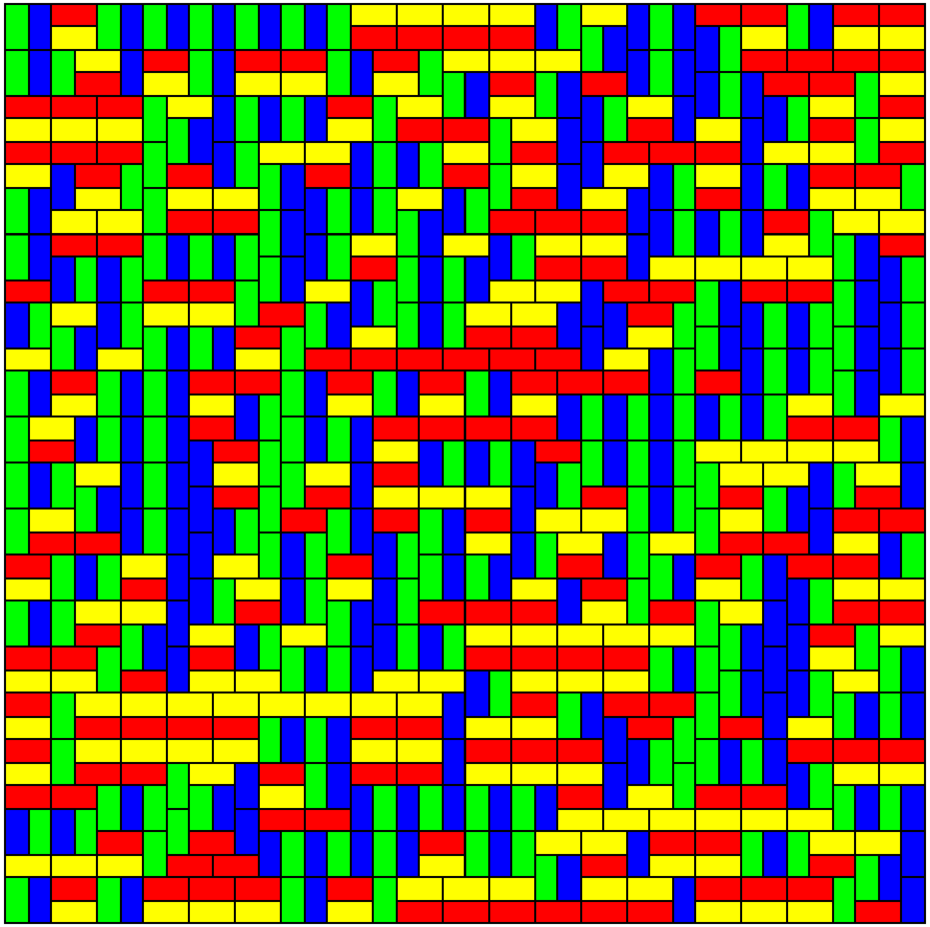
The limit shape phenomenon

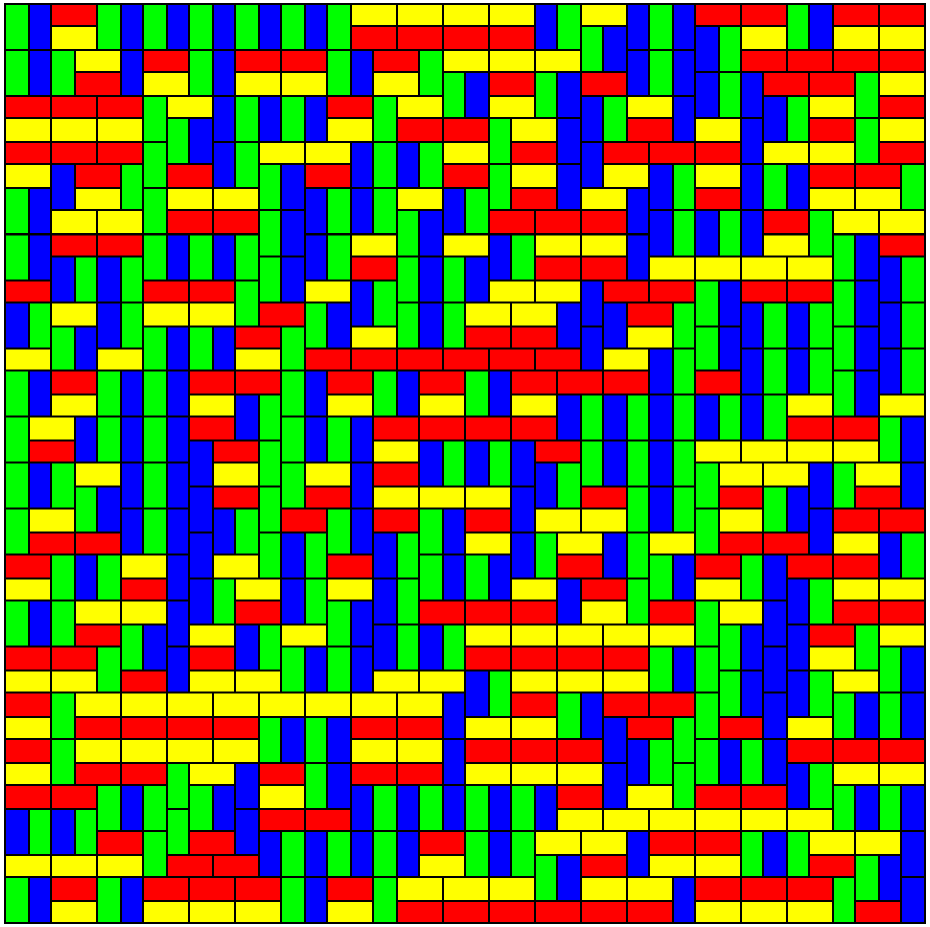
Consider the tiling by dominoes, i.e. by rectangular tiles of size 1×2, of a square region of size N. A typical tiling looks a follows:



http:/faculty.uml.edu/jpropp

The limit shape phenomenon

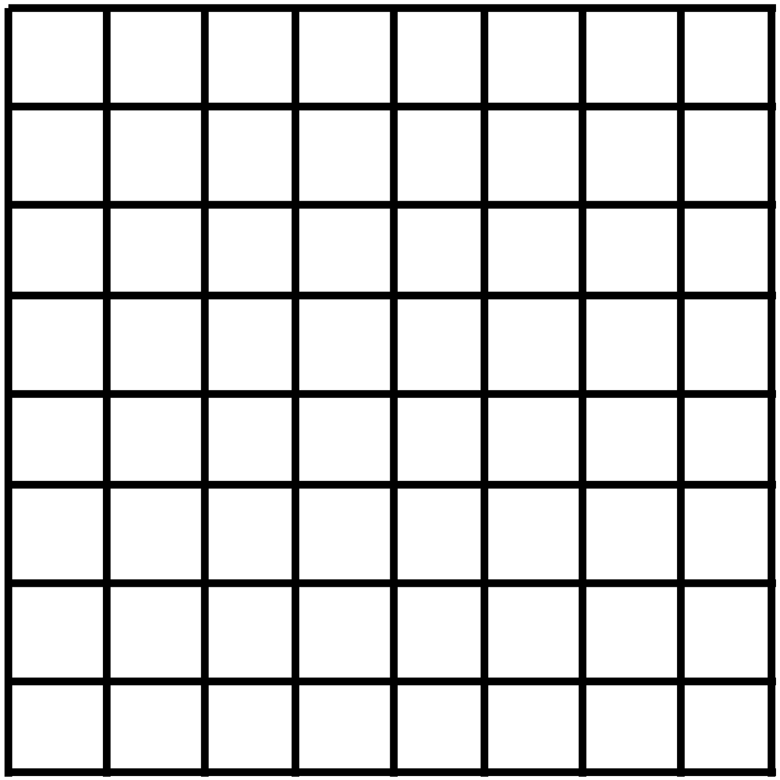
Consider the tiling by dominoes, i.e. by rectangular tiles of size 1×2, of a square region of size N. A typical tiling looks a follows:



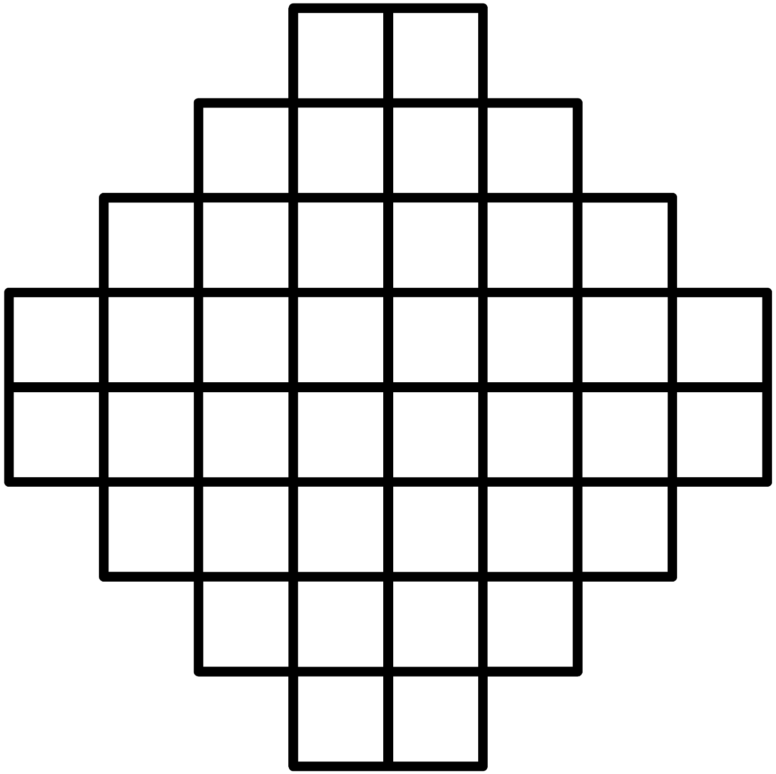
Dominoes arrange themselves disorderedly. Of course ordered configurations can in principle be observed, but with lower and lower probability as N increases.

Consider now, instead of a square region, the so-called Aztec Diamond

(http://en.wikipedia.org/wiki/Aztec_diamond)



$N \times N$ square

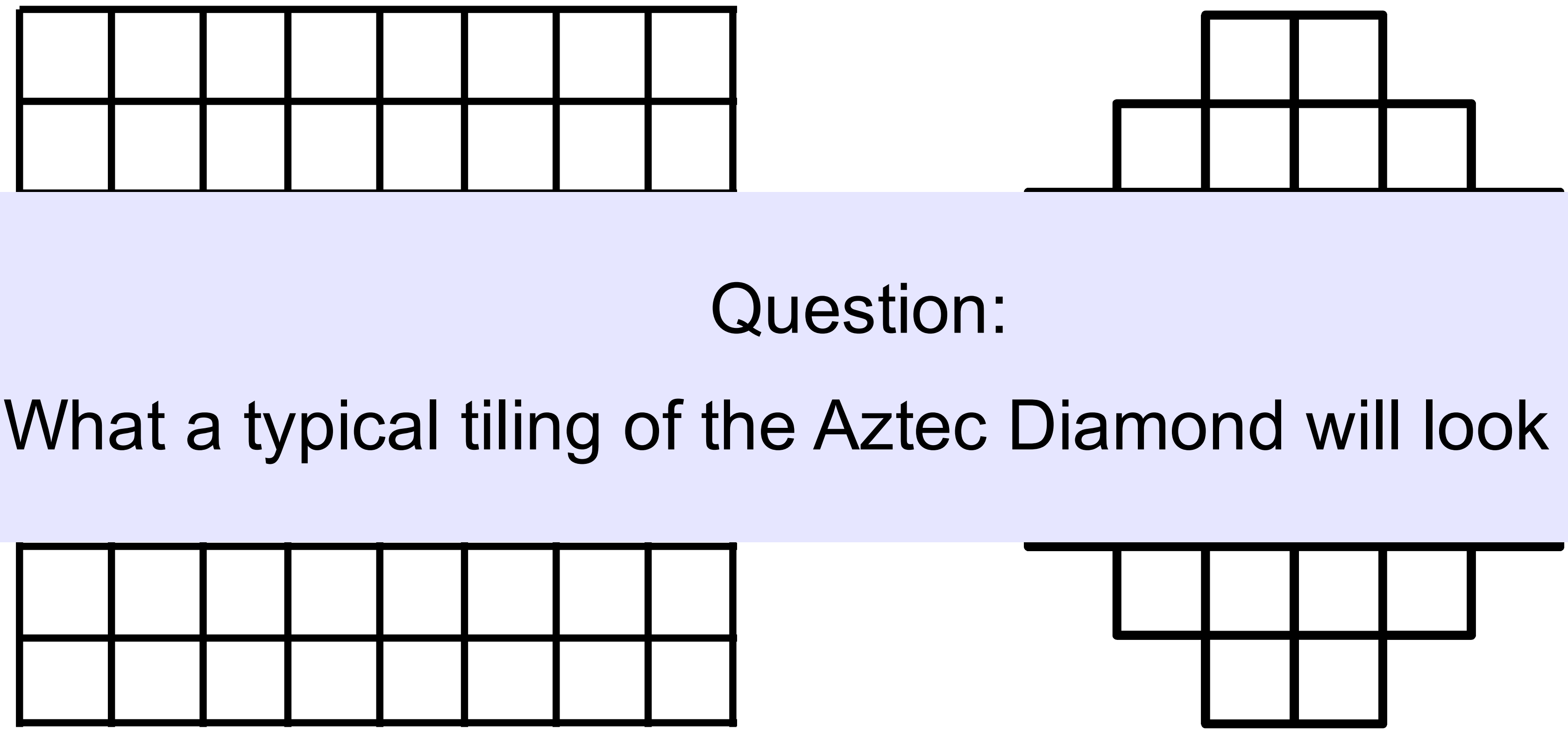


Aztec Diamond of order $N/2$

($N = 8$)

Consider now, instead of a square region, the so-called Aztec Diamond

(http://en.wikipedia.org/wiki/Aztec_diamond)



Question:

What a typical tiling of the Aztec Diamond will look like?

$N \times N$ square

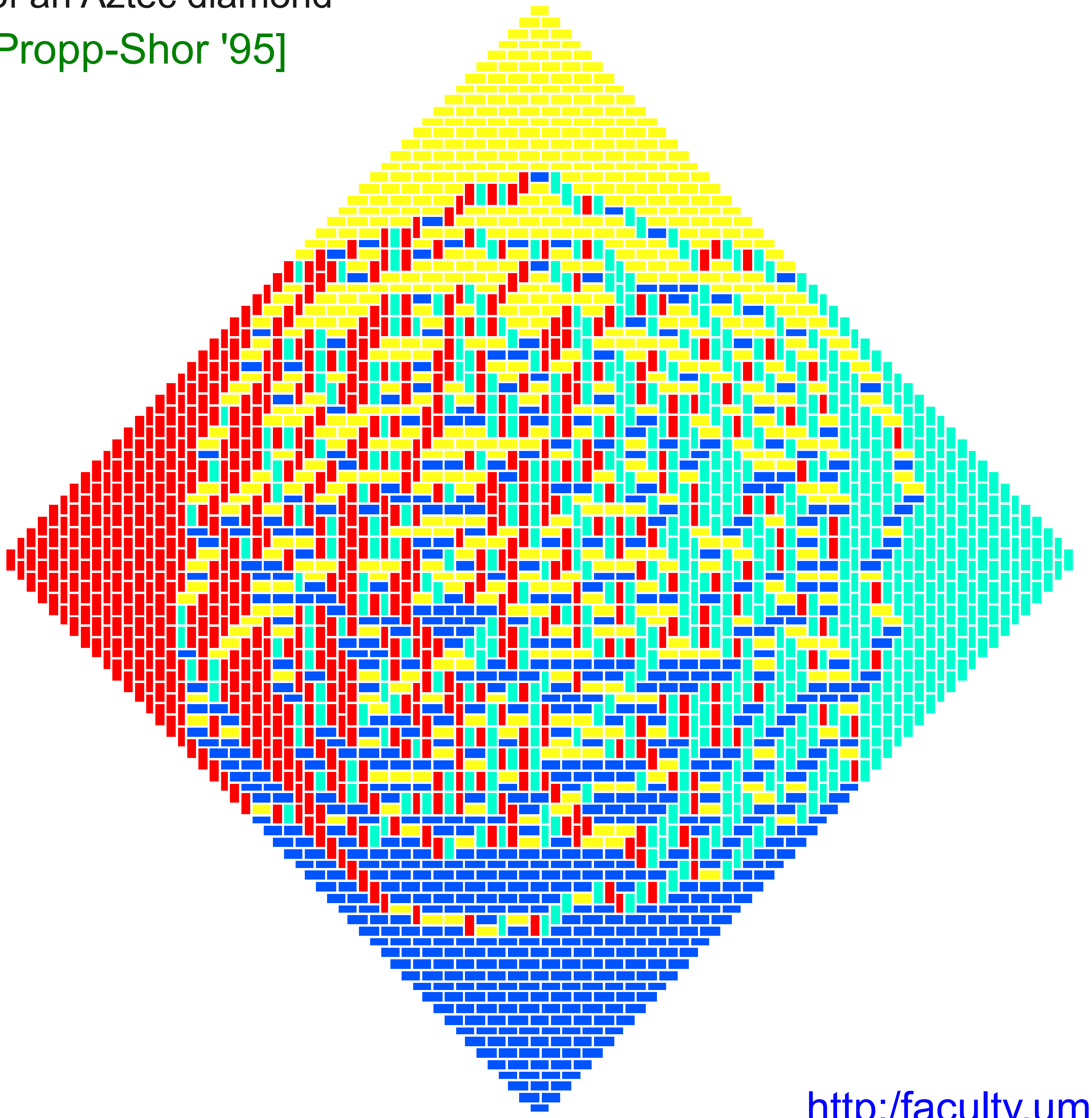
Aztec Diamond of order $N/2$

($N = 8$)

Domino tiling of an Aztec diamond

[Jockush-Propp-Shor '95]

$N = 64$

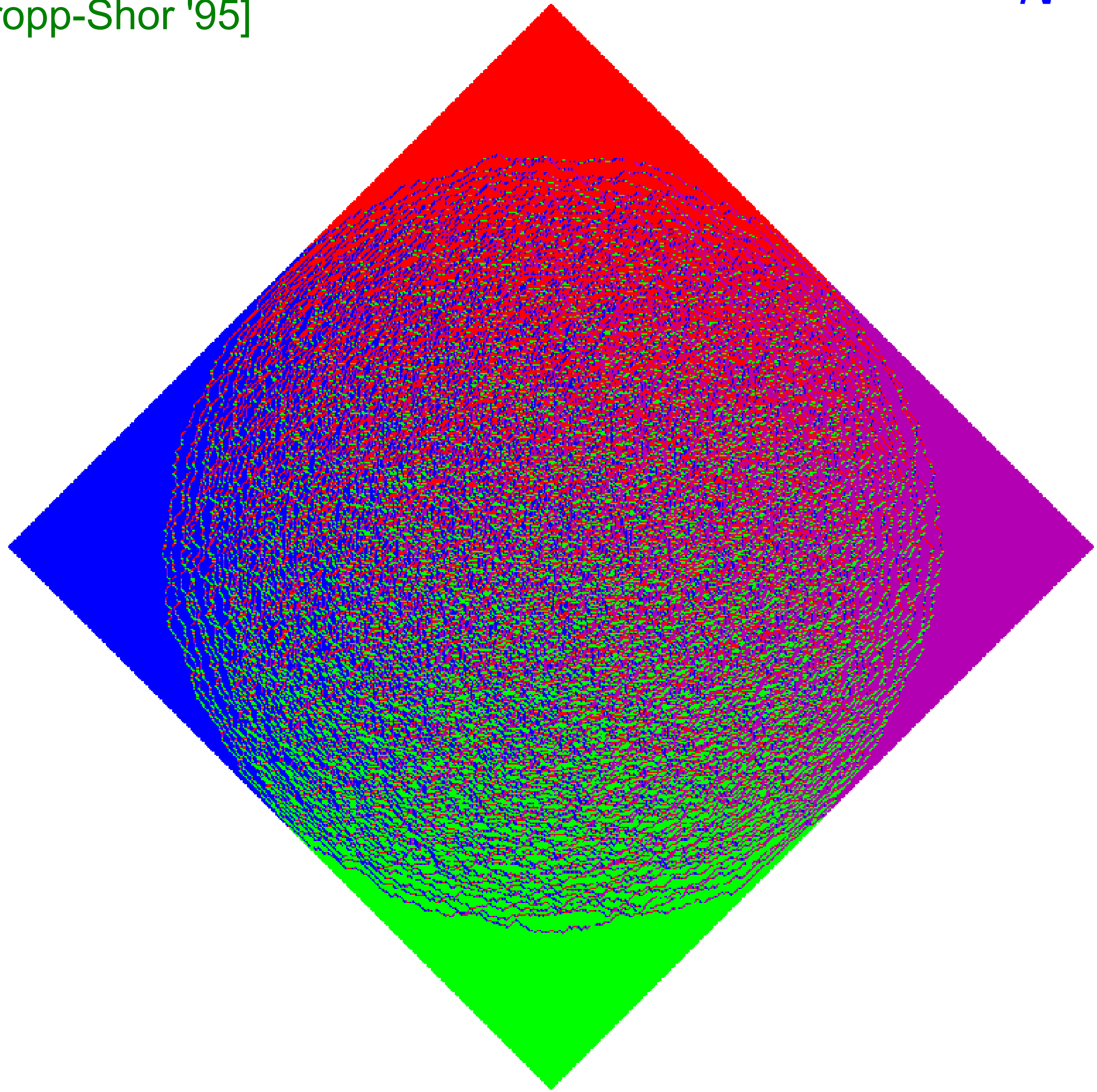


http:/faculty.uml.edu/jpropp

Domino tiling of an Aztec diamond

[Jockush-Propp-Shor '95]

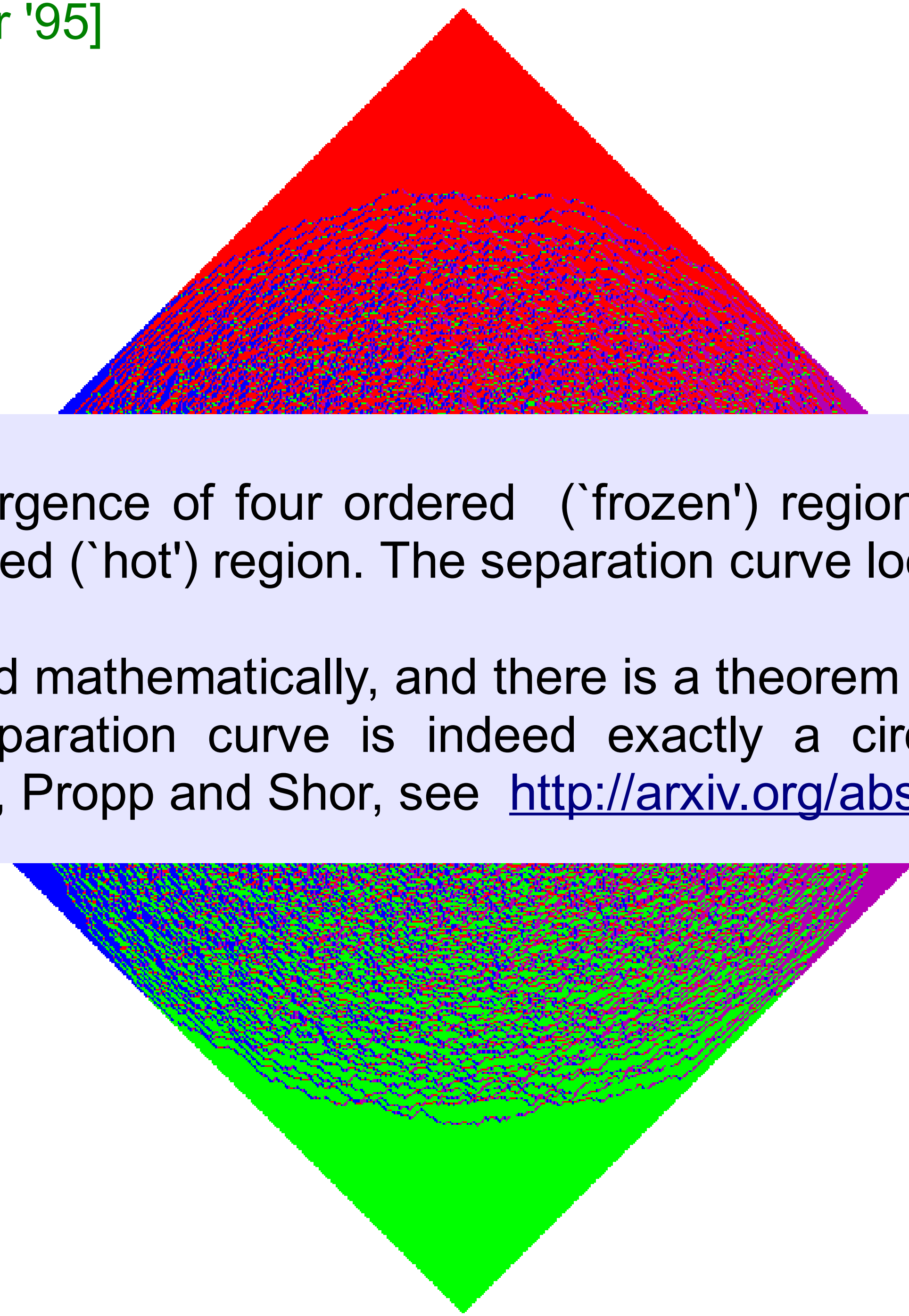
$N = 500$



Domino tiling of an Aztec diamond

[Jockush-Propp-Shor '95]

$N = 500$



We observe the emergence of four ordered (`frozen') regions in the corners, and a central disordered (`hot') region. The separation curve looks like a circle.

This can be formalized mathematically, and there is a theorem stating that in the large N limit the separation curve is indeed exactly a circle (Arctic Circle Theorem, by Jockush, Propp and Shor, see http://arxiv.org/abs/math/9801068)

Continua