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Determinant Representation for the Time Dependent Correlation Functions in the XX0 Heisenberg Chain

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Abstract

Time dependent correlation functions in the Heisenberg XX0 chain in the external transverse magnetic field are calculated. For a finite chain normalized mean values of local spin products are represented as determinants of $N \times N$ matrices, N being the number of quasiparticles in the corresponding eigenstate of the Hamiltonian. In the thermodynamical limit (infinitely long chain), correlation functions are expressed in terms of Fredholm determinants of linear integral operators.

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1 Introduction

Recently essential developments have been made in the theory of quantum correlation functions showing that correlators of quantum exactly solvable models satisfy classical completely integrable differential equations [1]-[6] (for the case of the nonrelativistic Bose gas, this program is fulfilled, see, for instance, [7]). In order to obtain these differential equations, an important preliminary step consists in representing correlation functions as the determinants of Fredholm linear integral operators. For the nonrelativistic Bose gas these representations were given both for the time independent case [8, 9] and for the time dependent one [10].

In this paper determinant representations of this kind are obtained for the distance, time and temperature dependent two-point correlation functions of the XX0 Heisenberg chain, both for the finite lattice and in the thermodynamical limit. In order to write differential equations and to calculate their asymptotics, our further plan is to construct and solve a matrix Riemann problem, similarly to the case of the nonrelativistic Bose gas [5], [11]-[13] (see also Ref. [7]).

The XX0 chain is the isotropic case of the XY model [14], being also the free fermions point for the XXZ chain. The Hamiltonian describing the nearest neighbour interaction of local spins $\frac{1}{2}$ situated at the sites of the one-dimensional lattice in a constant transverse magnetic field h is

$$H(h) = -\sum_{m=1}^{M} \left[\sigma_x^{(m)}\sigma_x^{(m+1)} + \sigma_y^{(m)}\sigma_y^{(m+1)} + h\sigma_z^{(m)}\right] . \tag{1.1}$$

The total number M of sites is supposed to be even and periodical boundary conditions, $\sigma_s^{(M+1)} = \sigma_s^{(1)}$ $(s = x, y, z)$, are imposed. Pauli matrices are normalized as $\left(\sigma_s^{(m)}\right)^2 = 1$.

The ferromagnetic state $|\, 0\rangle \equiv \otimes_{m=1}^{M} |\uparrow\rangle_m$ (all spins up) is an eigenstate of the Hamiltonian. All the other $2^M - 1$ eigenstates can be constructed by filling this ferromagnetic state with N quasiparticles $(N = 1, 2, ..., M)$ possessing quasimomenta p_a $(a = 1, 2, ..., N)$ and energies $\varepsilon(p_a)$,

$$\varepsilon(p) \equiv \varepsilon(p, h) = -4\cos p + 2h . \tag{1.2}$$

Due to periodical boundary conditions, one has the following condition for the permitted values of quasimomenta

$$e^{iMp_a} = (-1)^{N+1} , \qquad a = 1, ..., N . \tag{1.3}$$

All the momenta of the quasiparticles in a given eigenstate should be different, so that, *e.g.*, for $N = M$, one gets in fact only one eigenstate (which is just the other ferromagnetic state with all spins down, $| \, 0'\rangle = \otimes_{m=1}^{M} |\downarrow\rangle_m$).

Due to the similarity transformation,

$$\begin{aligned} &H(h) \;\to\; H(-h) = UH(h)U^{-1}; \qquad U = \prod_{m=1}^{M} \sigma_x^{(m)} , \\ &| \, 0\rangle \;\to | \, 0'\rangle = U \, | \, 0\rangle , \end{aligned} \tag{1.4}$$

it is sufficient to consider only nonnegative magnetic fields, $h \geq 0$. Furthermore the choice of the minus sign at the r.h.s. of eq. (1.1) is just a matter of convenience due to the property

$$H(h) \;\to\; -H(-h) = VH(h)V^{-1}; \qquad V = \prod_{m=1}^{M/2} \sigma_z^{(2m)} .$$

The model in the thermodynamic limit ($M \to \infty$, h fixed) is the most interesting. For $h \geq h_c \equiv 2$, the ground state of the Hamiltonian is just the ferromagnetic state $| \, 0\rangle$. For magnetic field smaller than the critical value, $0 \leq h < h_c$, the ground state $| \, \Omega\rangle$ is obtained by filling the ferromagnetic state with quasiparticles possessing all the allowed values of momenta inside the Fermi zone, $-k_F \leq p_a \leq k_F$, where the Fermi momentum k_F is defined by the requirement $\varepsilon(k_F) = 0$:

$$k_F = \arccos\left(\frac{h}{2}\right) \qquad 0 \leq h < h_c . \tag{1.5}$$

In the thermodynamical limit ($M \to \infty$) the number N_0 of quasiparticles in the ground state is going to infinity, $N_0 \to \infty$, "density" $D \equiv N_0/M$ remaining fixed. At zero magnetic field $k_F = \frac{\pi}{2}$, and there are $\frac{M}{2}$ quasiparticles in the ground state, magnetization being equal to zero.

At non zero temperatures $T > 0$, the distribution of quasiparticles in the momentum space is $\vartheta(p)/2\pi$ where $\vartheta(p)$ is the Fermi weight

$$\vartheta(p) \equiv \vartheta(p, h, T) = \frac{1}{1 + \exp\left[\frac{\varepsilon(p)}{T}\right]} . \tag{1.6}$$

Temperature and time dependent correlators of local spins $\sigma_s^{(m)}(t)$,

$$\begin{aligned} \sigma_s^{(m)}(t) &\equiv e^{iHt}\sigma_s^{(m)}e^{-iHt} , \\ \sigma_s^{(m)} &\equiv \sigma_s^{(m)}(0) , \qquad\qquad s = x, y, z, \end{aligned} \tag{1.7}$$

are defined as usual,

$$\begin{aligned} g_{sr}^{(T)}(m,t) &\equiv \langle \sigma_s^{(n_2)}(t_2)\sigma_r^{(n_1)}(t_1)\rangle_T = \\ &= \frac{\mathrm{Sp}\left\{\exp[-H/T]\sigma_s^{(n_2)}(t_2)\sigma_r^{(n_1)}(t_1)\right\}}{\mathrm{Sp}\left\{\exp[-H/T]\right\}} . \end{aligned} \tag{1.8}$$

Due to translation invariance, the correlators depend only on differences,

$$m = n_2 - n_1, \qquad\qquad t = t_2 - t_1 . \tag{1.9}$$

At zero temperature, only the ground state $\mid \Omega\rangle$ contributes to the traces in (1.8):

$$g_{sr}^{(0)}(m,t) \equiv \frac{\langle \Omega \mid \sigma_s^{(n_2)}(t_2)\sigma_r^{(n_1)}(t_1) \mid \Omega\rangle}{\langle \Omega \mid \Omega\rangle} . \tag{1.10}$$

In Ref. [14] the equal-time correlators ($t = 0$) of XY model (of which the XX0 model is a particular case) were calculated at zero magnetic field ($h = 0$). The simple answer for the correlator $g_{zz}^{(T)}(m, 0)$ of the third spin components was given; for the XX0 chain it reduces in essential to the square modulus of the Fourier transform of the Fermi weight. This result was generalized to the case of the time-dependent correlator with nonzero transverse magnetic field [15]. Properties of correlator $g_{zz}^{(T)}$ were considered in much detail [14]-[17]. Real systems for experimental comparisons were found [18].

On the contrary, correlators of the other local spin components are much more complicated. In Ref. [14] these correlators (for the XY model at $t = 0$, $h = 0$) were represented as the determinants of $m \times m$ matrices (m is the distance between correlating spins). This representation was investigated in detail in [16] (see also [19]). In Ref. [20] the structure of the time-dependent correlators was investigated on the basis of an extension of the thermodynamic Wick theorem. In Ref. [2], a representation (different from the one obtained below in this particular case) of the autocorrelator ($m = 0$, $t \neq 0$) in the transverse Ising chain in critical magnetic field (closely related to correlators in the XX0 model at $h = 0$) were given as the Fredholm determinant of a linear integral operator. This representation was used in [6] to produce differential equations for the autocorrelator.

In the present paper the time dependent correlators (see (1.8), (1.9) for notations; $\sigma_\pm^{(m)} \equiv \frac{1}{2}[\sigma_x^{(m)} \pm i\sigma_y^{(m)}]$)

$$g_+^{(T)}(m,t) \equiv \langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_T \,, \tag{1.11}$$

$$g_-^{(T)}(m,t) \equiv \langle \sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_T \,, \tag{1.12}$$

for the XX0 model in a transverse magnetic field are given (in the thermodynamical limit) as Fredholm determinants of linear integral operators. These representations, different from those of paper [14], are similar to the representations of two-point correlators previously obtained for the one-dimensional impenetrable Bose gas (see [8, 9] in the equal-time case and [10] for time dependent correlators). In Ref. [21], the Fredholm determinant representation for the time-independent generating functional of currents in the sine-Gordon model at the free-fermion point was obtained. Representations of this kind proved to be extremely useful in obtaining the integrable differential equations for correlation functions in the case of the impenetrable Bose gas (the V Painlevé transcendent in the equal time zero temperature case [1] and integrable partial differential equations for time and temperature dependent correlation functions [4, 5]). In turn, this fact allowed to construct exact asymptotics for the correlators [1, 12, 13]. Corresponding results are expected to be obtained also for the XX0 chain.

The further contents of this paper is as follows. In Section 2 the detailed description of the XX0 model on the finite lattice is given. In Section 3 the mean value of the generating functional of the equal-time third spin components correlators with respect to any eigenfunction of the Hamiltonian is calculated. It is represented as the determinant of a $N \times N$ matrix, N being the number of quasiparticles in the corresponding eigenstate. The mean value of the time-dependent product of two third spin components on a finite lattice is calculated in Section 4. In Section 5, form factors of operators $\sigma_\pm^{(m)}(t)$ (*i.e.* their matrix elements between eigenstates of the Hamiltonian) are represented as determinants of $N \times N$ matrices. The representation of the normalized mean values of products $\sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)$, $\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)$, with respect to any eigenstate of the Hamiltonian containing N quasiparticles, as the determinants of $N \times N$ matrices (for finite M) are given in Section 6. In Section 7, these representations are proved. The answers for the correlators in the thermodynamical limit ($M \to \infty$, h fixed) are given in Section 8 (for zero temperature) and in Section 9 (for non zero temperature). Some details in performing the thermodynamical limit are considered in the Appendix.

2 The Model on the Finite Lattice

The Hamiltonian describing the XX0 model in transverse magnetic field was given in (1.1):

$$H(h) = H_0 - 2hS_z \, . \tag{2.1}$$

Here H_0 describes the nearest neighbour interaction of spins $\frac{1}{2}$ situated at the sites of the lattice,

$$H_0 = -\sum_{m=1}^{M} [\sigma_x^{(m)}\sigma_x^{(m+1)} + \sigma_y^{(m)}\sigma_y^{(m+1)}] \, , \tag{2.2}$$

and S_z is the third spin component of the total spin,

$$S_z = \frac{1}{2}\sum_{m=1}^{M} \sigma_z^{(m)} \, . \tag{2.3}$$

The space $\mathcal{H}$ where these operators act is a tensor product of local spaces, $\mathcal{H} = \otimes_{m=1}^{M} \mathcal{H}_{(m)}$, $\mathcal{H}_{(m)}$ being the linear space $\mathbf{C}^2$ corresponding to the m^{th} local spin spanned by the basis vectors $|\uparrow\rangle_m$ (spin up), $|\downarrow\rangle_m$ (spin down)

$$\begin{aligned} &\sigma_z^{(m)} \mid\uparrow\rangle_m = \mid\uparrow\rangle_m \, ; \qquad &&\sigma_z^{(m)} \mid\downarrow\rangle_m = \mid\downarrow\rangle_m \, ; \\ &{}_m\langle\uparrow|\uparrow\rangle_m = {}_m\langle\downarrow|\downarrow\rangle_m = 1 \, ; \qquad &&{}_m\langle\uparrow|\downarrow\rangle_m = 0 \, . \end{aligned} \tag{2.4}$$

Local spin operators $\sigma_p^{(m)}$ ($p = x, y, z$) with commutation relations

$$\left[\sigma_p^{(m)}, \sigma_q^{(n)}\right] = 2i\delta_{mn}\epsilon_{pqr}\sigma_r^{(m)} \tag{2.5}$$

are Pauli matrices acting non trivially in $\mathcal{H}_{(m)}$. The complete set of mutual eigenstates of $H(h)$ (H_0) and S_z is obtained by applying lowering operators $\sigma_-^{(m)}$ (as usual, $\sigma_\pm^{(m)} = \frac{1}{2}\left[\sigma_x^{(m)} \pm i\sigma_y^{(m)}\right]$) to the ferromagnetic state $\mid 0\rangle$ (all spins up):

$$\begin{aligned} &\mid 0\rangle = \otimes_{m=1}^{M} \mid\uparrow\rangle_m, \, ; \\ &S_z \mid 0\rangle = \frac{M}{2} \mid 0\rangle \, ; \qquad H \mid 0\rangle = -Mh \mid 0\rangle \, . \end{aligned} \tag{2.6}$$

The explicit form for the eigenfunctions of Hamiltonian (2.1) is well known, being just the simplest case of eigenfunctions of the XXZ model [22] with the two-particle scattering phases equal to zero. For the XX0 model they have the following form:

$$\begin{aligned} \mid \Psi_N(\{p\})\rangle \ &\equiv \ \mid \Psi_N(p_1, ..., p_N)\rangle = \\ &= \ \frac{1}{\sqrt{N!}} \sum_{m_1,...,m_N}^{M} \chi_{_N}(\{m\} \mid \{p\})\, \sigma_-^{(m_1)} ... \sigma_-^{(m_N)} \mid 0\rangle \ , \end{aligned} \tag{2.7}$$

with wave function $\chi_{_N}$ given as

$$\begin{aligned} \chi_{_N}(\{m\} \mid \{p\}) \ &\equiv \ \chi_{_N}(m_1, ..., m_N \mid p_1, ..., p_N) = \\ &= \ \frac{1}{\sqrt{N!}} \left[\prod_{1\le a<b\le N} \epsilon(m_b - m_a) \right] \cdot \sum_Q (-1)^{[Q]} \exp\left[i \sum_{a=1}^{N} m_a p_{Q_a} \right] \ , \end{aligned} \tag{2.8}$$

where $\epsilon(m)$ is the sign-function, defined as

$$\epsilon(m) = \begin{cases} 1, & m > 0 \, ; \\ -1, & m < 0 \, ; \\ 0, & m = 0 \, . \end{cases} \tag{2.9}$$

The sum in (2.8) is taken over all the permutations of N numbers, $Q : \ (1, 2, ..., N) \ \rightarrow (Q_1, Q_2, ..., Q_N)$; $[Q]$ denotes the parity of the permutation.

Due to the periodical boundary conditions, quasimomenta p_a $(-\pi < p_a \le \pi)$ satisfy equations

$$\exp\left[i p_a M \right] = (-1)^{N+1} \ , \qquad a = 1, ..., N \ , \tag{2.10}$$

i.e., the permitted values for the momenta are

$$\begin{aligned} p_a &= \frac{2\pi}{M} n_a \ , \\ n_a &= -\tfrac{M}{2} + j \ , \qquad j = 1, 2, ..., M \qquad \text{for } N \text{ odd}, \\ n_a &= -\tfrac{M+1}{2} + j \ , \qquad j = 1, 2, ..., M \qquad \text{for } N \text{ even}. \end{aligned} \tag{2.11}$$

The sum over the permutations in (2.8) is just the Slater determinant. Hence wave function $\chi_{_N}$ is symmetric in "coordinates" m_a and antisymmetric in quasimomenta p_a, being equal to zero if some of the coordinates or momenta coincide. In particular, all the momenta p_a $(a = 1, ..., N)$ of the quasiparticles in the eigenstate $\mid \Psi_{_N}(\{p\})\rangle$ should be different, otherwise the wave function is identically equal to zero ("Pauli principle").

E.g., for $N = M$ only one eigenstate $\mid \Psi_M\rangle \equiv \mid 0'\rangle = \otimes_{m=1}^{M} \mid\downarrow\rangle_m$ does exist, which is just the other ferromagnetic state with all spins down.

Eigenvalues of operators $H(h)$ and S_z for eigenstate $\mid \Psi_{_N}(\{p\})\rangle$ are

$$
\begin{aligned}
H \mid \Psi_N(\{p\})\rangle &= \left(\sum_{a=1}^{N} \varepsilon(p_a)\right) \mid \Psi_N(\{p\})\rangle \; , \\
S_z \mid \Psi_N(\{p\})\rangle &= \left(\frac{M}{2} - N\right) \mid \Psi_N(\{p\})\rangle \; ,
\end{aligned}
\tag{2.12}
$$

with the one particle energy ("dispersion law") given as

$$
\varepsilon(p) = -4\cos p + 2h \; . \tag{2.13}
$$

Eigenvectors (2.7) with different numbers of quasiparticles are orthogonal,

$$
\langle \Psi_{N_1} \mid \Psi_{N_2}\rangle = 0 \; , \qquad N_1 \neq N_2 \; ,
$$

as well as eigenvectors with the same number of particles, but different sets of momenta:

$$
\langle \Psi_N(\{p\}) \mid \Psi_N(\{p\})\rangle = 0 \; , \qquad \{p\} \neq \{p'\} \; ;
$$

(more exactly, this is valid if the set $\{p'\}$ cannot be obtained from the set $\{p\}$ by means of permutations of quasimomenta p_a). The normalization is given as

$$
\begin{aligned}
\langle \Psi_N(\{p\}) \quad &| \quad \Psi_N(\{p'\})\rangle = \\
&= \sum_{m_1,\ldots,m_N=1}^{M} \chi_N^*(m_1,\ldots,m_N \mid p_1,\ldots,p_N)\, \chi_N(m_1,\ldots,m_N \mid p_1,\ldots,p_N) = \\
&= M^N \; .
\end{aligned}
\tag{2.14}
$$

Let us discuss the dependence on the external magnetic field h. We consider the model only in the case $h \geq 0$ (which is sufficient, due to property (1.4)). For strong magnetic fields, $h \geq h_c \equiv 2$, the ground state of the Hamiltonian is just the ferromagnetic state $\mid 0\rangle$, with normalized mean value $\langle \sigma_z^{(m)}\rangle$ (magnetization) equal to one. For magnetic field smaller than the critical value, $0 \leq h < h_c$, the ferromagnetic state is not the ground state. The ground state $\mid \Omega\rangle$ in this case is constructed by filling the ferromagnetic state with quasiparticles occupying all the permitted vacancies, see (2.11), in the Fermi zone, $-k_F \leq p_a \leq k_F$, where $k_F = \arccos(h/2)$ is the Fermi momentum (1.5).

Magnetization $\sigma_{_N}$ in the arbitrary eigenstate $\mid \Psi_{_N}(\{p\})\rangle$ is easily computed to be

$$\sigma_{_N} \equiv \langle \sigma_z^{(m)} \rangle_{_N} = \frac{2}{M} \langle S_z \rangle_{_N} = \frac{\langle \Psi_N(\{p\}) \mid \sigma_z^{(m)} \mid \Psi_N(\{p\})\rangle}{\langle \Psi_N(\{p\}) \mid \Psi_N(\{p\})\rangle} = 1 - \frac{2N}{M} . \tag{2.15}$$

Let us conclude this Section by discussing the correspondence between the XX0 model and free fermions on a one-dimensional lattice [14]. Introducing operator $\hat{q}_n$ of the number of quasiparticles at the n^{th} site of the lattice,

$$\hat{q}_n = \frac{1}{2}\left(1 - \sigma_z^{(n)}\right) = \sigma_-^{(n)} \sigma_+^{(n)} , \qquad \exp\left[2\pi i \hat{q}_n\right] = 1 , \tag{2.16}$$

and the operator $Q(m)$ of the number of quasiparticles in the first m sites,

$$Q(m) \equiv \sum_{n=1}^{m} \hat{q}_n ; \qquad \exp\left[2\pi i Q(m)\right] = 1 , \tag{2.17}$$

one constructs fermionic fields $\psi(m)$, $\psi^\dagger(m)$ through the Jordan-Wigner transformation [23]:

$$\begin{aligned} \psi(m) &= \exp\left[i\pi Q(m)\right] \; \sigma_+^{(m)} , \\ \psi^\dagger(m) &= \sigma_-^{(m)} \; \exp\left[i\pi Q(m)\right] , \end{aligned} \tag{2.18}$$

with the following anticommutation relations:

$$\begin{aligned} \left\{\psi(m), \psi^\dagger(n)\right\} &\equiv \psi(m)\psi^\dagger(n) + \psi^\dagger(n)\psi(m) \; = \; \delta_{mn} , \\ \left\{\psi(m), \psi(n)\right\} &= \left\{\psi^\dagger(m), \psi^\dagger(n)\right\} \; = \; 0 . \end{aligned}$$

Operators $\hat{q}_n$, $Q(m)$ and $\sigma_\pm^{(m)}$ may be rewritten in terms of the fermionic fields as

$$\hat{q}_n = \psi^\dagger(n)\psi(n) ; \qquad Q(m) = \sum_{n=1}^{m} \psi^\dagger(n)\psi(n) , \tag{2.19}$$

and

$$\begin{aligned} \sigma_+^{(m)} &= \exp\left[i\pi Q(m)\right] \; \psi(m) , \\ \sigma_-^{(m)} &= \psi^\dagger(m) \; \exp\left[i\pi Q(m)\right] . \end{aligned} \tag{2.20}$$

It is therefore possible to express the Hamiltonian H_0 (2.2) and the total spin S_z (2.3) in terms of the fermionic fields:

$$\begin{aligned} H_0 &= -2 \sum_{m=1}^{M} \left[\psi^\dagger(m+1)\psi(m) + \psi^\dagger(m)\psi(m+1)\right] , \\ S_z &= \frac{M}{2} - \sum_{m=1}^{M} \psi^\dagger(m)\psi(m) , \end{aligned} \tag{2.21}$$

which indeed describe free fermions on the lattice (one should also take into account that periodical boundary conditions in the XX0 model generate "a-cyclic" boundary conditions for free fermions [14]). Hence the problem of computing correlation functions of local operators, (as *e.g.* $\langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle\rangle$ in the XX0 model is equivalent to calculating correlators of "tailed" ("disordered") operators (2.20) for free fermions.

In [14], the correspondence with free fermions was exploited to compute correlators. Our calculations below are done directly in the frame of the XX0 model itself.

3 Normalized Mean Value of Operator $\exp[\alpha Q(m)]$ on the Finite Lattice

Let us consider the normalized mean value

$$\langle \exp[\alpha Q(m)]\rangle_N \equiv \frac{\langle \Psi_N(\{p\}) \mid \exp[\alpha Q(m)] \mid \Psi_N(\{p\})\rangle}{\langle \Psi_N(\{p\}) \mid \Psi_N(\{p\})\rangle} . \tag{3.1}$$

in the XX0 model on the finite periodical lattice with M sites. Here $Q(m)$ (2.17) is the operator of the number of quasiparticles in the first m sites of the lattice; α is a complex parameter. The mean value (3.1) is taken with respect to some eigenstate $\mid \Psi_N(\{p\})\rangle$ (2.7) of Hamiltonian (2.1). As explained in the end of this Section, the quantity $\exp[\alpha Q(m)]$ defined in (3.1) generates equal-time mean values of products of operators $\sigma_z^{(m)}$; so it is called "generating functional" for these mean values. The value of generating functional (3.1) itself at $\alpha = -\infty$ has a clear physical meaning giving the probability that there are no quasiparticles in the first m sites of the lattice in the eigenstate $\mid \Psi_N(\{p\})\rangle$.

The generating functional can be represented in the following explicit form:

$$\langle \exp[\alpha Q(m)]\rangle_N = \det_N \mathcal{M}(m) . \tag{3.2}$$

Here the r.h.s. is the determinant of the $N \times N$ matrix $\mathcal{M} \equiv \mathcal{M}(m)$ with matrix elements given by

$$(\mathcal{M})_{ab} = \delta_{ab}\left(1 + \frac{e^\alpha - 1}{M} m\right) + (1 - \delta_{ab})\frac{e^\alpha - 1}{M} \cdot \frac{\sin\frac{m}{2}(p_a - p_b)}{\sin\frac{1}{2}(p_a - p_b)} , \qquad a, b = 1, 2, ..., N , \tag{3.3}$$

where $\{p\} = \{p_1, ..., p_N\}$, $p_a = \frac{2\pi}{M} n_a$ is the set of quasimomenta (2.11) defining eigenstate $\mid \Psi(\{p\})\rangle$.

Let us explain how this expression for the generating functional is obtained. First, using commutation relation

$$
\begin{aligned}
&\exp\left[\alpha Q(m)\right]\ \sigma_-^{(n)} = \varphi(n,m)\sigma_-^{(n)}\ \exp\left[\alpha Q(m)\right]\ , \\
&\varphi(n,m) = \begin{cases} 1, & n > m, \\ e^{\alpha}, & n \le m, \end{cases}
\end{aligned}
\tag{3.4}
$$

and also equations (2.7), (2.4), as well as commutation relations (2.5) between the Pauli matrices, one gets

$$
\begin{aligned}
\langle \exp\left[\alpha Q(m)\right] \rangle_{N} &= \frac{1}{M^N} \sum_{m_1,...,m_N}^{M} \chi^*(m_1, ..., m_N \mid p_1, ..., p_N)\, \chi(m_1, ..., m_N \mid p_1, ..., p_N) \cdot \\
&\cdot \varphi(m_1, m)\ \varphi(m_2, m)\ ...\ \varphi(m_N, m)\ .
\end{aligned}
\tag{3.5}
$$

Now one uses equation (2.8) for χ_{N}, and performs explicitly the summations over $m_1, ..., m_N$ by means of formula

$$
\sum_{n=1}^{M} e^{in(p_a - p_b)} \varphi(n,m) = (M + (e^{\alpha} - 1)m)\, \delta_{ab} + (1 - \delta_{ab})(e^{\alpha} - 1) \frac{e^{im(p_a - p_b)} - 1}{1 - e^{-i(p_a - p_b)}}
$$

with the result

$$
\langle \exp\left[\alpha Q(m)\right] \rangle_{N} = \frac{1}{N!} \sum_{Q,Q'} (-1)^{[Q]+[Q']} \prod_{a=1}^{N} \left(\tilde{\mathcal{M}}\right)_{Q_a Q'_a} = \det{}_{N} \tilde{\mathcal{M}}\ , \tag{3.6}
$$

where the sum is taken over permutations Q, Q' of N numbers. Matrix elements $(\tilde{\mathcal{M}})_{ab}$ are

$$
\begin{aligned}
\left(\tilde{\mathcal{M}}\right)_{ab} &= \delta_{ab} \left(1 + \frac{e^{\alpha} - 1}{M} m\right) + \\
&+ (1 - \delta_{ab}) \frac{e^{\alpha} - 1}{M} \cdot \frac{\exp\left[im(p_a - p_b)\right] - 1}{1 - \exp\left[-i(p_a - p_b)\right]}\ , \\
&\qquad a, b = 1, ..., N.
\end{aligned}
$$

Now it is easy to see that the determinants of matrices $\tilde{\mathcal{M}}$ and $\mathcal{M}$ (3.3) are equal, the matrices being similar. So one comes to representation (3.2) for the generating functional.

Normalized mean values for operators $Q(m)$ and $\hat{q}_m$ can be derived from the generating functional (3.1) as follows:

$$
\begin{aligned}
\langle Q(m) \rangle_{N} &= \frac{\partial}{\partial\alpha} \langle \exp\left[\alpha Q(m)\right] \rangle_{N} \Big|_{\alpha=0} = \frac{Nm}{M} \; ; \\
\langle \hat{q}_m \rangle_{N} &= \mathcal{D}_1 \langle Q(m) \rangle_{N} = \frac{N}{M} \; ,
\end{aligned}
\tag{3.7}
$$

where $\mathcal{D}_1$ is the "first lattice derivative" acting on functions $f(m)$ as $(\mathcal{D}_1 f)(m) \equiv f(m) - f(m-1)$. Relation (2.15) is obviously reproduced for magnetization:

$$
\sigma_{N} \equiv \langle \sigma_z^{(m)} \rangle_{N} = 1 - 2\langle \hat{q}_m \rangle_{N} = 1 - \frac{2N}{M} \; . \tag{3.8}
$$

For the mean value of operator $Q^2(m)$ we readily get

$$
\begin{aligned}
\langle Q^2(m) \rangle_{N} &= \frac{\partial^2}{\partial\alpha^2} \langle \exp\left[\alpha Q(m)\right] \rangle_{N} \big|_{\alpha=0} = \\
&= \frac{N(N-1)}{M^2} m^2 + \frac{Nm}{M} - \frac{1}{M^2} \sum_{\substack{a,b=1 \\ a \neq b}}^{N} \frac{\sin^2 \frac{m}{2}(p_a - p_b)}{\sin^2 \frac{1}{2}(p_a - p_b)} \; .
\end{aligned}
\tag{3.9}
$$

Using translation invariance one expresses normalized mean values $\langle \hat{q}_{m+1} \hat{q}_1 \rangle_{N} = \langle \hat{q}_{n_2} \hat{q}_{n_1} \rangle_{N}$ $(m \equiv n_2 - n_1)$ as

$$
\begin{aligned}
\langle \hat{q}_{n_2} \hat{q}_{n_1} \rangle_{N} &= \frac{1}{2} \mathcal{D}_2 \langle Q^2(m) \rangle_{N} \; , \qquad m \geq 2 \; , \\
\langle \hat{q}_{n+1} \hat{q}_{n} \rangle_{N} &= \frac{1}{2} \langle Q^2(m=2) \rangle_{N} - \langle \hat{q}_n \rangle_{N} \; , \\
\langle \hat{q}_n^2 \rangle_{N} &= \langle \hat{q}_n \rangle_{N} \; ,
\end{aligned}
$$

where $\mathcal{D}_2$ is the second derivative on the lattice acting on $f(m)$ as $(\mathcal{D}_2 f)(m) = f(m+1) + f(m-1) - 2f(m)$. Using relation (2.16), $\hat{q}_m = \frac{1}{2}(1 - \sigma_z^{(m)})$, one obtains for the normalized mean value of the third spin components from (3.9):

$$
\langle \sigma_z^{(n_2)} \sigma_z^{(n_1)} \rangle_{N} = \sigma_{N}^2 - \frac{4}{M^2} \left| \sum_{a=1}^{N} e^{imp_a} \right|^2 \; , \qquad m \equiv n_2 - n_1 \neq 0 \; . \tag{3.10}
$$

4 Time-Dependent Normalized Mean Value of Operator $\sigma_z \; \sigma_z$ on the Finite Lattice

We consider here the simplest time-dependent two-point normalized mean value,

$$
\langle \sigma_z^{(n_2)}(t_2) \sigma_z^{(n_1)}(t_1) \rangle_{N} \equiv \frac{\langle \Psi_N(\{p\}) \mid \sigma_z^{(n_2)}(t_2) \sigma_z^{(n_1)}(t_1) \mid \Psi_N(\{p\}) \rangle}{\langle \Psi_N(\{p\}) \mid \Psi_N(\{p\}) \rangle} . \tag{4.1}
$$

Here

$$\begin{aligned}\sigma_z^{(n)}(t) &= \exp\left[iHt\right]\sigma_z^{(n)}(0)\exp\left[-iHt\right] \;, \\ \sigma_z^{(n)}(0) &\equiv \sigma_z^{(n)} \;, \end{aligned} \tag{4.2}$$

is the Heisenberg time-dependent operator of the third spin component at the n^{th} lattice site; $\mid \Psi_{_N}(\{p\})\rangle$ is any eigenfunction of the XX0 Hamiltonian (with periodical boundary conditions) parametrized by quasimomenta p_a $(a = 1, ..., N)$, see (2.7)-(2.9). The result of calculating mean value (4.1) is

$$\begin{aligned}\langle\sigma_z^{(n_2)}(t_2)\sigma_z^{(n_1)}(t_1)\rangle_{_N} &= \sigma_{_N}^2 - \frac{4}{M^2}\left|\sum_{a=1}^{N}\exp\left[imp_a - it\varepsilon(p_a)\right]\right|^2 + \\ &+\frac{4}{M^2}\left(\sum_{a=1}^{N}\exp\left[-imp_a + it\varepsilon(p_a)\right]\right)\cdot\left(\sum_{j=1}^{M}\exp\left[imq_j - it\varepsilon(q_j)\right]\right) \;, \\ &m \equiv n_2 - n_1 \;; \qquad t \equiv t_2 - t_1 \;. \end{aligned} \tag{4.3}$$

Here $\sigma_{_N} = 1 - \frac{2N}{M}$ is the magnetization (2.15), and $\varepsilon(p) = -4\cos p + 2h$ is the energy (1.2) of a quasiparticle. The last sum over j in (4.3) is taken over all the allowed values

$$q_j = \frac{2\pi}{M}\left(-\frac{M}{2} - \frac{1}{4}\left[1 + (-1)^N\right] + j\right) \;, \qquad j = 1, ..., M \;, \tag{4.4}$$

of quasimomenta. It should be noted that for $t = 0$, $m \neq 0$ the sum over j in (4.3) is equal to zero, so that the equal-time correlator (3.10) is reproduced. As mentioned in Introduction, the zz correlations were already studied in Ref. [14, 15]. Here we present the derivation of eq. (4.3) directly in the frame of the XX0 model.

Let us explain briefly the derivation of formula (4.3). Inserting in the r.h.s. of eq. (4.1) the complete set of N-quasiparticle normalized eigenstates,

$$\frac{\mid \Psi_{_N}(\{q\})\,\rangle\langle\,\Psi_{_N}(\{q\})\mid}{\langle\Psi_{_N}(\{q\})\mid\Psi_{_N}(\{q\})\rangle} \;, \tag{4.5}$$

(since operators $\sigma_z^{(n)}$ does not change the number of quasiparticles, only states with N quasiparticles do contribute) and taking into account normalization (2.14), one gets

$$\begin{aligned}\langle\sigma_z^{(n_2)}(t_2)\sigma_z^{(n_1)}(t_1)\rangle_{_N} &= \frac{1}{M^{2N}}\sum_{\{q\}} Z_{_N}^*(n_2, \{q\}, \{p\})\, Z_{_N}(n_1, \{q\}, \{p\})\,\cdot \\ &\qquad\cdot\exp\left\{-it\sum_{a=1}^{N}\left[\varepsilon(q_a) - \varepsilon(p_a)\right]\right\} \;, \\ &t \equiv t_2 - t_1. \end{aligned} \tag{4.6}$$

Here $Z_{_N}$ is the time-independent form factor of operator $\sigma_z^{(n)} \equiv \sigma_z^{(n)}(t=0)$:

$$Z_{_N}(n_1, \{q\}, \{p\}) \equiv \langle \Psi_{_N}(\{q\}) \mid \sigma_z^{(n)} \mid \Psi_{_N}(\{p\}) \rangle. \tag{4.7}$$

The sum in (4.6) is taken over all the different eigenstates parametrized by different momenta $\{q\} = q_1, ..., q_{_N}$. State $\mid \Psi_{_N}(\{q\})\rangle$ is antisymmetric under permutations of quasimomenta q_a's, and states $\mid \Psi_{_N}(\{q\})\rangle$ and $\mid \Psi_{_N}(\{q'\})\rangle$ differ at most in the sign if set $\{q'\}$ can be obtained from set $\{q\}$ by permutations of momenta q. Such $N!$ states are essentially the same, and only one of them (anyone, due to the symmetry in q's of expression (4.5)) should be inserted as intermediate state.

Using commutation relations (2.5) between Pauli matrices, the symmetry in arguments m of wave function $\chi_{_N}$ (2.8), the fact that $\chi_{_N} = 0$ if any two (or more) m's coincide, and relation $\sigma_z^{(n)} \mid 0\rangle = \mid 0\rangle$, one calculates for the form factor substituting expressions (2.7) for the eigenstates:

$$\begin{aligned} Z_{_N}(n, \{q\}, \{p\}) &\equiv \langle \Psi_N(\{q\}) \mid \sigma_z^{(n)} \mid \Psi_N(\{p\}) \rangle = \\ &= \sum_{m_1, ..., m_N = 1}^{M} \chi^*_{_N}(m_1, ..., m_N; \{q\})\, \chi_{_N}(m_1, ..., m_N; \{p\}) \\ &\quad -2N \sum_{m_1, ..., m_{N-1} = 1}^{M} \chi^*_{_N}(m_1, ..., m_{N-1}, n; \{q\})\, \chi_{_N}(m_1, ..., m_{N-1}, n; \{p\}). \end{aligned} \tag{4.8}$$

Using now the explicit form (2.8) of the eigenfunctions, one gets

$$\begin{aligned} Z_{_N}(n, \{q\}, \{p\}) &= \frac{1}{N!} \sum_{m_1, ..., m_N = 1}^{M} \sum_{Q', Q} (-1)^{[Q]+[Q']} \exp\left[-i \sum_{a=1}^{N} m_a (q_{Q'_a} - p_{Q_a})\right] - \\ &\quad - \frac{2N}{N!} \sum_{m_1, ..., m_{N-1} = 1}^{M} \sum_{Q', Q} (-1)^{[Q]+[Q']} \exp\left[-in(q_{Q'_N} - p_{Q_N})\right] \cdot \\ &\quad \cdot \exp\left[-i \sum_{a=1}^{N-1} m_a (q_{Q'_a} - p_{Q_a})\right]. \end{aligned} \tag{4.9}$$

It is to be mentioned that though, *e.g.*, $\epsilon^2(m_a - m_b) = 1 - \delta_{m_a, m_b} \neq 1$, terms proportional to δ_{m_a, m_b} do not contribute, since the sums over permutations in (4.9) are equal to zero at $m_a = m_b$, not depending on the value $\epsilon(0)$. The sums over m_a may be performed explicitly,

$$\sum_{m=1}^{M} e^{-im(q_a - p_b)} = \begin{cases} M, & q_a = p_b, \\ 0, & q_a \neq p_b, \end{cases} \tag{4.10}$$

(periodical boundary conditions (2.10), $e^{iMq_a} = e^{iMp_b} = (-1)^{N+1}$, should be taken into account). It is therefore evident that the r.h.s. of (4.9) does not vanish only if sets $\{q\}$ and $\{p\}$ differs at most in one momentum. In such cases, we obtain

$$Z_{_N}(n; p_1, ..., p_{_N}; p_1, ..., p_{_N}) = M^N \left(1 - \frac{2N}{M}\right) , \qquad (4.11)$$

$$Z_{_N}(n; p_1, ..., p_{_{N-1}}, q; p_1, ..., p_{_{N-1}}, p) = -2M^{N-1} e^{-in(q-p)} , \qquad q \neq p . \quad (4.12)$$

All the non zero form factors, being antisymmetric under permutations of q's and permutations of p's, can be easily obtained from (4.11), (4.12) just by prescribing the corresponding sign. Turning now to the normalized mean value (4.6) and taking into account that the product $Z^*_{_N} Z_{_N}$ is symmetrical under permutations of q's and of p's, being equal to zero if any two q's (or any two p's) coincide, one gets

$$\langle \sigma_z^{(n_2)}(t_2) \sigma_z^{(n_1)}(t_1) \rangle_{_N} = \left(1 - \frac{2N}{M}\right)^2 + \qquad (4.13)$$
$$+ \frac{4}{M^2} \sum_{a=1}^{N} \sum_{\substack{j=1 \\ q_j \neq p_1, \ldots, p_N}}^{M} \exp\left[im(q_j - p_a) - it[\varepsilon(q_j) - \varepsilon(p_a)]\right] .$$

Finally, taking into account that

$$\sum_{\substack{j=1 \\ q_j \neq p_1, \ldots, p_N}}^{M} f(q_j) = \sum_{j=1}^{M} f(q_j) - \sum_{a=1}^{N} f(p_a) , \qquad (4.14)$$

one comes immediately to representation (4.3).

5 Form factors of Operators $\sigma_\pm$ on the Finite Lattice

In this Section form factors of operators $\sigma_\pm^{(m)}(t)$,

$$\sigma_\pm^{(m)}(t) = e^{iHt} \, \sigma_\pm^{(m)} \, e^{-iHt} .$$

i.e. their matrix elements between eigenstates of Hamiltonian (2.1) are calculated. This is, in particular, necessary to calculate the corresponding correlators. Form factors are defined as

$$F_{_N}(m, t, \{q\}, \{p\}) \equiv \langle \Psi_{N+1}(\{q\}) \mid \sigma_-^{(m)}(t) \mid \Psi_N(\{p\}) \rangle , \qquad (5.1)$$

$$G_{_N}(m, t, \{p\}, \{q\}) \equiv \langle \Psi_N(\{p\}) \mid \sigma_+^{(m)}(t) \mid \Psi_{N+1}(\{q\}) \rangle . \qquad (5.2)$$

The form factors $F_{_N}$, $G_{_N}$ are related by complex conjugation,

$$F^*_{_N}(m,t,\{q\},\{p\}) = G_{_N}(m,t,\{p\},\{q\}) \;, \tag{5.3}$$

so that in the following only form factor F_N is considered. In the previous formulae, $\{q\} = q_1,...,q_{_{N+1}}$ ($\dim\{q\} = N+1$) and $\{p\} = p_1,...,p_{_N}$ ($\dim\{p\} = N$) are the sets of different quasimomenta parametrizing eigenvectors $\mid \Psi_{N+1}\rangle$ and $\mid \Psi_N\rangle$ (if $\dim\{q\} \neq \dim\{p\}+1$, then the form factors are equal to zero). It should be emphasized that, due to periodical boundary conditions (2.10)

$$e^{iMq_a} = (-1)^N \;, \qquad\qquad e^{iMp_b} = (-1)^{N+1} \;, \tag{5.4}$$

quasimomenta q_a and p_b never coincide.

Form factor $F_{_N}$ (5.1) can be represented (up to the factor obvious from translational invariance) as the determinant of a $N\times N$ matrix, namely

$$\begin{aligned} F_{_N}(m,t,\{q\},\{p\}) &= i^N \exp\left[-im\{\sum_{a=1}^{N+1} q_a - \sum_{b=1}^{N} p_b\} + it\{\sum_{a=1}^{N+1}\varepsilon(q_a) - \sum_{b=1}^{N}\varepsilon(p_b)\}\right] \cdot \\ &\quad \cdot \mathcal{F}_N(\{q\},\{p\}) \;, \\ \mathcal{F}_{_N}(\{q\},\{p\}) &= \mathcal{F}^*_{_N}(\{q\},\{p\}) \;, \end{aligned} \tag{5.5}$$

where function $\mathcal{F}_N$ does not depend on m, t, and can be represented as the determinant

$$\begin{aligned} \mathcal{F}_N(\{q\},\{p\}) &= \left(1+\frac{\partial}{\partial z}\right) \det_{_N} A(z)\Big|_{z=0} = \\ &= \det_{_N} A(z=1) \;. \end{aligned} \tag{5.6}$$

Here $A(z)$ is a $N\times N$ matrix with elements depending linearly on the complex parameter z:

$$\begin{aligned} A(z) &\equiv A^{(1)} - zA^{(2)} \;, \\ A^{(1)}_{ab} &= \cot\frac{1}{2}(q_a - p_b) \;, \\ A^{(2)}_{ab} &= \cot\frac{1}{2}(q_{_{N+1}} - p_b) \;, \qquad a,b = 1,...,N \;. \end{aligned} \tag{5.7}$$

All the rows of matrix $A^{(2)}$ being the same, the rank of this matrix is equal to one. Hence, $\det_{_N} A(z)$ is itself a linear function of z. For generic linear functions $f(z)$, the following obvious relation holds

$$\left[1+\frac{\partial}{\partial z}\right] f(z)\Big|_{z=0} = f(1)\,, \qquad\qquad f(z) = a + bz\,, \tag{5.8}$$

so that the second equation in (5.6) is evident.

Let us explain briefly the derivation of representation (5.6), which is similar to the one of paper [10] for the nonrelativistic Bose gas. Starting from definition (5.1) and using explicit expression (2.7), (2.8) for the eigenfunctions involved, one gets for the form factor

$$\begin{aligned}
&F_{N}\cdot\exp\left([-it[\sum_{a}^{N+1}\varepsilon(q_a)-\sum_{b=1}^{N}\varepsilon(p_b)]\right) = \\
&= \sqrt{N+1}\sum_{m_1,\dots,m_N=1}^{M}\chi^{*}_{N+1}(m_1,\dots m_N, m\mid\{q\})\,\chi_N(m_1,\dots,m_N\mid\{p\}) = \\
&= \frac{1}{N!}\sum_{m_1,\dots,m_N=1}^{M}\sum_{Q,P}(-1)^{[Q]+[P]}\exp\left[-imq_{Q_{N+1}}\right]\prod_{b=1}^{N}\epsilon(m-m_b)\,\cdot \\
&\quad\cdot\exp\left[-i\sum_{a=1}^{N}(q_{Q_a}-p_{P_a})\right]\,.
\end{aligned}\tag{5.9}$$

Here Q: $(1,\dots,N+1)\to(Q_1,\dots,Q_{N+1})$ and P: $(1,\dots,N)\to(P_1,\dots,P_N)$ are all the permutations of $N+1$ and N numbers, respectively. The summations over m_i's are performed explicitly by means of the formula

$$\sum_{n=1}^{M}\epsilon(m-n)\,e^{-in(q_a-p_b)} = i\cot\frac{1}{2}(q_a-p_b)\cdot e^{-im(q_a-p_b)}\,, \tag{5.10}$$

(relation $e^{iM(q_a-p_b)}=-1$, following from periodical boundary conditions (5.4) should be taken into account). Moreover, noticing that the form factor is antisymmetric under permutations of quasimomenta p_b, one gets

$$F_N = i^N\exp\left[-im[\sum_{a=1}^{N+1}q_a-\sum_{b=1}^{N}p_b]+it[\sum_{a=1}^{N+1}\varepsilon(q_a)-\sum_{b=1}^{N}\varepsilon(p_b)]\right]\cdot\mathcal{F}_N(\{q\},\{p\})\,, \tag{5.11}$$

with

$$\mathcal{F}_N(\{q\},\{p\}) = \sum_{Q}(-1)^{[Q]}\prod_{a=1}^{N}\cot\frac{1}{2}(q_{Q_a}-p_a)\,, \tag{5.12}$$

(the permutations of p_a's have been summed over, cancelling the factor $\frac{1}{N!}$ in (5.9)). The sum in the r.h.s. is just the determinant of the $(N+1)\times(N+1)$ matrix B:

$$\begin{aligned}
\mathcal{F}_N(\{q\},\{p\}) &= \det{}_{N+1}B\,; \\
B_{ab} &= \cot\frac{1}{2}(q_a-p_b)\,, \qquad a=1,\dots,N+1, \\
B_{a,N+1} &= 1\,, \qquad\qquad\qquad b=1,\dots,N;
\end{aligned}\tag{5.13}$$

Subtracting the last row of matrix B from the first N rows (which does not change the value of the determinant) and then expanding the determinant of the obtained $(N+1) \times (N+1)$ matrix in the elements of the $(N+1)^{th}$ column, we come exactly to to the determinant of the $N \times N$ matrix $A(z=1)$ defined in (5.7):

$$\det_{N+1} B = \det_N A(z=1) \,, \tag{5.14}$$

so that representation (5.6) for the form factor is proved.

Let us conclude by mentioning a useful property of representation (5.6): it is possible to introduce a complex parameter into the r.h.s. of (5.6) without changing the result. Let us define matrix $\tilde{A}(z,c)$ with matrix elements

$$\tilde{A}_{ab}(z,c) \equiv A_{ab}(z) + c(1-z) \,, \qquad\qquad c \in \mathbf{C} \,. \tag{5.15}$$

It is obvious that $\det_N \tilde{A}(z,c)$ is a linear function of z, and moreover, that

$$\det_N \tilde{A}(z=1,c) = \det_N A(z=1) \,, \tag{5.16}$$

so that we can also write for $\mathcal{F}_N$:

$$\mathcal{F}_N(\{q\},\{p\}) = \left[1 + \frac{\partial}{\partial z}\right] \det_N \tilde{A}(z,c)\Big|_{z=0} = \det_N \tilde{A}(z=1,c) \,, \tag{5.17}$$

(the r.h.s does not in fact depend on c). The possibility of introducing this arbitrary parameter c is related to the possibility of prescribing any value $\epsilon(0)$ for function $\epsilon(m)$ in (2.9), (5.10) without changing the final results of calculations. This is a consequence of the fact that wave function χ defined in (2.8) is equal to zero if $m_j = m_k$ $(j \neq k)$ for any choice of the value $\epsilon(0)$.

6 Time Dependent Normalized Mean Values of Products of Operators σ_+, σ_- on the Finite Lattice

In this Section the determinant representations for the normalized mean values of the product of local operators $\sigma_+^{(m)}$, $\sigma_-^{(n)}$ on the finite lattice are given. These representations are proved in the next Section.

Let us consider first the normalized mean value

$$\langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_{_N} \equiv \frac{\langle \Psi_N(\{p\}) \mid \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1) \mid \Psi_N(\{p\})\rangle}{\langle \Psi_N(\{p\}) \mid \Psi_N(\{p\})\rangle} \tag{6.1}$$

with respect to some eigenfunction $\mid \Psi_N(\{p\})\rangle$ of the XX0 Hamiltonian (2.1). Due to translational invariance this quantity depends only on relative distance $m = m_2 - m_1$ and time $t = t_2 - t_1$. It is also easily seen that

$$\langle \sigma_+^{(n_1)}(t_1)\sigma_-^{(n_2)}(t_2)\rangle_{_N} = \langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_{_N}^* \,, \tag{6.2}$$

so that in the following it is sufficient to consider mean value (6.1) in region $m \geq 0$, $-\infty < t < +\infty$.

The following representation is obtained in the next Section for quantity (6.1):

$$\begin{aligned} \langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_{_N} &= e^{-2iht}\left[g(m,t) + \frac{\partial}{\partial z}\right] \det{}_{_N} \left[S - zR^{(+)}\right]\Big|_{z=0} = \\ &= e^{-2iht}\left\{[g(m,t) - 1]\det{}_{_N} S + \det{}_{_N}\left[S - R^{(+)}\right]\right\} \,, \end{aligned} \tag{6.3}$$

Here matrix elements of $N \times N$ matrices $S = S(m,t,\{p\})$ and $R^{(+)} = R^{(+)}(m,t,\{p\})$ are

$$\begin{aligned} S_{ab} &= \delta_{ab}\, d(m,t,p_a) \exp\left[-imp_a - 4it\cos p_a\right] + \\ &\quad + (1-\delta_{ab})\, \frac{e_+(m,t,p_a)e_-(m,t,p_b) - e_-(m,t,p_a)e_+(m,t,p_b)}{M \tan\frac{1}{2}(p_a - p_b)} - \\ &\quad - \frac{1}{M} g(m,t) e_-(m,t,p_a) e_-(m,t,p_b) \,; \end{aligned} \tag{6.4}$$

$$R_{ab}^{(+)} = \frac{1}{M} e_+(m,t,p_a)e_+(m,t,p_b) \,. \tag{6.5}$$

Functions $e_\pm$ are defined as

$$\begin{aligned} e_-(m,t,p_a) &\equiv \exp\left[-\frac{im}{2}p_a - 2it\cos p_a\right] \,, \\ e_+(m,t,p_a) &\equiv e_-(m,t,p_a)e(m,t,p_a) \,, \end{aligned} \tag{6.6}$$

and functions g, e, d are given as the sums:

$$g(m,t) \equiv \frac{1}{M}\sum_q \exp\left[imq + 4it\cos q\right] \,, \tag{6.7}$$

$$e(m,t,p_a) \equiv \frac{1}{M}\sum_q \frac{\exp\left[imq + 4it\cos q\right]}{\tan\frac{1}{2}(q - p_a)} \,, \tag{6.8}$$

$$d(m,t,p_a) \equiv \frac{1}{M^2}\sum_q \frac{\exp\left[imq + 4it\cos q\right]}{\sin^2\frac{1}{2}(q - p_a)} \,, \tag{6.9}$$

Let us explain notations in more detail. Momenta p_a ($a = 1, ..., N$) are the momenta of quasiparticles in state $\mid \Psi_N(\{p\})\rangle$ satisfying equations

$$\begin{aligned}
e^{iMp_a} &= (-1)^{N+1}\,, \qquad -\pi < p_a \leq \pi\,,\\
p_a &= \begin{cases} \frac{2\pi}{M}\left(-\frac{M+1}{2}+n_a\right)\,, & N \text{ even},\\ \frac{2\pi}{M}\left(-\frac{M}{2}+n_a\right)\,, & N \text{ odd},\end{cases}\\
n_a &= 1, ..., M\,, \qquad a = 1, ..., N\,.
\end{aligned} \tag{6.10}$$

The sums over q's are taken over all the permitted values of momenta of quasiparticles in a $(N+1)$-particle state, *i.e*,

$$\sum_q f(q) \equiv \sum_{j=1}^{M} f(q_j)\,, \tag{6.11}$$

where momenta q_j satisfy equations

$$\begin{aligned}
e^{iMq_j} &= (-1)^{N+2}\,,\\
q_j &= \begin{cases} \frac{2\pi}{M}\left(-\frac{M}{2}+j\right)\,, & N \text{ even},\\ \frac{2\pi}{M}\left(-\frac{M+1}{2}+j\right)\,, & N \text{ odd},\end{cases}\\
j &= 1, ..., M\,.
\end{aligned} \tag{6.12}$$

Due to (6.10), (6.12), q_j never coincide with any of p_a's, so that the sums (6.7)-(6.9) are well defined.

The rank of matrix $R^{(+)}$ being equal to one, the first determinant in (6.3) is a linear function of z, and due to property (5.8) the second equality in (6.3) is obvious.

Representation (6.3) is simplified in the equal-time case ($t = 0$), where functions g, e, d can be computed explicitly:

$$\begin{aligned}
g(m, 0) &= \delta_{m,0}\,, \qquad\qquad (m = 0, 1, ..., M-1)\\
e(m, 0, p_a) &= i\,(1-\delta_{m,0})\,e^{imp_a}\,,\\
d(m, 0, p_a) &= \left(1-\frac{2m}{M}\right) e^{imp_a}\,.
\end{aligned} \tag{6.13}$$

Then for $m = 0$ one reproduces the obvious answer

$$\langle\sigma_+^{(m)}\sigma_-^{(m)}\rangle_N = \frac{1}{2} + \frac{1}{2}\langle\sigma_z^{(m)}\rangle_N = 1 - \frac{N}{M}\,. \tag{6.14}$$

For $m > 0$ the equal-time mean value is represented as follows:

$$\begin{aligned} \langle\sigma_+^{(n_2)}\sigma_-^{(n_1)}\rangle_{_N} &= \frac{\partial}{\partial z}\,\mathrm{det}_{_N}\left[s + zr^{(+)}\right]\Big|_{z=0} = \\ &= \mathrm{det}_{_N}\left[s + r^{(+)}\right] - \mathrm{det}_{_N}\left[s\right]\,, \qquad\qquad m > 0\,, \end{aligned} \tag{6.15}$$

where matrix elements of $N \times N$ matrices s and $r^{(+)}$ are

$$s_{ab} = \delta_{ab}\left(1 - \frac{2m}{M}\right) - \frac{2}{M}\,(1 - \delta_{ab})\,\frac{\sin\frac{m}{2}(p_a - p_b)}{\tan\frac{1}{2}(p_a - p_b)}\,, \tag{6.16}$$

$$r_{ab}^{(+)} = \frac{1}{M}e^{\frac{im}{2}(p_a + p_b)}\,, \tag{6.17}$$

Analogous representations are valid also for mean value $\langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_N}$; as

$$\langle\sigma_-^{(n_1)}(t_1)\sigma_+^{(n_2)}(t_2)\rangle_{_N} = \langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_N}^*\,, \tag{6.18}$$

it is again considered in region $m \geq 0$, $-\infty < t \equiv (t_2 - t_1) < +\infty$. In the time-dependent case, one has

$$\begin{aligned} \langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_N} &= e^{2iht}\frac{\partial}{\partial z}\,\mathrm{det}_{_N}\left[S + zR^{(-)}\right]\Big|_{z=0} = \\ &= e^{2iht}\left\{\mathrm{det}_{_N}\left[S + R^{(-)}\right] - \mathrm{det}_{_N}\left[S\right]\right\}\,, \qquad m \geq 0\,, \end{aligned} \tag{6.19}$$

where $N \times N$ matrix S is just the same as in (6.3), (6.4) and the matrix elements of $N \times N$ matrix $R^{(-)}$ are given as

$$R_{ab}^{(-)} = \frac{1}{M}e_-(m, t, p_a)e_-(m, t, p_b)\,, \tag{6.20}$$

function e_- being defined in (6.6). In the equal-time case, the obvious relations

$$\langle\sigma_-^{(m)}\sigma_+^{(m)}\rangle_{_N} = \frac{1}{2} - \frac{1}{2}\langle\sigma_z^{(m)}\rangle_{_N} = \frac{N}{M} \tag{6.21}$$

and

$$\langle\sigma_-^{(n_2)}\sigma_+^{(n_1)}\rangle_{_N} = \langle\sigma_+^{(n_1)}\sigma_-^{(n_2)}\rangle_{_N} = \langle\sigma_+^{(n_2)}\sigma_-^{(n_1)}\rangle_{_N}^* \qquad (n_2 > n_1) \tag{6.22}$$

are reproduced.

So the determinant representations for time-dependent (and equal-time) normalized mean values of products of operators σ_+, σ_- on the finite lattice are given.

To conclude this Section let us discuss the relation of the determinant representation (6.15) for the equal-time normalized mean value of operators σ_+, σ_- to representation

(3.2) for the normalized mean value of operator $\exp[\alpha Q(m)]$. Due to formulae (2.18) transforming the XX0 model to the free fermion model on a lattice, it is quite natural to expect that equal-time correlator (6.15) could be expressed in terms of the first minors of matrix $\mathcal{M}(m-1)$ entering representation (3.2), with parameter α set equal to $i\pi$. In fact, as shown in the next Section, due to the possibility of introducing an arbitrary parameter c into the determinant representation of the form factors (see (5.17)), we can also rewrite representation (6.15) as

$$\langle \sigma_+^{(n_2)} \sigma_-^{(n_1)} \rangle_{_N} = \frac{\partial}{\partial z} \det_{_N} \left[\tilde{s} + z r^{(+)} \right] \Big|_{z=0} , \qquad m \equiv n_2 - n_1 > 0 , \tag{6.23}$$

with the same matrix $r^{(+)}$ (6.17) and

$$\tilde{s}_{ab} = s_{ab} + \frac{ic_1}{M} e^{-\frac{im}{2}(p_a - p_b)} + \frac{ic_2}{M} e^{\frac{im}{2}(p_a - p_b)} \tag{6.24}$$

(the r.h.s. of (6.23) does not depend on arbitrary complex parameters c_1, c_2). Chosing, *e.g.*, $c_1 = c_2 = -i$ one gets

$$\tilde{s}_{ab} = \delta_{ab} \left[1 - \frac{2(m-1)}{M} \right] - \frac{2}{M} (1 - \delta_{ab}) \frac{\sin \frac{m-1}{2}(p_a - p_b)}{\sin \frac{1}{2}(p_a - p_b)} , \tag{6.25}$$

which are exactly the matrix elements of matrix $\mathcal{M}(m-1)$ appearing in (3.2) after setting $\alpha = i\pi$.

7 Derivation of Representations for Time Dependent Mean Values of σ_+, σ_- on the Finite Lattice

In this Section representations (6.3) and (6.19) for the normalized mean values of operators $\sigma_+\sigma_-$ and $\sigma_-\sigma_+$ are proved. We begin with the normalized mean value (6.1) inserting the complete set of states $|\,\Psi_{N+1}(\{q\})\rangle$ between operators $\sigma_+^{(n_2)}(t_2)$ and $\sigma_-^{(n_1)}(t_1)$. Taking into account the normalization (2.14) of eigenstates and representation (5.5) for form factors (5.1)-(5.3), we readily get

$$\langle \sigma_+^{(n_2)}(t_2) \sigma_-^{(n_1)}(t_1) \rangle_{_N} = \frac{1}{M^{2N+1}} \sum_{\{q\}} \exp \left[im(\sum_1^{N+1} q_a - \sum_1^{N} p_b) - it(\sum_1^{N+1} \varepsilon(q_a) - \sum_1^{N} \varepsilon(q_b)) \right] \cdot \cdot \mathcal{F}_{_N}^2(\{q\}, \{p\}) . \tag{7.1}$$

Here $\varepsilon(p) = -4\cos p + 2h$ is the energy of quasiparticles and the sum is taken over all the different sets of momenta $\{q\} = q_1, ..., q_{N+1}$ of intermediate states (similarly to the summation in (4.6)). As the expression under the sum is symmetric under permutations of q's (being equal to zero whenever two of the q's coincide), one can change the sum in (7.1) for the sum over all the permitted values of each q_a:

$$\sum_{\{q\}} \longrightarrow \frac{1}{(N+1)!} \sum_{q_1} \cdots \sum_{q_{N+1}} , \tag{7.2}$$

where the sum over q_a's ($a = 1, ..., N+1$) is to be understood as in (6.11):

$$\sum_{q_a} f(q_a) = \sum_{j=1}^{M} f\left((q_a)_j\right) \tag{7.3}$$

and permitted values $(q_a)_j$ (the same for each q_a) are given by (6.12).

Let us now use representation (5.12)

$$\mathcal{F}_N(\{q\},\{p\}) = \sum_Q (-1)^{[Q]} \prod_{a=1}^{N} \cot\frac{1}{2}(q_{Q_a} - p_a) ,$$

for one of the two form factors $\mathcal{F}_N$ in (7.1). As in (7.1) there is the sum over all the q_a's, and form factor $\mathcal{F}_N(\{q\},\{p\})$ is antisymmetrical under permutations of q_a's ($a = 1, ..., N+1$), the sum over permutations Q: $(1, ..., N+1) \to (Q_1, ..., Q_{N+1})$ can be written as

$$\sum_Q (-1)^{[Q]} \prod_{a=1}^{N} \cot\frac{1}{2}(q_{Q_a} - p_a) \longrightarrow (N+1)! \prod_{a=1}^{N} \cot\frac{1}{2}(q_a - p_a)$$

without changing the l.h.s. of (7.1).

Now using representation (5.6)

$$\mathcal{F}_N(\{q\},\{p\}) = \left[1 + \frac{\partial}{\partial z}\right] \det_N A(z)\big|_{z=0}$$

for the remaining factor $\mathcal{F}_N$ and taking into account that $\det_N A(z)$ is a linear function of z, one rewrites (7.1) as

$$\begin{aligned} \langle \sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_N &= \exp\left[-2iht - 4it\sum_{b=1}^{N}\cos p_b - im\sum_{b=1}^{N} p_b\right] \cdot \\ &\cdot \sum_{q_1} \cdots \sum_{q_{N+1}} \left\{\frac{1}{M}\exp\left[4it\cos q_{N+1} + imq_{N+1}\right] + \frac{\partial}{\partial z}\right\} \cdot \\ &\cdot \det_N \mathcal{U}(z)\big|_{z=0} , \end{aligned} \tag{7.4}$$

where the $N \times N$ matrix $\mathcal{U}(z)$ is given as

$$\begin{aligned}
\mathcal{U}(z) &= \mathcal{U}^{(1)} - \frac{z}{M}\mathcal{U}^{(2)} , \\
\mathcal{U}^{(1)}_{ab} &= \frac{1}{M^2} \exp\left[4it\cos q_a + imq_a\right] \cot\frac{1}{2}(q_a - p_a) \cot\frac{1}{2}(q_a - p_b) , \\
\mathcal{U}^{(2)}_{ab} &= \frac{1}{M^2} \exp\left[4it\cos q_a + imq_a\right] \exp\left[4it\cos q_{N+1} + imq_{N+1}\right] \cdot \\
& \cdot \cot\frac{1}{2}(q_a - p_a) \cot\frac{1}{2}(q_{N+1} - p_b) . \qquad (7.5)
\end{aligned}$$

It should be noted that the rank of matrix $\mathcal{U}^{(2)}$ is equal to one (all its rows are proportional to each other); hence $\det_N \mathcal{U}(z)$ is a linear function of z.

Let us consider now summation in q_{N+1} in (7.4). As $\det_N \mathcal{U}(z = 0)$ does not contain q_{N+1} (all the dependence on q_{N+1} is in matrix $\mathcal{U}^{(2)}$), one has

$$\frac{1}{M} \sum_{q_{N+1}} \exp\left[4it\cos q_{N+1} + imq_{N+1}\right] \det_N \mathcal{U}(z)\Big|_{z=0} = g(m,t) \det_N \mathcal{U}(z)\big|_{z=0} , \qquad (7.6)$$

with function $g(m,t)$ defined in (6.7). On the other hand, using the fact that the rank of matrix $\mathcal{U}^{(2)}$ is equal to one, we conclude that

$$\sum_{q_{N+1}} \frac{\partial}{\partial z} \det_N \mathcal{U}(z) = \frac{\partial}{\partial z} \det_N \left[\mathcal{U}^{(1)} - \frac{z}{M}\tilde{\mathcal{U}}^{(2)}\right] , \qquad (7.7)$$

where matrix $\mathcal{U}^{(1)}$ is just the same as in (7.5) and

$$\tilde{\mathcal{U}}^{(2)}_{ab} = \frac{1}{M} \exp\left[4it\cos q_a + imq_a\right] \cot\frac{1}{2}(q_a - p_a)\, e(m,t,p_b) \qquad (7.8)$$

with function $e(m,t,p_b)$ defined in (6.8). So the summation over q_{N+1} can be done "inside" the determinant.

The summations over the remaining q_a's ($a = 1, ..., N$) can be also reduced to the summations of matrix elements of $\mathcal{U}(z)$, because momentum q_a enters only the a^{th} row of this matrix. Therefore one comes to the following expression:

$$\begin{aligned}
\langle \sigma_+^{(n_2)}(t_2) \sigma_-^{(n_1)}(t_1) \rangle_N &= \exp\left[-2iht - 4it\sum_{b=1}^{N} \cos p_b - im\sum_{b=1}^{N} p_b\right] \cdot \\
& \cdot \left[g(m,t) + \frac{\partial}{\partial z}\right] \det_N \left[\tilde{\mathcal{U}}^{(1)} - zR\right]\Big|_{z=0} , \qquad (7.9)
\end{aligned}$$

with

$$\tilde{\mathcal{U}}^{(1)}_{ab} = \frac{1}{M^2} \sum_q \exp\left[imq + 4it\cos q\right] \cot\frac{1}{2}(q-p_a) \cot\frac{1}{2}(q-p_b) \,, \tag{7.10}$$

$$R_{ab} = \frac{1}{M} e(m,t,p_a) e(m,t,p_b) \,. \tag{7.11}$$

The diagonal elements of matrix $\tilde{\mathcal{U}}^{(1)}$ can be written as

$$\tilde{\mathcal{U}}^{(1)}_{aa} = d(m,t,p_a) - \frac{1}{M} g(m,t) \,, \tag{7.12}$$

with function $d(m,t,p_a)$ defined in (6.9). Taking into account the identity

$$\cot\frac{1}{2}(q-p_a) \cot\frac{1}{2}(q-p_b) = \cot\frac{1}{2}(p_a-p_b) \left[\cot\frac{1}{2}(q-p_a) - \cot\frac{1}{2}(q-p_b)\right] - 1 \,,$$

off-diagonal elements of matrix $\tilde{\mathcal{U}}^{(1)}$ turn out to be:

$$\tilde{\mathcal{U}}^{(1)}_{ab} = \frac{1}{M} \frac{1}{\tan\frac{1}{2}(p_a-p_b)} \left[e(m,t,p_a) - e(m,t,p_b)\right] - \frac{1}{M} g(m,t) \,. \tag{7.13}$$

Let us now "insert" the exponential factor in the r.h.s. of (7.9) inside the determinant; this means to multiply the a^{th} row by

$$\exp\left[-\frac{im}{2} p_a - 2it\cos p_a\right] \,,$$

and the b^{th} column by

$$\exp\left[-\frac{im}{2} p_b - 2it\cos p_b\right] \,.$$

Taking into account definition (6.6) of functions e_+, e_-, one comes just to representation (6.3), which is thus proved.

It is to be mentioned that instead of representations (5.6) and (5.12) for the form factors in (7.1), one can also use representation (5.17)

$$\mathcal{F}_N(\{q\},\{p\}) = \left[1 + \frac{\partial}{\partial z}\right] \det{}_N A(z,c_1)\big|_{z=0}$$

for one of them, and the form analogous to (5.12)

$$\mathcal{F}_N(\{q\},\{p\}) = \sum_Q (-1)^{[Q]} \prod_{a=1}^{N} \left[\cot\frac{1}{2}(q_{Q_a} - p_a) + c_2\right]$$

for the other one; here c_1, c_2 are arbitrary complex constants; the answer, of course, does not depend on c_1, c_2. This alternative procedure gives, instead of (6.3), the following representation

$$\begin{aligned}\langle\sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle_{_N} &= e^{-2iht}\left[g(m,t)+\frac{\partial}{\partial z}\right]\cdot \\ &\cdot \det{}_{_N}\left[\tilde{S}(c_1,c_2)-z\tilde{R}^{(+)}(c_1,c_2)\right]\Big|_{z=0}\ , \end{aligned} \tag{7.14}$$

where matrix elements of $N\times N$ matrices $\tilde{S}$, $\tilde{R}^{(+)}$ are

$$\begin{aligned}\tilde{S}_{ab}(c_1,c_2) &= S_{ab}+\frac{c_1}{M}e_+(m,t,p_a)e_-(m,t,p_b)+\frac{c_2}{M}e_-(m,t,p_a)e_+(m,t,p_b)+ \\ &+\frac{c_1c_2-1}{M}g(m,t)e_-(m,t,p_a)e_-(m,t,p_b)\ ; \\ \tilde{R}_{ab}^{(+)}(c_1,c_2) &= \frac{1}{M}\left[e_+(m,t,p_a)+c_2e_-(m,t,p_a)g(m,t)\right]\cdot \\ &\cdot\left[e_+(m,t,p_b)+c_1e_-(m,t,p_b)g(m,t)\right]\cdot\end{aligned}$$

where $S_{ab}=\tilde{S}_{ab}(0,0)$ is given by (6.4). For $t=t_2-t_1=0$ and $m=n_2-n_1>0$ (since then $g(m,t=0)=0$) one gets just representation (6.23) for the equal-time correlator.

Let us now consider the other normalized mean value:

$$\begin{aligned}\langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_N} &= \frac{\langle\Psi_{_N}(\{p\})\mid\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\mid\Psi_{_N}(\{p\})\rangle}{\langle\Psi_{_N}(\{p\})\mid\Psi_{_N}(\{p\})\rangle}\ , \\ m\equiv n_2-n_1\geq 0; &\qquad -\infty<t\equiv t_1-t_2<+\infty.\end{aligned} \tag{7.15}$$

Inserting the complete set of $N-1$-particle states $\mid\Psi_{n-1}(\{q\})\rangle$ ($\{q\}=q_1,...,q_{_{N-1}}$) and using representation (5.5) for the appearing form factors, one gets

$$\begin{aligned}\langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_N} &= \frac{1}{M^{2N-1}}\sum_{\{q\}}\exp\left[-im(\sum_1^N p_a-\sum_1^{N-1}q_b)+it(\sum_1^N\varepsilon(p_a)-\sum_1^{N-1}\varepsilon(q_b))\right]\cdot \\ &\cdot\mathcal{F}^2_{_{N-1}}(\{p\},\{q\})\ .\end{aligned} \tag{7.16}$$

It is to be emphasized that now the number of external momenta p_a ($a=1,...,N$) is larger than the number of intermediate (summed over) momenta q_b ($b=1,...,N-1$); hence the order of arguments in $\mathcal{F}_{_N}(\{p\},\{q\})$. The sum over different sets $\{q\}$ in (7.16) can be changed (analogously to (7.2)) to independent sums over all the permitted values of each q_a:

$$\sum_{\{q\}}\longrightarrow\frac{1}{(N-1)!}\sum_{q_1}...\sum_{q_{N-1}}\ . \tag{7.17}$$

Representing form factor $\mathcal{F}_{_{N-1}}(\{p\},\{q\})$ by means of (5.12),

$$\mathcal{F}_{_{N-1}}(\{p\},\{q\}) = \sum_{Q} (-1)^{[Q]} \prod_{a=1}^{N-1} \cot \frac{1}{2}(p_{Q_a} - q_a) \; , \tag{7.18}$$

(where the sum is now over permutations Q: $(1, ..., N) \to (Q_1, ..., Q_N)$) one represents (7.16) as

$$\begin{aligned} \langle \sigma_-^{(n_2)}(t_2) \sigma_+^{(n_1)}(t_1) \rangle_{_N} \quad &= \quad \frac{1}{M^{2N-1}(N-1)!} \exp \left[2iht - im \sum_1^N p_a - 4it \sum_1^N \cos(p_a) \right] \cdot \\ &\cdot \sum_{P,Q} (-1)^{[P]+[Q]} \prod_{a=1}^{N-1} \mathcal{S}(p_{Q_a}, p_{P_a}, m, t) \; . \end{aligned} \tag{7.19}$$

Here function $\mathcal{S}$ is introduced,

$$\begin{aligned} \mathcal{S}(p_a, p_b, m, t) \quad &\equiv \quad \sum_q \exp\left[imq + 4it \cos q \right] \cot \frac{1}{2}(q - p_a) \cot \frac{1}{2}(q - p_b) \; ; \\ \mathcal{S}(p_a, p_a, m, t) \quad &= \quad M^2 d(m, t, p_a) - M g(m, t) \; , \\ \mathcal{S}(p_a, p_b, m, t) \quad &= \quad \frac{M}{\tan \frac{1}{2}(p_a - p_b)} \left[e(m, t, p_a) - e(m, t, p_b) \right] - M g(m, t) \; , \qquad \text{if } a \neq b \; , \end{aligned} \tag{7.20}$$

with functions g, e, d defined in (6.7)-(6.9). The sum over P, Q in (7.19) is the sum over all the permutations, P: $(1, ..., N) \to (P_1, ..., P_N)$ and Q: $(1, ..., N) \to (Q_1, ..., Q_N)$. It is not difficult to notice that this sum is just the sum of all the first minors (multiplied by $(N-1)!$) of $N \times N$ matrix $\tilde{\mathcal{S}}$ with matrix elements

$$\tilde{\mathcal{S}}_{ab} \equiv \mathcal{S}(p_a, p_b, m, t) \; , \tag{7.21}$$

and can therefore be represented in the form

$$\sum_{P,Q} (-1)^{[P]+[Q]} \prod_{a=1}^{N-1} \mathcal{S}(p_{Q_a}, p_{Q_b}, m, t) = (N-1)! \, \frac{\partial}{\partial z} \det{}_{_N} \left[\tilde{\mathcal{S}} + \tilde{z} \right] \Big|_{z=0} \; . \tag{7.22}$$

Here all the elements of matrix $\tilde{z}$ are the same:

$$\tilde{z}_{ab} = z \; , \tag{7.23}$$

(z is as usual a complex parameter) so that in fact $\det_{_N} \left[\tilde{\mathcal{S}} + \tilde{z} \right]$ is a linear function of z.

Representing now the sums over permutations in (7.19) by means of (7.22) one easily transform the representation obtained just to the form (6.19) which is hence proved.

8 Correlators in the Thermodynamic Limit; the Case of Zero Temperature

The most interesting from the physical point of view are correlators of the model in the thermodynamical limit where the total number M of sites of the lattice goes to infinity, $M \to \infty$. In this Section the thermodynamical limit at zero temperature is discussed, correlators being defined as the normalized mean values of corresponding operators with respect to the ground state of the model (see (1.10)).

The ground state $\mid \Omega_M \rangle$ for fixed magnetic field h ($0 \leq h < h_c \equiv 2$, see discussion in Section 2) is obtained by filling with quasiparticles all the permitted vacancies with momenta p_a inside the Fermi zone, $-k_F \leq p_a \leq k_F$, where k_F is the Fermi momentum (1.5) $k_F = \arccos(h/2)$. In the thermodynamical limit the permitted values of momenta corresponding to the ground state fill the whole interval $[-k_F, k_F]$, with the number of quasiparticles in the ground state, $N = M k_{_F}/\pi$, see (2.11), going to infinity. However, density $D = \frac{N}{M} = \frac{k_F}{\pi}$ of quasiparticles in the ground state remains finite, the magnetization $\langle \sigma_z \rangle$ being (see (2.15)):

$$\langle \sigma_z \rangle = \langle \sigma_z^{(m)} \rangle = 1 - 2D = 1 - \frac{2k_F}{\pi} \, . \tag{8.1}$$

Let us consider first the thermodynamical limit of generating functional (3.1)

$$\langle \exp\left[\alpha Q(m)\right] \rangle = \frac{\langle \Omega \mid \exp\left[\alpha Q(m)\right] \mid \Omega \rangle}{\langle \Omega \mid \Omega \rangle} \, , \tag{8.2}$$

where $\mid \Omega \rangle$ is the ground state in the limit: $\mid \Omega \rangle = \lim_{M\to\infty} \mid \Omega_M \rangle$, magnetic field h being fixed. Taking the corresponding limit in formula (3.2) one comes to the following representation for the generating functional:

$$\langle \exp\left[\alpha Q(m)\right] \rangle = \det(\hat{I} + \gamma \hat{M})\Big|_{\gamma = e^{\alpha} - 1} \tag{8.3}$$

In the r.h.s. there is the Fredholm determinant of linear integral operator $\hat{M}(m)$ acting on functions $f(p)$ on interval $[-k_F, k_F]$ according to the rule:

$$(\hat{M} f)(p) = \frac{1}{2\pi} \int_{-k_F}^{k_F} dq \; M(p, q) f(q) \tag{8.4}$$

with kernel $M(p,q)$ given as

$$M(p,q) = \frac{\sin\frac{m}{2}(p-q)}{\sin\frac{1}{2}(p-q)} \ , \tag{8.5}$$

Operator $\hat{I}$ in (8.3) is the unit operator, with kernel $2\pi\delta(p-q)$.

To derive this representation one has to notice that $N \times N$ matrix $\mathcal{M}$ in (3.2) acts on arbitrary N-component vector f_a $(a = 1, ..., N)$ as follows

$$\begin{aligned} (\mathcal{M}(m) f)_a &= f_a + \frac{\gamma m}{M} f_a + \frac{\gamma}{M} \sum_{\substack{b=1 \\ b\neq a}}^{N} \frac{\sin\frac{m}{2}(p_a - p_b)}{\sin\frac{1}{2}(p_a-p_b)} f_b \ , \\ &\gamma \equiv e^{\alpha} - 1 \ . \end{aligned} \tag{8.6}$$

In the thermodynamical limit, due to (2.11), one should replace the sum by the integral

$$\frac{1}{M}\sum_{a=1}^{N} \longrightarrow \frac{1}{2\pi}\int_{-k_F}^{k_F} dq \tag{8.7}$$

coming just to (8.3) (the second term in the r.h.s. of (8.6) is included naturally to operator $\hat{M}(m)$). Let us also mention that for $\alpha = -\infty$ $(\gamma = -1)$ the generating functional possesses a clear physical meaning giving the probability $P(m)$ for all spins in an interval of length m to be up in the ground state (*i.e.* that there are no quasiparticles on this interval):

$$P(m) = \langle \exp\left[\alpha Q(m)\right]\rangle_{\alpha=-\infty} = \det(\hat{I} - \hat{M}) \ . \tag{8.8}$$

Let us consider now the time-dependent correlator

$$g_{zz}^{(0)}(m,t) \equiv \langle \sigma_z^{(n_2)}(t_2)\sigma_z^{(n_1)}(t_1)\rangle \equiv \frac{\langle \Omega \mid \sigma_z^{(n_2)}(t_2)\sigma_z^{(n_1)}(t_1) \mid \Omega\rangle}{\langle \Omega \mid \Omega \rangle} \ , \tag{8.9}$$

in the thermodynamical limit. Replacing sums over a $(a = 1, ..., N)$ in formula (4.3) with integrals over the Fermi zone, as in (8.7), and sum over all the particles j $(j = 1, ..., M)$ as

$$\sum_{j=1}^{M} \longrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} dq \ , \tag{8.10}$$

one gets

$$\begin{aligned} g_{zz}^{(0)}(m,t) &= \langle\sigma_z\rangle^2 - \frac{1}{\pi^2}\left|\int_{-k_F}^{k_F} dp\ \exp\left[imp + 4it\cos p\right]\right|^2 + \\ &+\frac{1}{\pi^2}\int_{-k_F}^{k_F} dp\ \exp\left[-imp - 4it\cos p\right]\int_{-\pi}^{\pi} dq\ \exp\left[imq + 4it\cos q\right] \ , \end{aligned} \tag{8.11}$$

where $m = m_2 - m_1$, $t = t_2 - t_1$, and $\langle\sigma_z\rangle$ is the magnetization (8.1); explicit expression (1.2) for the energy, $\varepsilon(p) = -4\cos p + 2h$, has been taken into account. This last result has already been obtained from a different procedure in [15]. In the equal-time case ($t = 0$), the last integral is equal to zero for $m \neq 0$, while the other ones can be calculated explicitly, reproducing the well known answer

$$\langle\sigma_z^{(n_2)}\sigma_z^{(n_1)}\rangle = \langle\sigma_z\rangle^2 - \frac{4}{\pi^2}\frac{\sin^2 mk_F}{m^2}\,, \qquad m \equiv n_2 - n_1 \neq 0\,. \tag{8.12}$$

Let us turn now to correlators of operators σ_+, σ_-, considering first correlator (1.11)

$$g_+^{(0)}(m,t) \equiv \langle\sigma_+^{(n_2)}(t_2)\sigma_-^{(n_1)}(t_1)\rangle\,, \tag{8.13}$$

where as usual $m = n_2 - n_1$, $t = t_2 - t_1$. Due to the relations

$$g_+^{(0)}(m,t) = g_+^{(0)}(-m,t) = \left[g_+^{(0)}(-m,-t)\right]^*\,, \tag{8.14}$$

it is sufficient to consider the region

$$m \geq 0\,, \qquad t \geq 0\,. \tag{8.15}$$

The determinant representation for this correlator is obtained by performing the thermodynamical limit in representation (6.3) for finite number M of sites; the ground state of the model at finite M should be taken as state $\mid \Psi_N(\{p\})\rangle$ with respect to which the mean value is taken in eq. (6.1), (6.3).

As shown in Appendix, functions g, e, d (6.7)-(6.9) entering the representation for the finite lattice should be changed to functions G, E, D, respectively, in the thermodynamical limit:

$$\begin{aligned} G(m,t) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} dq\, \exp\left[imq + 4it\cos q\right] = \\ &= i^m J_m(4t)\,, \end{aligned} \tag{8.16}$$

(J_m is a Bessel function)

$$\begin{aligned} E(m,t,p) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} dq\, \frac{\exp\left[imq + 4it\cos q\right] - \exp\left[imp + 4it\cos p\right]}{\tan\frac{1}{2}(q-p)} \equiv \\ &\equiv \frac{1}{2\pi}\mathcal{P}\int_{-\pi}^{\pi} dq\, \frac{\exp\left[imq + 4it\cos q\right]}{\tan\frac{1}{2}(q-p)}\,, \end{aligned} \tag{8.17}$$

and

$$D(m,t,p)=\exp\left[imp+4it\cos p\right]+\frac{2}{M}\frac{\partial}{\partial p}E(m,t,p)\,. \tag{8.18}$$

Then, in analogy with the derivation of formula (8.3) for generating functional $\langle\exp\left[\alpha Q(m)\right]\rangle$, one obtains the representation for correlator $g_+^{(0)}(m,t)$ (8.13) in the thermodynamical limit in terms of the Fredholm determinant of a linear integral operator:

$$\begin{aligned} g_+^{(0)}(m,t) &= e^{-2iht}\left[G(m,t)+\frac{\partial}{\partial z}\right]\det\left[\hat{I}+\hat{V}-z\hat{R}^{(+)}\right]\Big|_{z=0} = \\ &= e^{-2iht}\left\{\left[G(m,t)-1\right]\det\left[\hat{I}+\hat{V}\right]+\det\left[\hat{I}+\hat{V}-\hat{R}^{(+)}\right]\right\}\,, \end{aligned} \tag{8.19}$$

where $\hat{I}$ is again the identity operator, and linear operators $\hat{V}$, $\hat{R}^{(+)}$ acting on functions $f(p)$ on segment $[-k_F,k_F]$

$$\begin{aligned} \left(\hat{V}f\right)(p) &= \frac{1}{2\pi}\int_{-k_F}^{k_F}dq\,V(p,q)f(q)\,, \\ \left(\hat{R}^{(+)}f\right)(p) &= \frac{1}{2\pi}\int_{-k_F}^{k_F}dq\,R^{(+)}(p,q)f(q)\,, \end{aligned} \tag{8.20}$$

possess kernels

$$V(p,q) = \frac{E_+(p)E_-(q)-E_-(p)E_+(q)}{\tan\frac{1}{2}(p-q)}-G(m,t)E_-(p)E_-(q)\,, \tag{8.21}$$

$$R^{(+)}(p,q) = E_+(p)E_+(q)\,. \tag{8.22}$$

Functions E_+, E_-, are defined analogously to e_+, e_- in (6.6):

$$\begin{aligned} E_-(p) &\equiv E_-(m,t,p)=\exp[-\frac{i}{2}mp-2it\cos p]\,, \\ E_+(p) &\equiv E_+(m,t,p)=E_-(p)E(m,t,p)\,. \end{aligned} \tag{8.23}$$

It is to mention that the second term in the right hand side of expression (8.18) for function D is included naturally to operator $\hat{V}$.

Thus the representation for correlator $g_+^{(0)}(m,t)$ (8.13) is obtained.

Let us consider now correlator (1.12)

$$g_-^{(0)}(m,t)\equiv\langle\sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle\,. \tag{8.24}$$

Again, due to the properties

$$g_-^{(0)}(m,t)=g_-^{(0)}(-m,t)=\left[g_-^{(0)}(-m,-t)\right]^*\,, \tag{8.25}$$

we consider it only in region $m \geq 0$, $t \geq 0$. Using expression (6.19) at finite M, one derives the following representation in the thermodynamical limit:

$$\begin{aligned} g_-^{(0)}(m,t) &= e^{2iht}\frac{\partial}{\partial z}\det\left[\hat{I}+\hat{V}+z\hat{R}^{(-)}\right]\Big|_{z=0} = \\ &= e^{2iht}\left\{\det\left[\hat{I}+\hat{V}+\hat{R}^{(-)}\right]-\det\left[\hat{I}+\hat{V}\right]\right\}, \end{aligned} \tag{8.26}$$

where $\hat{V}$ is just the same linear operator as in representation (8.19) for correlator $g_+^{(0)}$, see (8.21), and the kernel of operator $\hat{R}^{(-)}$ is

$$R^{(-)}(p,q) = E_-(m,t,p)\,E_-(m,t,q)\,. \tag{8.27}$$

In the equal-time case ($t=0$) functions G, E are calculated explicitly to be

$$\begin{aligned} G(m,0) &= \delta_{m,0}\,, \\ E(m,0) &= i\,[1-\delta_{m,0}]\,e^{imp}\,, \end{aligned} \tag{8.28}$$

so that for equal time correlator $g_+^{(0)}(m,0)$ one gets the representation (which is, of course, the thermodynamical limit of representation (6.15)):

$$\begin{aligned} g_+^{(0)}(m) &= \langle\sigma_+^{(n_2)}\sigma_-^{(n_1)}\rangle = \\ &= \frac{\partial}{\partial z}\det\left[\hat{I}+\hat{v}+z\hat{r}^{(+)}\right] = \\ &= \det\left[\hat{I}+\hat{v}+\hat{r}^{(+)}\right]-\det\left[\hat{I}+\hat{v}\right]\,, \end{aligned} \tag{8.29}$$

where the kernels of operator $\hat{v}$, $\hat{r}^{(+)}$ (acting on interval $[-k_F,k_F]$) are

$$v(p,q) = -2\frac{\sin\frac{m}{2}(p-q)}{\tan\frac{1}{2}(p-q)}\,, \tag{8.30}$$

$$r^{(+)}(p,q) = \exp\left[\frac{im}{2}(p+q)\right]\,. \tag{8.31}$$

This answer can also be put into the following form (see (6.25) and the discussion at the end of Section 6):

$$g_+^{(0)}(m) = \frac{\partial}{\partial z}\det\left[\hat{I}+\hat{w}+z\hat{r}^{(+)}\right]\,, \qquad m>0\,, \tag{8.32}$$

where operator $\hat{r}^{(+)}$ is the same as in (8.31) and the kernel of operator $\hat{w}$ is

$$W_0(p,q) = -2\frac{\sin\frac{m-1}{2}(p-q)}{\sin\frac{1}{2}(p-q)}\,, \tag{8.33}$$

so that in fact the representation of correlator $g_+^{(0)}(m)$ involves the first Fredholm minors of the same linear integral operator as in representation (8.3) of the generating functional $\langle\exp[\alpha Q(m)]\rangle$ at $\alpha = i\pi$ (compare (8.33) and (8.5)).

9 Correlators in the Thermodynamical Limit at Nonzero Temperature

In this Section we consider the correlators at non-zero temperature ($T > 0$) in the thermodynamical limit. Temperature dependent correlation functions are defined in the standard way (1.8). For some operator $\mathcal{O}$, the temperature mean value $\langle \mathcal{O} \rangle_T$ is

$$\langle \mathcal{O} \rangle_T = \frac{\mathrm{Sp}\left[e^{-\frac{H}{T}}\mathcal{O}\right]}{\mathrm{Sp}\left[e^{-\frac{H}{T}}\right]}. \tag{9.1}$$

For integrable systems, calculating mean values of this kind is rather simple [24]. From the practical point of view, one should only change the integration measure in the representations of correlators obtained in the zero temperature case, namely,

$$\int_{-k_F}^{k_F} dq \longrightarrow \int_{-\pi}^{\pi} dq\, \vartheta(q)\ . \tag{9.2}$$

Here $\vartheta(q) \equiv \vartheta(q, h, T)$ is the Fermi weight (1.6),

$$\vartheta(q) = \frac{1}{1 + \exp\left[\frac{\varepsilon(q)}{T}\right]} = \frac{1}{1 + \exp\left[\frac{-4\cos q + 2h}{T}\right]}\ , \tag{9.3}$$

describing the momenta distribution of particles at the thermodynamical equilibrium. Of course, for the XX0 chain, which is equivalent to the free fermion model, this procedure is quite obvious.

Taking this into account, it is straightforward to extend the representations of zero temperature correlators to finite temperatures.

For the generating functional $\langle \exp\left[\alpha Q(m)\right]\rangle_T$ one has (see (8.3)) the representation in terms of the Fredholm determinant:

$$\langle \exp\left[\alpha Q(m)\right]\rangle_T = \det\left[\hat{I} + \gamma \hat{M}_T\right]\Big|_{\gamma = e^{\alpha} - 1} \tag{9.4}$$

where now $\hat{M}_T$ is an integral operator acting on functions $f(p)$ on interval $[-\pi, \pi]$:

$$\left(\hat{M}_T f\right)(p) = \frac{1}{2\pi}\int_{-\pi}^{\pi} dq\, M_T(p, q) f(q)\ , \tag{9.5}$$

with kernel $M_T(p, q)$ obtained from kernel $M(p, q)$ (8.5) as

$$\begin{aligned} M_T(p,q) &= M(p,q)\,\vartheta(q) = \frac{\sin\frac{m}{2}(p-q)}{\sin\frac{1}{2}(p-q)}\,\vartheta(q)\ , \\ & -\pi \le p, q \le \pi\ . \end{aligned} \tag{9.6}$$

Since the similarity transformation,

$$M_{_T}(p,q) \;\longrightarrow\; \sqrt{\vartheta(p)}\; M_{_T}(p,q)\; \frac{1}{\sqrt{\vartheta(q)}}\;, \tag{9.7}$$

does not change the value of the Fredholm determinant, one can rewrite representation (9.4) as

$$\langle \exp\left[\alpha Q(m)\right]\rangle_{_T} = \det\left[\hat{I} + \gamma \hat{M}^{(S)}_{_T}\right]\Big|_{\gamma=e^{\alpha}-1} \tag{9.8}$$

where operator $\hat{M}^{(S)}_{_T}$ possesses the symmetrical kernel

$$M^{(S)}_{_T}(p,q) = \sqrt{\vartheta(p)}\;\frac{\sin\frac{m}{2}(p-q)}{\sin\frac{1}{2}(p-q)}\;\sqrt{\vartheta(q)}\,. \tag{9.9}$$

Consider now the time-dependent correlator of the third spin components. Representation (8.11) is rewritten for $T>0$ as [15]

$$\begin{aligned} g^{(T)}_{zz}(m,t) \equiv& \\ \equiv \langle \sigma^{(n_2)}_z(t_2)\sigma^{(n_1)}_z(t_1)\rangle_{_T} =& \langle\sigma_z\rangle^2 - \frac{1}{\pi^2}\left|\int_{-\pi}^{\pi} dp\,\vartheta(p)\exp\left[imp + 4it\cos p\right]\right|^2 + \\ +\frac{1}{\pi^2}&\left(\int_{-\pi}^{\pi} dp\,\vartheta(p)\exp\left[-imp-4it\cos p\right]\right)\left(\int_{-\pi}^{\pi} dq\;\exp\left[imq+4it\cos q\right]\right)\,. \end{aligned} \tag{9.10}$$

In the equal-time case, the known answer is reproduced:

$$g^{(T)}_{zz}(m) \equiv \langle \sigma^{(n_2)}_z\sigma^{(n_1)}_z\rangle_{_T} = \langle\sigma_z\rangle^2_T - \frac{1}{\pi^2}\left|\int_{-\pi}^{\pi} dp\vartheta(p)e^{imp}\right|^2\;, \qquad (m\equiv n_2-n_1\neq 0) \tag{9.11}$$

i.e. the correlator is given by the square modulus of the Fourier transform of Fermi weight. In (9.10), (9.11), $\langle\sigma_z\rangle_{_T}$ is the magnetization at temperature T,

$$\langle\sigma_z\rangle_{_T} = 1 - \frac{1}{\pi}\int_{-\pi}^{\pi} dq\,\vartheta(q)\;, \tag{9.12}$$

with $\langle\sigma_z\rangle_{_T} = 0$ at $h=0$.

Let us now consider correlators of operators σ_+, σ_-. For correlator (1.11) one gets (see (8.19)) the following representation at $T>0$

$$\begin{aligned} g^{(T)}_{+}(m,t) &\equiv \langle\sigma^{(n_2)}_+(t_2)\sigma^{(n_1)}_-(t_1)\rangle_{_T} = \\ &= e^{-2iht}\left[G(m,t) + \frac{\partial}{\partial z}\right]\det\left[\hat{I} + \hat{V}_T - z\hat{R}^{(+)}_T\right]\Big|_{z=0} = \\ &= e^{-2iht}\left\{\left[G(mt)-1\right]\det\left[\hat{I}+\hat{V}_T\right] + \det\left[\hat{I}+\hat{V}_T-\hat{R}^{(+)}_T\right]\right\}\;, \end{aligned} \tag{9.13}$$

where function $G(m,t)$ (8.16) is the same as in (8.19) and operators $\hat{V}_T$ and $\hat{R}_T^{(+)}$ (acting on interval $[-\pi,\pi]$, see (9.5)) possess kernels

$$\begin{aligned} V_{_T}(p,q) &= V(p,q)\vartheta(q)\ , \\ R_{_T}^{(+)} &= R^{(+)}(p,q)\vartheta(q)\ , \end{aligned} \tag{9.14}$$

with functions $V(p,q)$ and $R^{(+)}(p,q)$ defined (for $-\pi \le p,q \le \pi$) by formulae (8.21), (8.22). Making similarity transform, as in (9.7), one rewrites (9.14) as

$$\begin{aligned} g_+^{(T)}(m,t) &= e^{-2iht}\left[G(m,t)+\frac{\partial}{\partial z}\right] \det\left[\hat{I}+\hat{V}_{_T}^{(S)} - z\hat{R}_{_T}^{(+,S)}\right]\Big|_{z=0}\ , \\ & m \ge 0\ , \end{aligned} \tag{9.15}$$

where the kernels of operators are symmetrical,

$$V_{_T}^{(S)}(p,q) = \frac{E_+^T(p)E_-^T(q) - E_-^T(p)E_+^T(q)}{\tan\frac{1}{2}(p-q)} - G(m,t)E_-^T(p)E_-^T(q)\ , \tag{9.16}$$

$$R_{_T}^{(+,S)} = E_+^T(p)E_+^T(q)\ , \tag{9.17}$$

and functions $E_\pm^T(p) \equiv E_\pm^T(m,t,p)$ are defined as

$$E_\pm^T(p) \equiv \sqrt{\vartheta(p)}\, E_\pm(p)\ ; \tag{9.18}$$

for functions $E_\pm(m,t,p)$ see (8.23).

For correlator $g_-^{(T)}(m,t)$ (1.12) one has (see (8.26)):

$$\begin{aligned} g_-^{(T)}(m,t) &\equiv \langle \sigma_-^{(n_2)}(t_2)\sigma_+^{(n_1)}(t_1)\rangle_{_T} = \\ &= e^{2iht}\frac{\partial}{\partial z} \det\left[\hat{I}+\hat{V}_{_T} + z\hat{R}_{_T}^{(-)}\right]\Big|_{z=0} = \\ &= e^{2iht}\frac{\partial}{\partial z} \det\left[\hat{I}+\hat{V}_{_T}^{(S)} + z\hat{R}_{_T}^{(-,S)}\right]\Big|_{z=0} \end{aligned} \tag{9.19}$$

where $\hat{V}_{_T}$, $\hat{V}_{_T}^{(S)}$ are the same operators (9.14), (9.16) and kernels of operators $\hat{R}_T^{(-)}$, $\hat{R}_T^{(-,S)}$ are

$$R_T^{(-)}(p,q) = R^{(-)}(p,q)\,\vartheta(q)\ . \tag{9.20}$$

(function $R^{(-)}(p,q)$ was defined in (8.27)) and

$$R_{_T}^{(-,S)}(p,q) = E_-^T(p)\,E_-^T(q) \tag{9.21}$$

(for function E_-^T see (9.18)). So the representations for the temperature and time dependent correlators are given.

In the equal-time case these representations acquire more explicit form; for correlator $g_+^T(m, t=0)$ one gets

$$\begin{aligned} g_+^T(m) &\equiv \langle \sigma_+^{(n_2)} \sigma_-^{(n_1)} \rangle_{_T} = \\ &= \frac{\partial}{\partial z} \det \left[\hat{I} + \hat{v}_{_T} + z \hat{r}_{_T}^{(+)} \right] \Big|_{z=0} , \qquad m > 0 , \end{aligned} \tag{9.22}$$

with operators $\hat{v}_{_T}$, $\hat{r}_{_T}^{(+)}$ acting on interval $[-\pi, \pi]$, their kernels being given (in the symmetrical form) as

$$\begin{aligned} \hat{v}_{_T}(p,q) &= -2\sqrt{\vartheta(p)} \, \frac{\sin \frac{m}{2}(p-q)}{\tan \frac{1}{2}(p-q)} \, \sqrt{\vartheta(q)} , \\ \hat{r}_{_T}^{(+)}(p,q) &= \sqrt{\vartheta(p)} \, \exp \left[\frac{im}{2}(p+q) \right] \sqrt{\vartheta(q)} , \end{aligned} \tag{9.23}$$

which is the generalization for the non-zero temperature case of representation (8.29).

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Appendix: Thermodynamical Limit for Functions g, e and d.

Functions $g(m,t)$, $e(m,t,p_a)$ and $d(m,t,p_a)$ on the finite lattice are defined by formulae (6.7)-(6.9) as

$$g(m,t) \equiv \frac{1}{M} \sum_q \exp\left[imq + 4it\cos q\right] , \tag{A.1}$$

$$e(m,t,p_a) \equiv \frac{1}{M}\sum_q \frac{\exp\left[imq+4it\cos q\right]}{\tan\frac{1}{2}(q-p_a)} , \tag{A.2}$$

$$d(m,t,p_a) \equiv \frac{1}{M^2}\sum_q \frac{\exp\left[imq+4it\cos q\right]}{\sin^2\frac{1}{2}(q-p_a)} . \tag{A.3}$$

The sums are taken over all M different permitted values (6.12) of momentum q,

$$\sum_q f(q) = \sum_{j=1}^{M} f(q_j) , \tag{A.4}$$

(see (6.11)). As permitted values of momenta p_a never coincide with any q_j, there are no singularities in these finite sums, for finite M.

In the thermodynamical limit momenta q_j fill the interval $[-\pi,\pi]$. As $q_{j+1}-q_j=\frac{2\pi}{M}$, (see (6.12)), one makes the following change:

$$\frac{1}{M}\sum_q \longrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} dq . \tag{A.5}$$

For function $g(m,t)$ the thermodynamical limit is obtained according to this rule as

$$\begin{aligned} g(m,t) \longrightarrow G(m,t) &\equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} dq\ \exp\left[imq+4it\cos q\right] = \\ &= i^m J_m(4t) , \end{aligned} \tag{A.6}$$

where J_m is a Bessel function.

For functions e, d, however, poles on the integration contour appear which should be taken into account.

Let us first consider $e(m,t,p_a)$. As

$$\sum_q \frac{1}{\tan\frac{1}{2}(q-p_a)} = 0 , \tag{A.7}$$

(see (6.10), (6.12)), one can write for finite M instead of (A.1)

$$e(m,t,p_a) = \sum_q \frac{\exp\left[imq+4it\cos q\right]-\exp\left[imp_a+4it\cos p_a\right]}{\tan\frac{1}{2}(q-p_a)} , \tag{A.8}$$

so that in the thermodynamical limit we get

$$\begin{aligned} e(m,t,p_a) \longrightarrow\ & E(m,t,p) = \\ =\ & \frac{1}{2\pi}\int_{-\pi}^{\pi} dq \frac{\exp\left[imq+4it\cos q\right]-\exp\left[imp+4it\cos p\right]}{\tan\frac{1}{2}(q-p)} \equiv \\ \equiv\ & \frac{1}{2\pi}\mathcal{P}\int_{-\pi}^{\pi} dq \frac{\exp\left[imq+4it\cos q\right]}{\tan\frac{1}{2}(q-p)} , \end{aligned} \tag{A.9}$$

where $\mathcal{P}$ means "Principal Value", and p is some value of momentum in the interval $[-\pi, \pi]$.

Analogously, using relation (A.7) and the following equality

$$\frac{1}{M^2}\sum_q \frac{1}{\sin^2\frac{1}{2}(q-p_a)} = 1\ , \tag{A.10}$$

one writes for $d(m,t,p_a)$ for finite M:

$$d(m,t,p_a) = \exp\left[imp_a + 4it\cos p_a\right] + \tag{A.11}$$
$$+\frac{1}{M^2}\sum_q \frac{\exp\left[imq+4it\cos q\right] - \exp\left[imp_a + 4it\cos p_a\right]\left[1+(im-4it\sin p_a)\sin(q-p_a)\right]}{\sin^2\frac{1}{2}(q-p_a)}$$

The subtraction in the numerator is easily seen to be equivalent, for $q \sim p_a$, to the subtraction of the first two terms in the expansion of $\exp\left[imq + 4it\cos q\right]$ at $q = p_a$, so that the sum in (A.11) is nonsingular in the thermodynamical limit, and can be represented as an ordinary integral:

$$d(m,t,p_a) \ \longrightarrow\ D(m,t,p) = \exp\left[imp + 4it\cos p\right] + \tag{A.12}$$
$$+\frac{1}{2\pi M}\int_{-\pi}^{\pi} dq \frac{\exp\left[imq+4it\cos q\right] - \exp\left[imp+4it\cos p\right]\left[1+(im-4it\sin p)sin(q-p)\right]}{\sin^2\frac{1}{2}(q-p)}\ .$$

Finally, using the identity

$$\frac{\partial}{\partial p}E(m,t,p) \ = \ \frac{\partial}{\partial p}\frac{1}{2\pi}\mathcal{P}\int_{-\pi}^{\pi} dq \frac{\exp\left[imq+4it\cos q\right]}{\tan\frac{1}{2}(q-p)} =$$
$$= \ \frac{\partial}{\partial p}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} dq \frac{\exp\left[imq+4it\cos q\right] - \exp\left[imp+4it\cos p\right]}{\tan\frac{1}{2}(q-p)}\right) = \tag{A.13}$$
$$= \ \frac{1}{4\pi}\int_{-\pi}^{\pi} \frac{\exp\left[imq+4it\cos q\right] - \exp\left[imp+4it\cos p\right]\left[1+(im-4it\sin p)sin(q-p)\right]}{\sin^2\frac{1}{2}(q-p)}$$

(see eq. (A.9) for the definition of E), one rewrites (A.12) as

$$d(m,t,p_a) \ \longrightarrow\ D(m,t,p) =$$
$$= \ \exp\left[imp + 4it\cos p\right] + \frac{2}{M}\frac{\partial}{\partial p}E(m,t,p)\ . \tag{A.14}$$

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