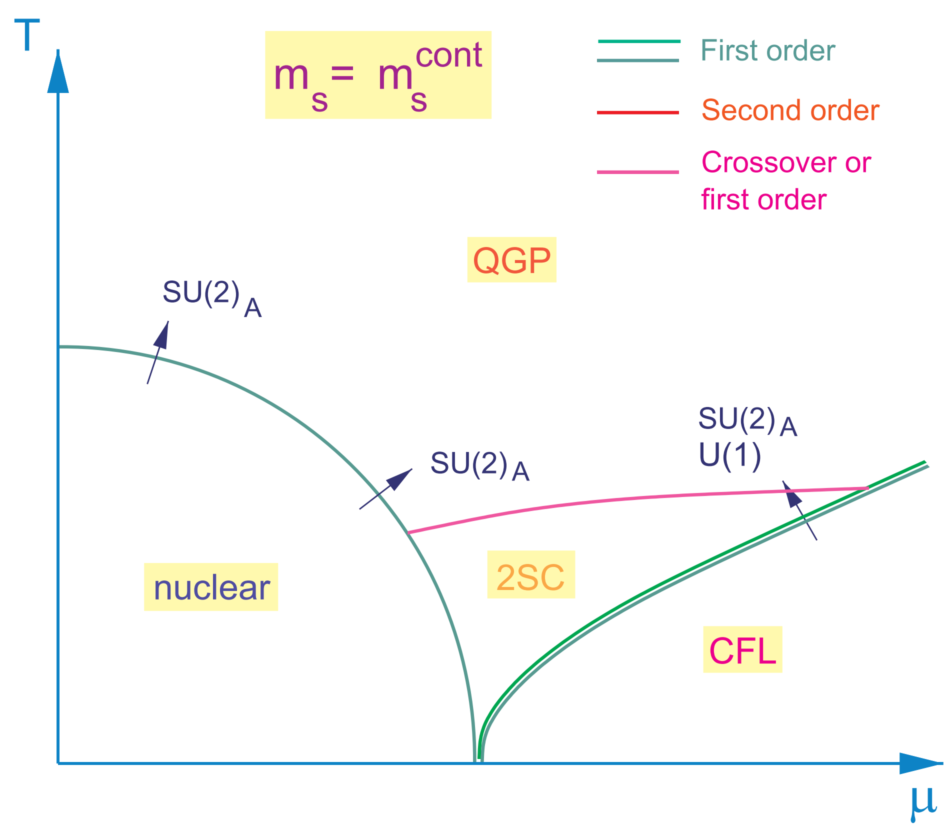


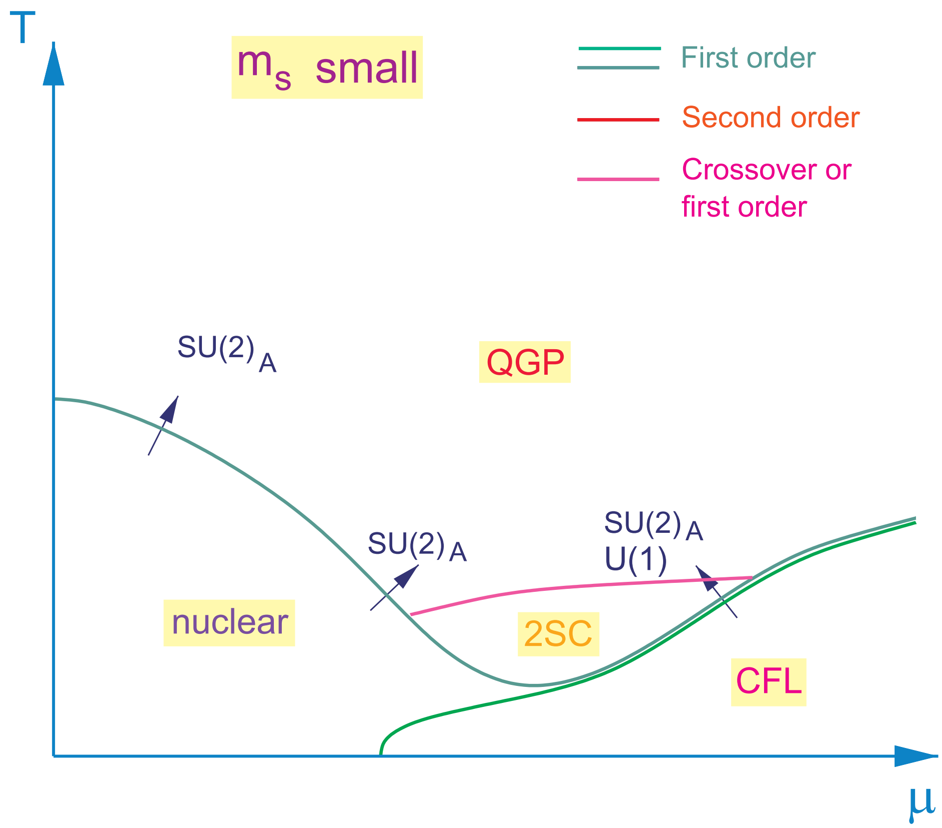
T
$m_s = \infty$
First order
Second order
Crossover or
first order
$SU(2)_A$
QGP
nuclear
$SU(2)_A$
2SC
gas
liquid
μ

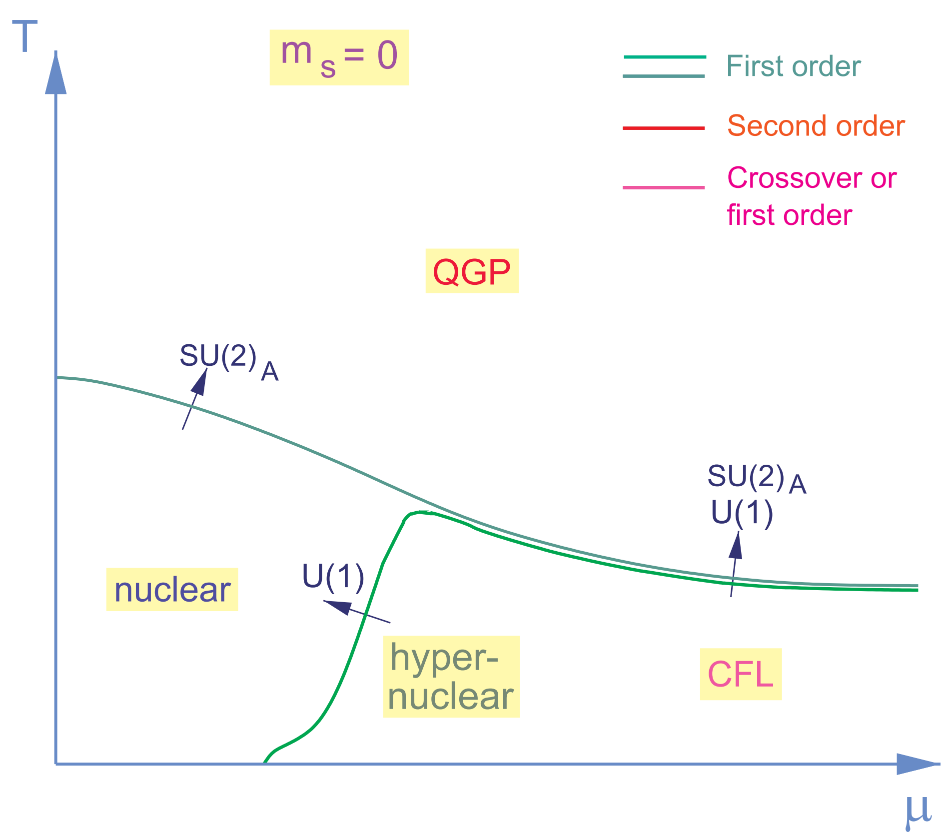


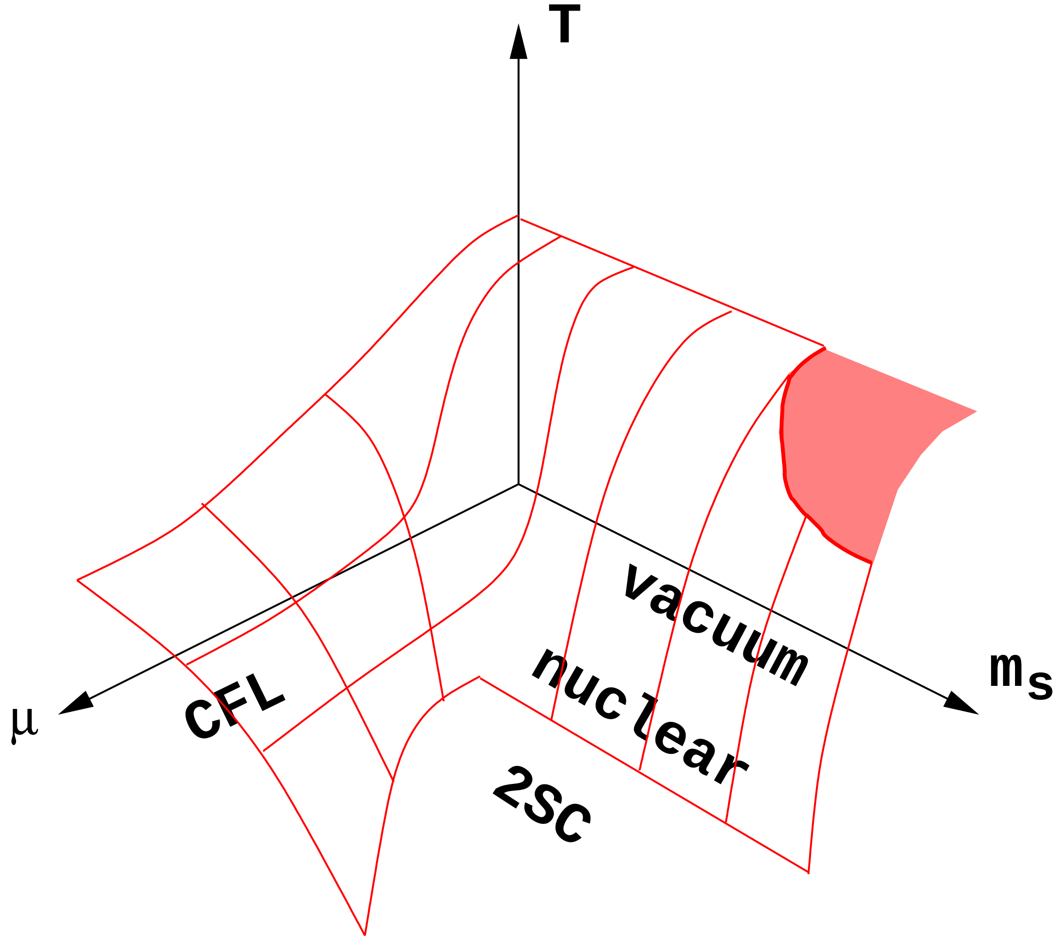
T
$m_s \gg m_s^{cont}$
First order
Second order
Crossover or
first order
QGP
$SU(2)_A$
2SC
$SU(2)_A$
$SU(2)_A$
U(1)
nuclear
CFL
2SC+s
μ


T
$m_s > m_s^{cont}$
$m_s = m_s^c$
First order
Second order
Crossover or
first order
QGP
SU(2)A
nuclear
SU(2)A
SU(2)A
U(1)
2SC
CFL
2SC+s
μ


T
$m_s = m_s^{cont}$
First order
Second order
Crossover or
first order
QGP
SU(2)A
SU(2)A
U(1)
SU(2)A
2SC
nuclear
CFL
μ


T
m_s small
First order
Second order
Crossover or
first order
$SU(2)_A$
QGP
$SU(2)_A$
$SU(2)_A$
$U(1)$
nuclear
2SC
CFL
μ


T
$m_s = 0$
First order
Second order
Crossover or
first order
QGP
$SU(2)_A$
$SU(2)_A$
$U(1)$
nuclear
$U(1)$
hyper-
nuclear
CFL
μ


T
μ
m_s
CFL
vacuum
nuclear
2SC

$\mathbf{T \neq 0}$

No comprehensive NJL study. From BCS the vanishing (through a second order phase transition) of quark pairing is expected at $T_c \approx 0.6\Delta$. Main features:

- The second order phase transition line (present at low density) shrinks as $m_s \to m_s^c$

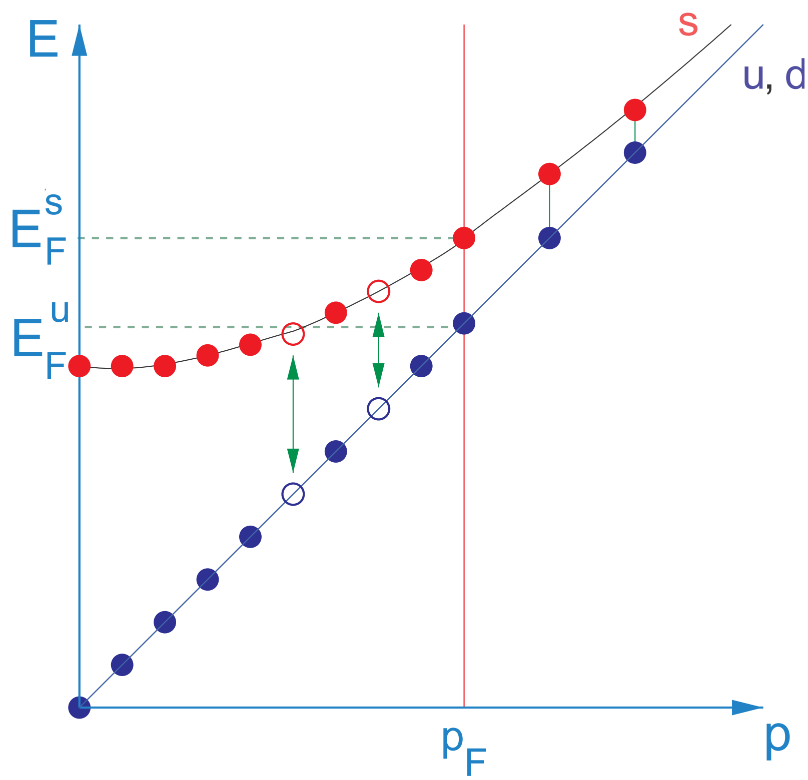
- The $U(1)_S$ and the χ-symmetry breaking lines do not coincide. At low T strange quarks gap breaking $U(1)_s$

- For $\mu \to \infty$ the CFL phase is always present

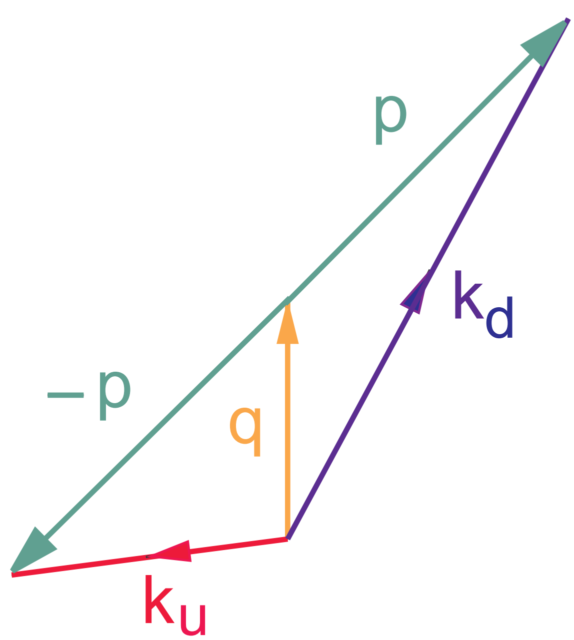
The transition between CFL and 2SC is first order. In the CFL phase the quark s is paired to u, whereas in 2SC is unpaired $\rightarrowtail$ $\Delta_{us} \to 0$ at the transition. But $s \to u$ by weak transition, unless the two quarks are paired. For the pair to be stable, the free energy gained in the decay must be smaller than the energy lost breaking the pair, therefore Δ_{us} cannot go to zero in a continuous way

$$\sqrt{\mu^2 + M_s^2(\mu)} - \sqrt{\mu^2 + M_u^2(\mu)}$$

$$\simeq \frac{M_s(\mu)^2 - M_u(\mu)^2}{2\mu} \simeq \frac{M_s(\mu)^2}{2\mu} \lesssim 2\Delta_{us}$$



- At the edge of s decoupling from light quarks, a further phase could exist the **LOFF phase** (Larkin, Ovchinnikov, 1964; Fulde, Ferrel, 1964). Here the pairing is between quarks with total momentum different from zero. **Rotational and translational invariance are broken**. The gap has a periodicity with a crystalline pattern



- To explore the phase diagram one can vary μ at different machines:

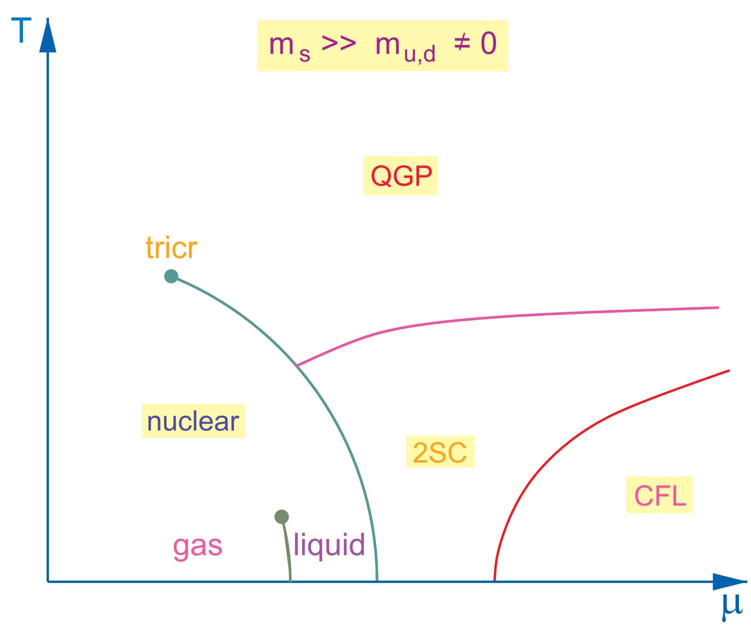
  AGS, $\sqrt{s} = 5\ AGeV,\ \mu \approx 500 \div 600\ MeV$

  SPS, $\sqrt{s} = 17\ AGeV,\ \mu \approx 200 \div 300\ MeV$

  RHIC, $\sqrt{s} = 56, 130, 200\ AGeV,\ \mu \lesssim 100\ MeV$

  Also, study of the quark gluon plasma at $T > T_{\text{transition}}$

- If $m_s^c \approx m_s/2$, the tricritical point is in the actual QCD phase diagram. However μ_{tric} is very hard to be predicted. At $m_s = 0$ there are indication of $\mu_{\text{tric}} \approx 600 \div 800\ MeV$ (within SPS each), but it could be $300 \div 400\ MeV$ as well



- To get signature of the tricritical point one looks at thermodynamical parameters which supposedly have large fluctuations near the singularity. One can change control parameters as ion size, centrality and rapidity selections. Also dialing $\sqrt{s}$ it is possible to vary μ

- Event by event fluctuations will be enhanced. The actual distribution of mean $p_\perp$ at NA49 is gaussian reflecting the thermodynamics character. This could be a good signature of critical dynamics as well as the multiplicity of soft pions. There are some indications that SPS could be exploring the crossover to the left of the tricritical point.

Conclusions

★ The recent investigations of QCD at large density and zero temperature let to a very rich picture of the phase diagram of QCD

★ Most of the features are not on a very firm theoretical basis, but there are strong suggestions that the qualitative picture might be of the type we have indicated. Of course many more theoretical studies will be necessary to assess this matter in a more quantitative way

★ A big problem is how to investigate the phase diagram from an experimental point of view. As we have seen, there are various suggestions to explore the area of small densities at existent and future accelerators. Informations about the area of small temperature and high density might be obtained through accurate analysis of compact stellar objects as neutron stars

Astrophysical consequences of color superconductivity

- **Equation of state**. Corrections are negligible, order $(\Delta/\mu)^2$

- **Cooling by neutrino emission**. There are practically no effects in the CFL case, whereas 2SC could affect the initial cooling

- **Supernova neutrinos**. Bursts of neutrinos from future supernova might suggest a transition to color superconductivity

- **r-mode instabilities**. Neutron star with large angular momentum, Ω, may become unstable to growth of mode oscillations radiating angular momentum via gravitational waves, reducing Ω

- **Magnetic field evolution**. Color superconductivity does not entail normal superconductivity due to the conservation of a rotated electric charge, preventing the Meissner effect

- **Glitches**. Glitches are sudden jumps in the Ω of a pulsar ($\Delta\Omega/\Omega \approx 10^{-6}$). The glitches are connected to rotational vortices in a neutron superfluid pinned up to the crystal structure of the crust. Vortices move outward rearranging the crust and producing glitches. This could happen in the LOFF phase