

The Prehistory of Strings

From current-algebra to the Veneziano formula

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From the beginning of the 60's there was a great activity in particle physics leading in the following 12 years to the construction of the **Standard Model**, first the electroweak theory of Weinberg and Salam in 1967, and later the **Quantum Chromodynamics (QCD)** around 1973.

In the same period there was the birth of the **Dual Models** (1968) and their interpretation in terms of the **String Model**, formulated by **Nambu** in 1969 and independently by **Nielsen, Susskind and Takabayashi** in 1970. The original model was reformulated as a theory of fundamental interactions including gravity by **Scherk and Schwarz** in 1974.

The String Model has been rapidly growing in the following years and has become a very complex and formal theory, well justifying the name of **String Theory**.

In this talk I would like to remember very briefly the beginning of the story, following the line starting from **SU(3)** and continuing with **current algebra** and **superconvergence**, up to the **Veneziano formula**.

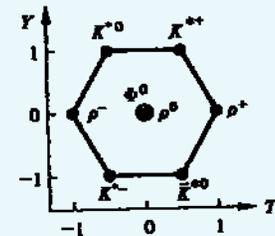
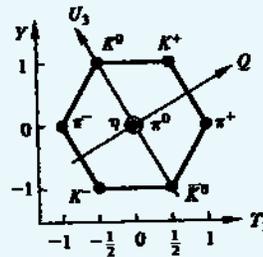
SU(3) symmetry

Let me start from 1961, which has been very important for at least three reasons. One is the first attempt by **Glashow** to unify the weak and electromagnetic interactions, with the introduction of the weak isospin.

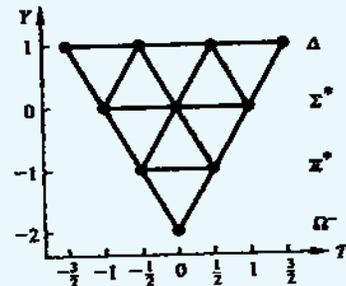
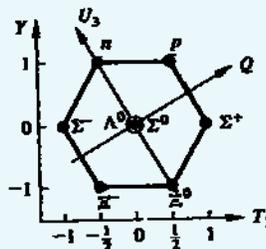
The second one is the introduction by **Ne'eman** and **Gell-Mann** of **SU(3)** as an approximate symmetry of strong interactions. This gave a simple scheme to classify the known mesons and baryons into octets (or nonets) and decuplets.

The third one is the first proposal by **Gell-Mann** of the **current algebra**.

Pseudoscalar and vector meson octets



Baryon octet and baryonic resonances decuplet



Vector and axial vector currents

In another paper of 1961 **Gell-Mann** proposed that the hadronic e.m. current and the vector currents of the weak interactions belong to the same SU(3) octet J^μ_i ($i= 1, \dots, 8$):

- electromagnetic current: $J^\mu_{\text{em}} = J^\mu_3 + \frac{1}{\sqrt{3}} J^\mu_8$
- weak current $\Delta S=0, \Delta Q=+1$: $J^\mu_{\Delta S=0} = J^\mu_1 + i J^\mu_2$
- weak current $\Delta S=\Delta Q=+1$: $J^\mu_{\Delta S=1} = J^\mu_4 + i J^\mu_5$

Similarly, the axial vector currents $J^{\mu 5}_i$ form another octet.

The weak interactions, responsible e.g. of the baryonic β -decay, were given by the **Feynman-Gell-Mann V–A** theory of 1958, in the current by current form $L_w = G \hat{A}^\mu \ell_\mu$, where \hat{A}^μ and ℓ_μ are the hadronic and the leptonic currents. In the framework of SU(3), the hadronic current was given by **Cabibbo** in 1963 by

$$\hat{A}^\mu = \cos \theta [(J^\mu_1 + i J^\mu_2) - (J^{\mu 5}_1 + i J^{\mu 5}_2)] \\ + \sin \theta [(J^\mu_4 + i J^\mu_5) - (J^{\mu 5}_4 + i J^{\mu 5}_5)]$$

The quark model

The absence of candidates for the fundamental 3 representation of SU(3) suggested Gell-Mann in 1964 the hypothesis of the quarks.

According to this model, all the known particles are bound states of three quarks, named **u** (for “up”), **d** (for “down”) and **s** (for “strange”) (They are the “light” quarks of today). They have fractional charges, namely $Q=+2/3, -1/3, -1/3$ (in e units) respectively, and baryonic charge $B=+1/3$.

- Mesons = $q\bar{q}$ (e.g. $\pi^+ = u\bar{d}$; $K^- = s\bar{d}$)
- Baryons = qqq (e.g. $p = uud$; $\Lambda = [ud]s$)

The quark model also supplies a field-theoretical expression for the currents:

$$J^\mu_i = 1/2 \bar{q} \gamma^\mu \lambda_i q; \quad J^{\mu 5}_i = 1/2 \bar{q} \gamma^\mu \gamma^5 \lambda_i q$$

where $q=q(x)$ is the quark 3x4 spinor field and λ_i are the 3x3 matrices of the SU(3) generators.

Current Algebra

In the paper of 1961 quoted before, Gell-Mann proposed that the vector charges $F_i(t) = \int J_i^0(x) d^3x$ and the axial charges $F_i^5(t) = \int J_i^{05}(x) d^3x$ satisfy the equal time commutation relations

$$[F_i(t), F_j(t)] = if_{ijk} F_k$$

$$[F_i(t), F_j^5(t)] = if_{ijk} F_k^5$$

$$[F_i^5(t), F_j^5(t)] = if_{ijk} F_k$$

representing the $SU(3) \otimes SU(3)$ Lie algebra, where f_{ijk} are the $SU(3)$ structure constants.

Gell-Mann pointed out that this algebra could give important predictions, because the equations are non-linear and therefore they fix the magnitude of the charges.

In 1964 Gell-Mann extended the charge algebra to the current algebra, i.e. the local equal time commutators given by the quark model

$$[J_i^0(\mathbf{x}, t), J_j^0(\mathbf{y}, t)] = if_{ijk} J_k^0(\mathbf{x}, t) \delta(\mathbf{x}-\mathbf{y}), \quad \text{etc.}$$

Sum rules

The first idea to exploit the CA relations is to take them between one particle states and to insert a complete set of states. Take e.g. the commutator

$$[F_+^{5}, F_-^{5}] = 2 F_3$$

where $F_{\pm}^{5} = F_1^{5} \pm F_2^{5}$ and $F_3 = I_3$ is the isotopic spin operator. We get:

$$\sum_n [|\langle a | J_+^{05}(0) | n \rangle|^2 - |\langle a | J_-^{05}(0) | n \rangle|^2] (2\pi)^3 \delta(\mathbf{p}_n - \mathbf{p}_a) = 4E_a I_3(a)$$

This formula (for finite p_a) has however two basic drawbacks:

a) $q^2 = (p_a - p_n)^2 = (E_a - E_n)^2 > 0$

and this forbids to relate the sum to a scattering process where $q^2 < 0$;

b) q^2 *increases without limit with the mass of n*

and this makes it difficult to guess the convergence of the series.

A good idea was devised by **Fubini** and **Furlan** and is to take the limit $p_a \rightarrow \infty$. The method has been called

The infinite momentum limit

For $\mathbf{p}_a = \mathbf{p}_n = \mathbf{p}$ and $|\mathbf{p}| \rightarrow \infty$ we have

$$q^2 = (E_a - E_n)^2 = [(\mathbf{p}^2 + m_a^2)^{1/2} - (\mathbf{p}^2 + m_n^2)^{1/2}]^2 \rightarrow 0$$

for any intermediate state. This changes the previous sum rule in a fixed $q^2=0$ sum rule.

An important ingredient is the PCAC. The matrix elements $\langle a | J_{\pm}^{05}(0) | n \rangle$ can be written in the form

$$\langle a | J_{\pm}^{05}(0) | n \rangle = -i(E_a - E_n)^{-1} \langle a | \partial_{\mu} J_{\pm}^{\mu 5}(0) | n \rangle$$

and the divergence of the axial current is related to the pion field by the PCAC relation of Gell-Mann and Levy (1960)

$$\partial_{\mu} J_{\pm}^{\mu 5}(x) = f_{\pi} \mathbf{f}_{\square}(x)$$

This allows one to express the matrix element of the axial current in terms of the $a n \pi$ vertex.

It is important to notice that the local commutator of the time components of the currents would lead to sum rules with $q^2 \neq 0$.

Some relevant sum rules

- Starting from the axial charge commutator, the PCAC relation and the infinite momentum sum rule, **Adler** and **Weisberger** in 1964 were able to calculate the renormalization factor of the axial vector coupling constant of the neutron β decay, in terms of the total cross section of pion-proton scattering. The result is correct within 5%.

Other important relations are:

- The **Callan-Treiman** relation (1965), connecting the leptonic decays of the K meson $K \rightarrow \pi l \nu$ and $K \rightarrow \pi \pi l \nu$.
- The **Cabibbo-Radicati** sum rule (1966), giving a combination of the e.m. form factors of the nucleon in terms of the photon-nucleon total cross section. This was the first tested sum rule at $q^2 \neq 0$.
- The **Weinberg** calculation (1966) of the $K \rightarrow \pi \pi l \nu$ form factors, from which the decay rate of $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$ results in excellent agreement with experiment.
- The **Weinberg** theory of multiple pion production and the calculation of the pion scattering lengths (1966).

Scattering and Superconvergence

A different subject developed in the years 1966-68: the **superconvergence**. This property was discovered by **De Alfaro, Fubini, Furlan and Rossetti** studying the sum rules from the local current commutators. However it has nothing to do with currents and is only concerned with strong interactions.

Consider the two-body scattering

$$a+b \rightarrow c+d$$

The scattering is described by a **scattering amplitude** defined as follows.

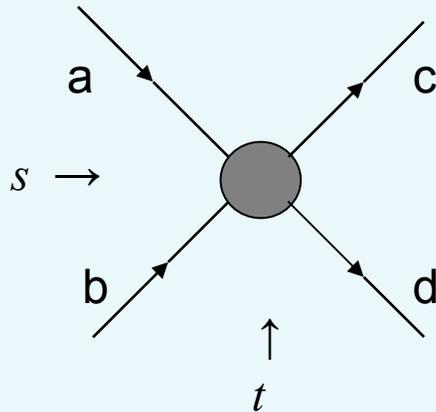
- The **scattering matrix** S is a unitary operator that transforms the initial state (at $t \rightarrow -\infty$) into the state evolved at $t \rightarrow +\infty$.
- The **transition matrix** T is defined by $S = I + iT$. The unitarity relation $S^\dagger S = I$ then gives $T - T^\dagger = iT^\dagger T$.
- For a scattering process $|i\rangle \rightarrow |f\rangle$ we define the **scattering amplitude** M_{fi} by

$$\langle f | T | i \rangle = (2\pi)^4 \delta(P_f - P_i) M_{fi}$$

- The amplitude M_{fi} is then expanded as

$$M_{fi} = \sum_r K_{fi}^r A_r(s, t, u)$$

where K_{fi}^r are **covariant factors** depending on spin, isotopic spin and momentum components of the external particles and $A_r(s, t, u)$ are **invariant amplitudes**, depending only on the Mandelstam variables s , t and u (only 2 independent).



$$\begin{aligned}
 s &= (p_a + p_b)^2 \\
 t &= (p_b - p_d)^2 \\
 u &= (p_a - p_d)^2 \\
 s + t + u &= \sum_i m_i^2
 \end{aligned}$$

The invariant amplitude $A(s, t, u)$ has the following fundamental properties.

- **Crossing symmetry.** The same amplitude (in different regions) describes the reactions $ab \rightarrow cd$, $bd \rightarrow ac$, $ad \rightarrow cb$ and their inverses (by CPT).
- **Unitarity.** The amplitude obeys the unitarity relation coming from that for M_{fi} :

$$M_{fi} - M_{if}^* = i(2\pi)^4 \sum_n M_{nf}^* M_{ni} \delta(P_n - P_i)$$

- **Analyticity.** $A(s, t, u)$ at fixed t is analytic in the complex plane of the variable $v = \frac{1}{4}(s - u)$, with singularities (poles and cuts) on the real axis and obeys a dispersion relation of the form

$$A(v, t) = \frac{1}{\pi} \int \frac{\text{Im} A(v', t)}{v' - v} dv' \quad \text{if } \text{Im} A(v', t) \rightarrow 0 \text{ for } v' \rightarrow \infty$$

- **Asymptotic behaviour.** In the Regge Pole Model, for $s \rightarrow \infty$ and fixed t we have

$$A(s, t) \rightarrow \beta(t) \xi(t) s^{\alpha(t)}$$

where $\alpha(t)$ is the Regge trajectory, $\beta(t)$ is an entire function of t and $\xi(t) = (1 \pm e^{-i\pi\alpha}) \Gamma(-\alpha)$.

In the following we shall always consider linear trajectories, i.e. with $\alpha(t) = \alpha_0 + \alpha' t$.

Superconvergence sum rules

Consider the scattering of particles with spin, for example the $\rho\pi$ forward ($t=0$) scattering. There are 3 kinds of invariant amplitudes, with helicity flip in the t channel of 0, 1 and 2, that we shall call A_0 , A_1 and A_2 . Their large s behaviours are s^α , $s^{\alpha-1}$, $s^{\alpha-2}$ respectively, where $\alpha=\alpha(0)$ is the leading Regge trajectory. Consider in particular the amplitudes with isospin 1 in the t channel, which are dominated by the ρ -trajectory, with $\alpha(0)\approx 0.5$. The amplitudes $A_1(v)$ and $A_2(v)$ are convergent for $v\rightarrow\infty$, are odd functions of v by crossing symmetry, hence with an even absorptive part, and obey the dispersion relation

$$A(v) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\text{Im } A(v')}{v'-v} dv'$$

Now for $A_2(v)$ we have for large v ($\approx \frac{1}{2} s$): $vA_2(v)\rightarrow 0$. Then from the above DR we obtain

$$\int \text{Im } A_2(v) dv = 0$$

which is the **superconvergence relation**. When $\text{Im } A_2$ is expressed by unitarity as a sum of contributions of the intermediate states in the s and u channels we get a **superconvergence sum rule (SSR)**.

First applications of SSR

- The first important example was the sum rule obtained by **Drell and Hearn** in 1966. They considered the spin-flip amplitude of the photon-proton Compton scattering and obtained a relation giving the anomalous magnetic moment of the proton (from the single proton intermediate state) in terms of the γp total cross section.
- **De Alfaro, Fubini, Furlan and Rossetti**, in the original SSR paper, studied the ρ - π forward scattering and considered two SSR, the one for A_2 seen before and the one for A_1 and isospin 2 in the t channel. Saturating the SSR only with the low-lying resonances ω and ϕ , they obtained a relation between the coupling constants $g_{\omega\rho\pi}$ and $g_{\phi\rho\pi}$, in good agreement with experiment.
- **Igi and Matsuda** and independently **Logunov, Soloviev and Tavkhelidze** and **Raoul Gatto** in 1967 analyzed the πN scattering with charge exchange ($\pi^- p \rightarrow \pi^0 n$). This is not a SSR, because the integral does not converge, due to asymptotic ρ -trajectory contribution. Taking this out, they obtained a relation between the πp total cross sections and the parameters of the ρ -trajectory.

Finite energy sum rules

An important step forward was made by Dolen, Horn and Schmid in 1967. Inspired by the previous Authors, they extended the SSR to the following relation, called a **finite energy sum rule (FESR)**:

$$\int_0^N \text{Im } A(\nu, t) d\nu = \sum_i \frac{\beta_i(t) N^{\alpha_i+1}}{\Gamma(\alpha_i+2)}$$

Here N is a *finite* value of ν , $\alpha_i = \alpha_i(t)$ and the r.h.s. is the sum of all Regge terms. Notice that the SSR is reobtained in the limit $N \rightarrow \infty$, if $\alpha_i < -1$. This relation, however, holds for any value of α_i .

The Authors applied the FESR to the scattering $\pi^- p \rightarrow \pi^0 n$. From the low-energy πN data they predicted the parameters of the ρ -meson Regge trajectory as functions of t .

The importance of the FESR lies just in this property, that it relates the low-energy data in the s and u channels to the Regge trajectories in the t channel. We can say that such a relation represents a sort of **bootstrap**.

SSR for two meson scattering

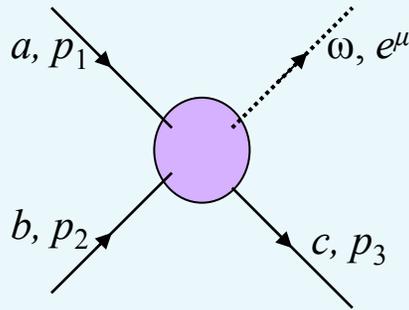
Let me now come to my personal reminiscences. In the spring of 1967, i.e. exactly 40 years ago, I went for two weeks to Israel, at the Weizmann Institute, to collaborate with **Gabriele Veneziano** to the new idea of superconvergence. There I met **Hector Rubinstein** and **Miguel Virasoro**, and together we started a collaboration that went on for more than one year.

In a first paper we discussed the saturation of the SSR at $t \neq 0$. Separating the resonance from the Regge contribution we obtained a FESR that can be analytically continued to the region where the integral of the SSR would diverge. Studying in particular the $\rho\pi$ scattering, we recognized the necessity of the Regge contribution to satisfy the SSR at all t .

We then analyzed the saturation of SSR for several processes of meson-meson scattering, of the type $PP \rightarrow PV$, $PV \rightarrow PV$, $PP \rightarrow PT$, where P, V, T stand for pseudoscalar, vector and tensor (spin 2) mesons.

The scattering $\pi\pi \rightarrow \pi\omega$

The most interesting case was the $\pi\pi \rightarrow \pi\omega$ scattering, where the s , t and u channels are identical. There is only one invariant amplitude $A(s, t, u)$, fully symmetric in the Mandelstam variables s , t and u and dominated by the ρ -meson.



$$\langle e, c, p_3 | M | a, p_1; b, p_2 \rangle = \varepsilon_{abc} \varepsilon_{\mu\nu\rho\sigma} e^\mu p_1^\nu p_2^\rho p_3^\sigma A(s, t, u)$$

a, b, c = pion isospin components

p_i = pion momenta; e^μ = ω -polarization

We take the amplitude $A(v, t)$ as a function of the independent variables $v = 1/4(s-u)$ and t . The behaviour for large v is given by

$$A(v, t) \rightarrow \beta(t) \xi(\alpha(t)) (v/v_1)^{\alpha(t)-1}$$

where

$$\xi(\alpha) = (1 - e^{-i\pi\alpha}) / \sin(\pi\alpha)$$

$$\beta(t) = \beta_0 / \Gamma(\alpha)$$

Bootstrap of the ρ -trajectory

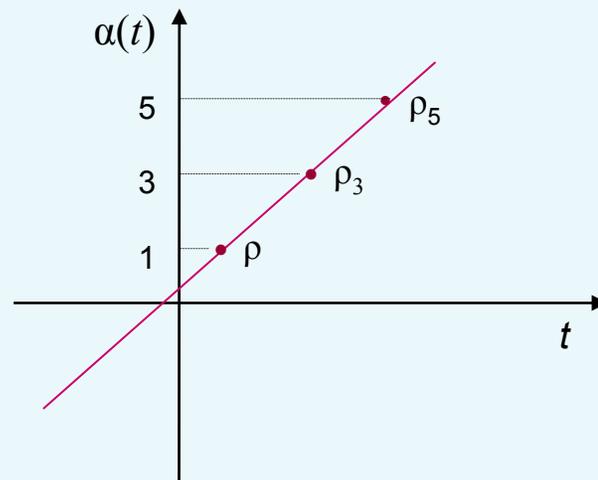
We considered the sum rule

$$\int_0^{\bar{v}} v^n \text{Im} A(v, t) dv = \frac{\beta(t)}{\alpha(t) + n} \left(\frac{\bar{v}}{v_1} \right)^{\alpha(t)-1} \bar{v}^{n+1}$$

for a general moment n , where \bar{v} is the limit of the resonance region and v_1 is an arbitrary scale parameter.

Since $A(v, t)$ is an **even** function of v by s - u crossing symmetry, and then $\text{Im} A(v, t)$ is **odd**, the first nontrivial moment in the sum rule is $n=1$.

For the resonance part on the l.h.s. we considered the ρ -meson and the higher resonances ρ_3 and ρ_5 with spin $j=3$ and $j=5$ respectively, lying on the leading ρ -trajectory, and for the high energy on the r.h.s. the ρ -trajectory itself.



The preceding sum rule represents a consistency condition for the parameters of the ρ -trajectory and can be seen as a true **bootstrap of the ρ -trajectory** itself. The numerical results were very good even with the insertion of the ρ alone and became better with the higher resonances, as shown in the figures.

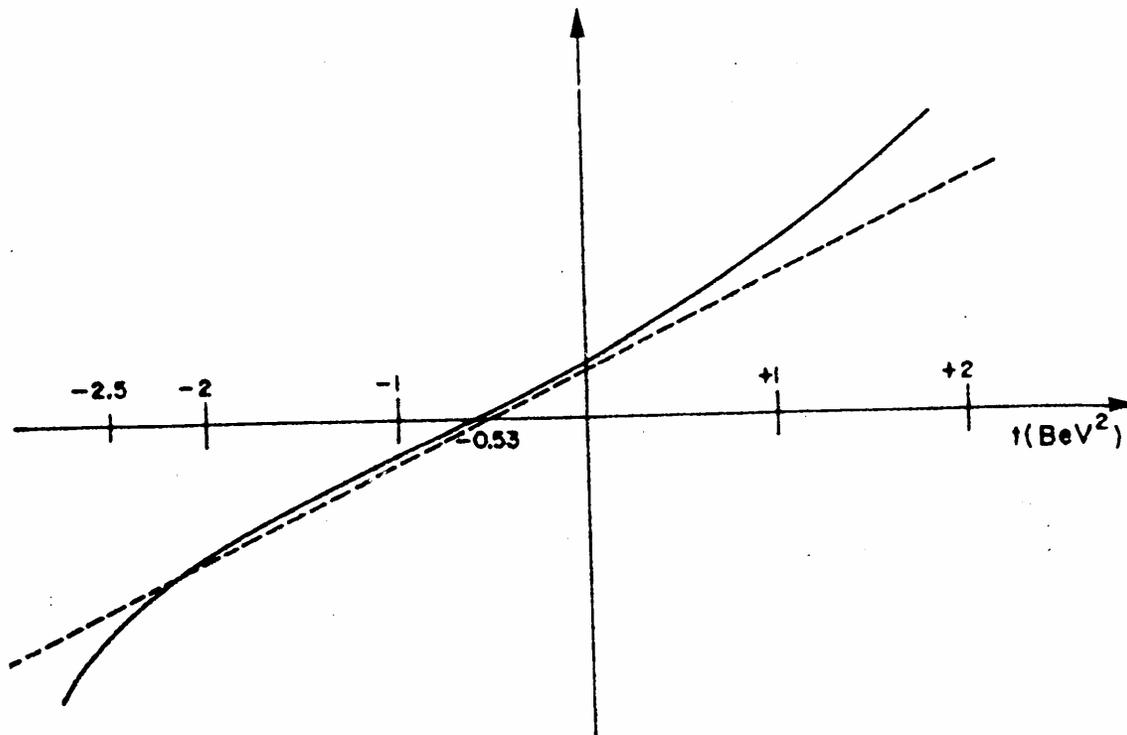


FIG. 2. Saturation of the $\pi\pi \rightarrow \pi\omega$ sum rules with the ρ resonance alone. Dashed line represents the resonance side and full line the Regge side. Ordinates in arbitrary units.

Saturation of the SR with ρ and ρ_3 :

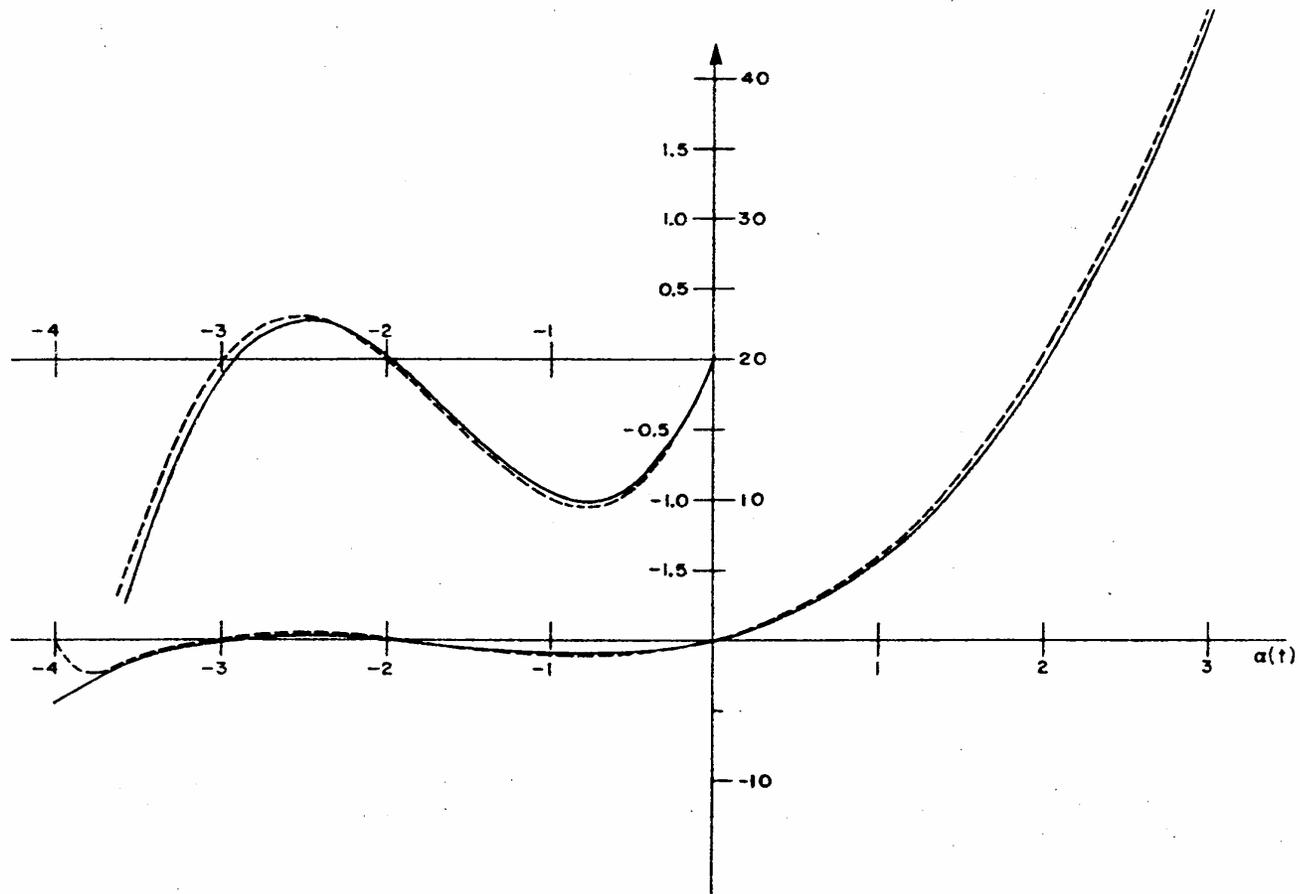


FIG. 4. Saturation of the same sum rule as in Fig. 2 with the ρ and the $\rho_3(3^-)$ in the resonance side. On the upper left side the most important region is shown on a larger scale. Here the Regge side is represented by the dashed line.

Saturation of the SR with $\rho(1^-)$, $\rho_3(3^-)$ and $\rho_5(5^-)$:

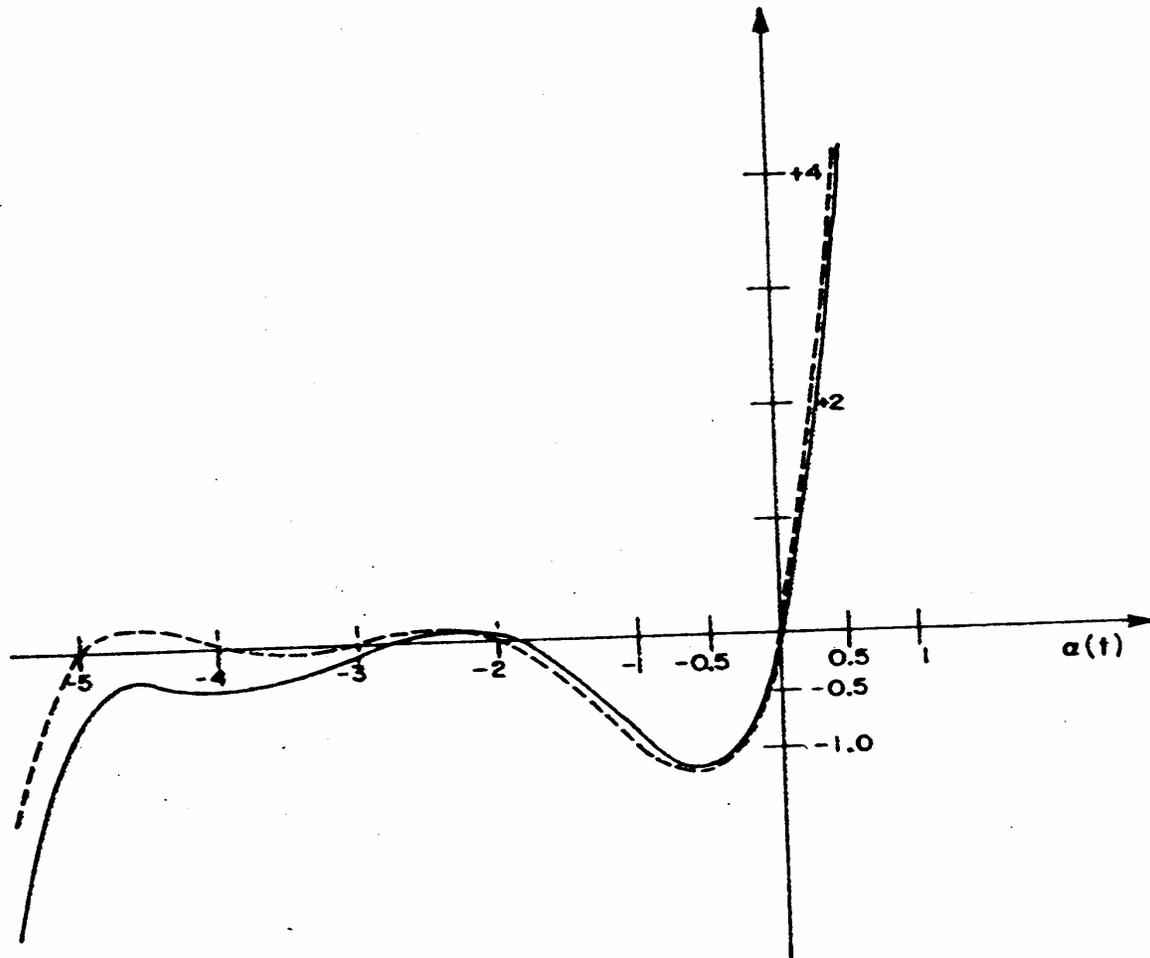


FIG. 5. Same as in Figs. 2 and 4 with the $\rho_5(5^-)$ included in the resonance side. Here the Regge side is represented by a dashed line.

The concept of “duality”

The **bootstrap**, however, is not yet **duality**. We can define a **dual model** as a model of scattering theory based on the **narrow resonance approximation**, i.e. in which only **single particles** are exchanged.

Consider the scattering of $2 \rightarrow 2$ spinless particles. In a dual model the invariant amplitude **at fixed t** can be expanded in terms of **s and u poles**:

$$A(s, t) = \sum_n \frac{R_n(t)}{s - s_n} + \sum_m \frac{\tilde{R}_m(t)}{u - u_m}, \quad u = -s - t + \sum m_i^2$$

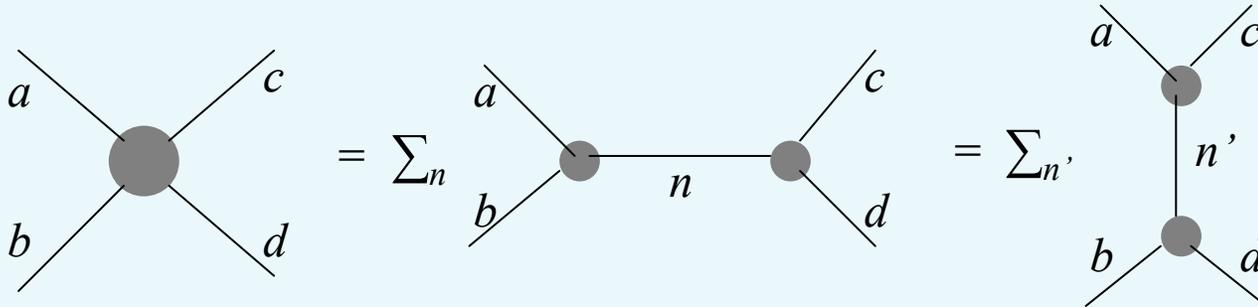
where $R_n(t)$ and $\tilde{R}_m(t)$ are polynomials in t and s_n, u_m are the m^2 values of the resonances. Furthermore $R_n(t)$ will be the product of the vertices (abn) and (ncd) and similarly for $\tilde{R}_m(t)$. The same amplitude **at fixed s** can also be expanded in terms of the **t and u poles**:

$$A(s, t) = \sum_n \frac{S_n(s)}{t - t_n} + \sum_m \frac{\tilde{S}_m(s)}{u - u_m}$$

In the case that there are no resonances in the u channel, as in $\pi^+\pi^-$ scattering, the duality relation has the simple form

$$A(s,t) = \sum_n \frac{R_n(t)}{s - s_n} = \sum_{n'} \frac{S_{n'}(s)}{t - t_{n'}}$$

This relation can be represented graphically as follows:



General comments

Some general comments are needed.

- A dual amplitude requires an **infinite number of particles**. Consider the scattering of two spinless particles of equal mass in the c.m.s. of the s channel. The contribution of an intermediate state of **spin l** is of the form

$$A_l(s) P_l(\cos \theta); \quad \cos \theta = 1 + 2t/(s - 4m^2)$$

where θ is the scattering angle. Since P_l is a polynomial, a finite number of such contributions still gives a polynomial in t and cannot give a pole.

Furthermore the asymptotic behaviour in s would be s^{-1} , and not $s^{\alpha(t)}$.

- The dual amplitude **is quite different** from the one of perturbative field theory at lowest order, where the pole (Born) terms in the s, t and u channels have to be added. Furthermore, field theory at higher orders gives many-particle intermediate states, as is required by unitarity.
- In fact **the dual model is not unitary**. This can be seen at least in two ways.
(i) The unitarity relation gives $\text{Im } A$ as a sum of contributions of **all** the physical intermediate states, including many-particle states. (ii) The intermediate particles are coupled to the external ones; therefore they can decay (when allowed) with a certain rate. This gives the resonance a finite width and places a pole in the complex s plane out of the real axis.

Difficulties of strong interaction theories

At the end of the 60's, i.e. before QCD, a fundamental theory of strong interactions did not exist. There were some general theories, but with a limited use.

- **Field theory** was very successful in describing e.m. and (after 1967) weak interactions. For strong interactions, however, there are some basic difficulties. (i) There are too many hadrons and too many forces to construct a sensible interaction Lagrangian. Infinite component field theories have also been considered, but the arbitrariness is too big to build a useful theory. On the other hand, the idea to take few elementary particles (the quarks!) to explain the spectrum of the hadrons and their interactions was at the time far from feasibility. (ii) The theory is based on perturbative expansion by Feynman diagrams, but here the coupling constants are large, then the higher order corrections are also large and the perturbative expansion becomes meaningless.
- **S-matrix theory**. The ambitious program enounced by **Chew** (1960-65) can be condensed in the principle of the **bootstrap of strong interactions**. It affirms that the experimental data are self-determined through the principles of the S-matrix. This would give an infinite set of coupled non-linear equations, whose solution is of course hopeless. Therefore, even if the principles are correct, in practice some drastic approximations are necessary.

The Veneziano formula

Soon after our collaboration on superconvergence and the bootstrap, **Gabriele Veneziano** had the brilliant idea of his famous formula. He considered the simple case of the scattering $\pi\pi \rightarrow \pi\omega$ and was looking for an amplitude with the following analytic properties:

- (i) an infinite set of poles in the s and u channels at fixed t ;
- (ii) asymptotic behaviour for large s and fixed t like $s^{\alpha(t)}$;
- (iii) complete crossing symmetry in the three channels.

He started from the large s behaviour of the amplitude we considered for the bootstrap, that can be written in the form

$$A(s, t, u) \rightarrow \beta \Gamma(1 - \alpha(t)) (-\alpha(s))^{\alpha(t) - 1}$$

The idea was to hold the first factor, that gives the poles for $\alpha(t) = n \geq 1$, and to consider the second factor as valid only asymptotically. Therefore he replaced this factor by

$$(-\alpha(s))^{\alpha(t) - 1} \rightarrow \Gamma(1 - \alpha(s)) / \Gamma(2 - \alpha(s) - \alpha(t))$$

This has the following nice properties: (i) it has poles for $\alpha(s) = n \geq 1$; (ii) it avoids simultaneous poles in s and t ; (iii) it has the right asymptotic behaviour.

Finally, the complete crossing-symmetric **Veneziano** amplitude is given by

$$A(s,t,u) = \beta \mathcal{R} [B(1-\alpha(t), 1-\alpha(s)) + B(1-\alpha(t), 1-\alpha(u)) + B(1-\alpha(s), 1-\alpha(u))]$$

where $B(x,y)$ is the Euler Beta function, given by

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = \int_0^1 t^{x-1}(1-t)^{y-1} dt$$

This amplitude has all the required properties for a dual amplitude, i.e. in the narrow resonance approximation. In particular it has infinite simple poles in each channel and the right asymptotic behaviour.

It turns out that the poles are located not only on the leading Regge trajectory, but also on the daughter trajectories.

The **spectrum** problem is very interesting, but it goes beyond the limits of this talk.

References

EW

S. L. Glashow, Nucl. Phys. 22, 579 (1961).

SU(3)

Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

M. Gell-Mann, Report CTSL 20 (1961), unpublished.

N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

QM

M. Gell-Mann, Phys. Lett. 8, 214 (1964).

G. Zweig, CERN Report no. 8182 (1964), unpublished.

CA

M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

M. Gell-Mann, Physics 1, 63 (1964).

S. Fubini and G. Furlan, Physics 1, 229 (1965).

S. L. Adler, Phys. Rev. Lett. 14, 1051 (1965); Phys. Rev. 140, B736 (1965).

W. I. Weisberger, Phys. Rev. Lett. 14, 1047 (1965); Phys. Rev. 143, 1302 (1966).

C. G. Callan and S. B. Treiman, Phys. Rev. Lett. 16, 153 (1966).

N. Cabibbo and L. A. Radicati, Phys. Letters 19, 697 (1966).

S. Weinberg, Phys. Rev. Lett. 17, 336 (1966).

S. Weinberg, Phys. Rev. Lett. 16, 879 (1966).

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).

SSR

V. De Alfaro, S. Fubini, G. Furlan and C. Rossetti, Phys. Lett. 21, 576 (1966).

S. D. Drell and A. C. Hearn, Phys. Rev. Lett. 16, 908 (1966).

K. Igi and S. Matsuda, Phys. Rev. Lett. 18, 625 (1967).

A. A. Logunov, L. D. Soloviev and A. N. Tavkhelidze, Phys. Lett. 24 B, 181 (1967).

R. Gatto, Phys. Rev. Lett. 18, 803 (1967).

R. Dolen, D. Horn and C. Schmid, Phys. Rev. Lett. 19, 402 (1967); Phys. Rev. 166, 1768 (1968).

ARVV

M. Ademollo, H. R. Rubinstein, G. Veneziano and M. A. Virasoro, Nuovo Cimento 51 A, 227 (1967).

Idem, Phys. Rev. Lett. 19, 1402 (1967).

idem, Phys. Lett. 27 B, 99 (1968).

idem, Phys. Rev. 176, 1904 (1968).

G.V.

G. Veneziano, Nuovo Cimento 57 A, 190 (1968).



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Edi and Gabriele at Akko



Miguel, Gabriele and Edi with other people at Kibbutzim "Maagan Michael"



Edi and Gabriele on the border of the Dead Sea



The ruins of Massada over the Dead Sea