The limit shape phenomenon

Consider the tiling by dominoes, i.e. by rectangular tiles of size $1 \times 2$, of a square region of size $N$. A typical tiling looks as follows:

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Dominoes arrange themselves disorderedly. Of course ordered configurations can in principle be observed, but with lower and lower probability as $N$ increases.
Consider now, instead of a square region, the so-called Aztec Diamond

([http://en.wikipedia.org/wiki/Aztec_diamond](http://en.wikipedia.org/wiki/Aztec_diamond))

\[ N \times N \] square

Aztec Diamond of order \( \frac{N}{2} \)

\[ (N = 8) \]
Consider now, instead of a square region, the so-called Aztec Diamond

(http://en.wikipedia.org/wiki/Aztec_diamond)

Question:
What a typical tiling of the Aztec Diamond will look like?

$N \times N$ square  Aztec Diamond of order $N/2$

($N = 8$)
Domino tiling of an Aztec diamond

[Jockush-Propp-Shor '95]

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Domino tiling of an Aztec diamond

[Jockush-Propp-Shor '95]
We observe the emergence of four ordered ('frozen') regions in the corners, and a central disordered ('hot') region. The separation curve looks like a circle.

This can be formalized mathematically, and there is a theorem stating that in the large $N$ limit the separation curve is indeed exactly a circle (Arctic Circle Theorem, by Jockush, Propp and Shor, see http://arxiv.org/abs/math/9801068)
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