The Spectrum of Physical States of the Dual Resonance Model

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Foreword

- This talk is based on
  - P. Di Vecchia, "The Birth of String Theory", arXiv:0704.0101.
  - PdV and A. Schwimmer, "The Beginning of String Theory: a Historical Sketch".
- Contributions to the Gabriele Veneziano celebrative volume "String theory and fundamental interactions", Ed.s M. Gasperini and J. Maharana, Springer.

Plan of the talk

1. *N*-point amplitude
2. Factorization
3. Problem with ghosts
4. QED
5. The Virasoro conditions
6. Characterization of physical states
7. Scattering amplitudes for physical states
8. DDF states and no ghosts
9. From DRM to String Theory
10. Conclusions

N-point amplitude

- Following the principle of planar duality and the axioms of S-matrix theory the scattering amplitude $B_N(p_1, p_2, \dots p_N)$ for the scattering of N particles was constructed:

$$B_N = \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{i=1}^{N} \left[(z_i - z_{i+1})^{\alpha_0 - 1} \right] \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

- There is a Koba-Nielsen variable z_i for each external particle.
- Invariance under the projective group : $z_i \to \frac{Az_i + B}{Cz_i + D}$.
  Three of the variables z_i can be fixed: $z_1 = \infty, z_2 = 1, z_N = 0$.
- Only simple poles lying on linearly rising Regge Trajectories:

$$\alpha(s) = \alpha_0 + \alpha' s$$

- What is the meaning of this amplitude?
  What is the spectrum of particles?

Factorization

- Since a particle corresponds to a pole in the scattering amplitude with factorized residue, the "obvious" thing to do was to study the factorization properties of the amplitude at each pole.
  [Fubini and Veneziano + Bardaçki and Mandelstam, 1969]
- Introduce an infinite set of harmonic oscillators
  [Fubini, Gordon and Veneziano; Nambu, Susskind, 1969]

$$[a_{n\mu}, a^{\dagger}_{m\nu}] = \eta_{\mu\nu}\delta_{nm} \quad ; \quad [\hat{q}_{\mu}, \hat{p}_{\nu}] = i\eta_{\mu\nu} \quad ,$$

the Fubini-Veneziano operator
[Fubini and Veneziano, 1969 and 1970]:

$$Q_{\mu}(z) = Q^{(+)}_{\mu}(z) + Q^{(0)}_{\mu}(z) + Q^{(-)}_{\mu}(z)$$

where

$$Q^{(+)} = i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\frac{a_n}{\sqrt{n}}z^{-n} \quad ; \quad Q^{(-)} = -i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\frac{a^{\dagger}_n}{\sqrt{n}}z^{n}$$

$$Q^{(0)} = \hat{q} - 2i\alpha'\hat{p}\log z$$

- and the vertex operator;

$$V(z;p) =: e^{ip\cdot Q(z)} :\equiv e^{ip\cdot Q^{(-)}(z)} e^{ip\hat{q}} e^{+2\alpha'\hat{p}\cdot p \log z} e^{ip\cdot Q^{(+)}(z)}$$

- In terms of them we can rewrite the N-point amplitude using this operator formalism:

$$A_N \equiv (2\pi)^d \delta^{(d)}(\sum_{i=1}^{N} p_i) B_N = \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \times$$

$$\times \prod_{i=1}^{N} \left[(z_i - z_{i+1})^{\alpha_0 - 1} \right] \langle 0,0| \prod_{i=1}^{N} V(z_i, p_i) |0,0\rangle$$

- or introducing the propagator ($L_0 = \alpha' \hat{p}^2 + \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n$):

$$D = \int_0^1 dx x^{L_0 - 1 - \alpha_0} (1-x)^{\alpha_0 - 1} = \frac{1}{L_0 - 1} = \frac{1}{\alpha' \hat{p}^2 + R - 1} \quad if \quad \alpha_0 = 1$$

- we get

$$A_N \equiv \langle 0, p_1 | V(1, p_2) D \ldots V(1, p_M) D V(1, p_{M+1} \ldots D V(1, p_{N-1}) | 0, p_N \rangle$$

- that can be rewritten as follows:

$$A_N(p_1, p_2 \ldots p_N) = \langle p_{(1,M)} | D | p_{(M+1,N)} \rangle$$

where

$$\langle p_{(1,M)} | = \langle 0, p_1 | V(1, p_2) D V(1, p_3) \ldots V(1, p_M)$$

and

$$| p_{(M+1,N)} \rangle = V(1, p_{M+1}) D \ldots V(1, p_{N-1}) | p_N, 0 \rangle$$

- At the pole the amplitude can be factorized by introducing two complete set of states:

$$A_N = \sum_{\lambda,\mu} \langle p_{(1,M)} | \lambda, P \rangle \langle \lambda, P | \frac{1}{R - \alpha(s)} | \mu, P \rangle \langle \mu, P | p_{(M+1,N)} \rangle$$

- The propagator develops a pole when ($R = \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n$)

$$\alpha(s) \equiv 1 - \alpha' P^2 \equiv 1 - \alpha' (p_1 + \cdots + p_M)^2 = \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n = m$$

  is a non-negative integer ($m \geq 0$).
- The residue at the pole $\alpha(s) = m$ factorizes in a finite sum of terms corresponding to the states $|\mu, P\rangle$ satisfying the condition:

$$R|\mu, P\rangle \equiv \sum_{n=1}^{\infty} n a_n^\dagger \cdot a_n |\mu, P\rangle = m|\mu, P\rangle$$

- The lowest state, corresponding to $m = 0$, is the vacuum of oscillators: $|0, P\rangle$ with $1 - \alpha' P^2 = 0$.
  This is a tachyon because $\alpha_0 = 1$.
- The next state with $m = 1$ is the state: $a^{\dagger}_{1\mu}|0, P\rangle$ corresponding to a massless vector.
- At the level $m = 2$ we have the following states ($1 - \alpha' P^2 = 2$):

$$a^{\dagger}_{1\mu} a^{\dagger}_{1\nu}|0, P\rangle \ \ ; \ \ a^{\dagger}_{2\mu}|0, P\rangle$$

- At the level $m = 3$ we have the following states ($1 - \alpha' P^2 = 3$):

$$a^{\dagger}_{1\mu} a^{\dagger}_{1\nu} a^{\dagger}_{1\rho}|0, P\rangle \ \ ; \ \ a^{\dagger}_{2\mu} a^{\dagger}_{1\nu}|0, P\rangle \ \ ; \ \ a^{\dagger}_{3\mu}|0, P\rangle$$

- and so on

Problems with ghosts

- The N-point amplitude is Lorentz invariant.
- This forces to factorize the amplitude by introducing a space that is not positive definite:

$$[a_{n\mu}, a^{\dagger}_{m\nu}] = \eta_{\mu\nu}\delta_{nm} \;\; ; \;\; \eta_{\mu\nu} = (-1, 1, \ldots, 1)$$

- Therefore the states with an odd number of time components have a negative norm.
- This is in contradiction with the fact that in a quantum theory the Hilbert space must be positive definite due to the probabilistic interpretation of the norm of a state.
- General problem: how to put together

Quantum theory $\Leftrightarrow$ Special Relativity

QED

- Consider a scattering amplitude in QED near a photon pole. We can write it as follows:

$$A^{\mu}(p_1, \ldots p_M, P)\ \frac{\eta_{\mu\nu}}{P^2}\ B^{\nu}(P, p_{M+1} \ldots p_N)\ \ ;\ \ \eta_{\mu\nu} = (-1, 1, 1, 1)$$

  Naively it seems that the residue consists of four terms and one of them is a ghost corresponding to a negative norm state.
- But gauge invariance implies:

$$P_{\mu} A^{\mu} = P_{\mu} B^{\mu} = 0$$

- In the frame where $P_{\mu} = E(1, 0, 0, 1)$ gauge invariance implies:

$$A_3 - A_0 = B_3 - B_0 = 0$$

- They imply that the residue at the photon pole has only two terms:

$$\sum_{i,j=1}^{2} A^i(p_1, \ldots p_M, P) \frac{\delta_{ij}}{P^2} B^j(P, p_{M+1} \ldots p_N) \ ; \ \ i,j = 1,2$$

  corresponding to the two helicities ± 1 of the photon.
- In this way QED solves the potential conflict between special relativity and quantum theory.
- We can write everything in a covariant way in a space containing negative norm states,
- but then we know that gauge invariance eliminates the unwanted states,
- and the spectrum of physical states is positive definite.
- The physical states are characterized by the "Fermi condition"

$$\partial^\mu A_\mu^{(+)} |Phys.\rangle = 0$$

- Do we have similar relations in the DRM?

The Virasoro conditions

- One such condition was immediately found:

$$W_1 | p_{(1,M)} \rangle = 0 \quad ; \quad W_1 = L_1 - L_0$$

L_0 and L_1 can be written in terms of harmonic oscillators.

- It was used to show that there was no negative norm state at the first excited level [Fubini and Veneziano, 1970].
- But it was not enough to eliminate all the non-positive norm states.
- Then Virasoro realized that, if $\alpha_0 = 1$, one can find an infinite number of such conditions:

$$W_n | p_{1 \ldots M} \rangle = 0 \quad ; \quad n = 1 \ldots \infty \quad ; \quad W_n = L_n - L_0 - (n-1)$$

[Virasoro , 1969]

- and hope that they can cancel all the non-positive norm states.

Characterization of physical states

- Virasoro found the analogous of the condition imposed by gauge invariance.
- But what is the condition that is the analogous of the Fermi condition in QED?
- Those conditions were found proceeding as in QED

$$L_n|Phys., P\rangle = (L_0 - 1)|Phys., P\rangle = 0 \;\; ; \;\; 1 - \alpha' P^2 = m$$

[Del Giudice and PDV, 1970]

- At the level $m = 1$ the analysis reduces to the one in QED.
- At the level $m = 2$ the physical states are a spin 2:

$$|Phys >_1 = [a^\dagger_{1,i} a^\dagger_{1,j} - \frac{1}{(d-1)} \delta_{ij} \sum_{k=1}^{d-1} a^\dagger_{1,k} a^\dagger_{1,k}]|0, P\rangle$$

with positive norm (i, j are space indices),

- and a spin 0

$$|Phys\rangle_2 = \left[\sum_{i=1}^{d-1} a^\dagger_{1,i} a^\dagger_{1,i} + \frac{d-1}{5} (a^{\dagger 2}_{1,0} - 2a^\dagger_{2,0}) \right] |0, P\rangle$$

- with norm equal to

$$2(d-1)(26-d) \quad (1)$$

  that is positive if d > 26.
- The state decouples from the physical spectrum if d =26.
- But the original analysis was done taking for grant that $d = 4$.... as was...obvious...at that time....
- The absence of ghosts was also shown at the level $m = 3$, but it was difficult to proceed further.
- The remaining question was: Is the DRM free of ghosts?
- But we had to wait few years to get an answer.

Scattering amplitudes for physical states

- In the meantime it became clear that the "Virasoro operators" L_n satisfy the algebra of the conformal group in two dimensions:

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{d}{24} n(n^2 - 1)\delta_{n+m;0}$$

[Fubini and Veneziano, 1970]
including the central charge [Weis, 1970].

- The vertex operators corresponding to the physical states are conformal (primary) fields with conformal dimension $\Delta = 1$:

$$[L_n, V_\alpha(z, p)] = \frac{d}{dz}\left(z^{n+1} V_\alpha(z, p)\right)$$

- They are related to the corresponding physical states by the relations:

$$\lim_{z\to 0} V_\alpha(z;p)|0,0\rangle \equiv |\alpha;p\rangle \;\; ; \;\; \langle 0;0| \lim_{z\to\infty} z^2 V_\alpha(z;p) = \langle \alpha, p|$$

[Campagna, Fubini, Napolitano and Sciuto, 1970]

- They satisfy the hermiticity relation:

$$V_\alpha^\dagger(z, P) = V_\alpha(\frac{1}{z}, -P)(-1)^{\alpha(-P^2)}$$

- In terms of these vertices one can write the most general amplitude involving physical states:

$$(2\pi)^4 \delta(\sum_{i=1}^{N} p_i) B_N^{ex} = \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \langle 0, 0| \prod_{i=1}^{N} V_{\alpha_i}(z_i, p_i) |0, 0\rangle$$

- Complete democracy among physical states.
- A special excited vertex is the one associated to the massless gauge field. It is given by:

$$V_\epsilon(z, k) \equiv \epsilon \cdot \frac{dQ(z)}{dz} e^{ik\cdot Q(z)} \quad ; \quad k \cdot \epsilon = k^2 = 0$$

DDF states and no ghosts

- Using the vertex operator corresponding to the massless gauge field one can define the DDF operator:

$$A_{i,n} = \frac{i}{\sqrt{2\alpha'}} \oint_0 dz \epsilon_i^{\mu} P_{\mu}(z) e^{ik \cdot Q(z)} \; ; \; 2\alpha' p \cdot k = n$$

p_μ is the four-momentum of the states on which it acts

- and

$$P(z) \equiv \frac{dQ(z)}{dz} = -i\sqrt{2\alpha'} \sum_{n=-\infty}^{\infty} \alpha_n z^{-n-1}$$

- They are physical operators

$$[L_m, A_{n;i}] = 0$$

- and they satisfy the algebra of the harmonic oscillators:

$$[A_{n,i}, A_{m,j}] = n\delta_{ij}\delta_{n+m;0} \; ; \; i,j = 1 \ldots d-2$$

[Del Giudice, DV and Fubini, 1971]

- In terms of this infinite set of transverse oscillators we can construct an orthonormal set of states:

$$|i_1, N_1; i_2, N_2; \ldots i_m, N_m\rangle = \prod_h \frac{1}{\sqrt{\lambda_h!}} \prod_{k=1}^{m} \frac{A_{i_k,-N_k}}{\sqrt{N_k}} |0, p\rangle$$

- Is it complete? Does it span the entire space of physical states?
- This was checked for $d = 4$ and in this case the DDF states are not complete.
- There are additional states that were called Brower states.
- They are complete if $d = 26$.
- They span a positive definite Hilbert space: no ghosts if $d = 26$.
- The proof of no ghosts was then extended to any $d \leq 26$. [Brower and Goddard and Thorn, 1972]

- This number ($d = 26$) had already appeared a couple of years before [Lovelace, 1970].
- It was required in order to avoid a violation of unitarity in the twisted loop.
- But almost nobody took it seriously.
- It was very difficult (also psicologically at that time) to think of a theory for strong interactions in $d \neq 4$!!!
- Now after the proof of the no ghost theorem everybody started to accept it.
- After about four years of hard work the basic properties of the DRM were understood.
- Also loop diagrams to implement unitarity were constructed using the sewing procedure. Functions well defined on Riemann surfaces were generated by the sewing procedure. [Alessandrini and Amati, 1971]
- But it was still unclear in 1972 what the underlying structure was.

From DRM to String Theory

- The existence of an infinite number of harmonic oscillators brought already in 1969 some people to suggest that the underlying structure was that of a relativistic string. [Nambu, Nielsen, Susskind, 1969]
- A Lagrangian was written that was a generalization to two dimensions of the one for a pointlike particle in the proper time gauge:

$$L \sim \frac{1}{2}\frac{dX}{d\tau}\cdot\frac{dX}{d\tau} \Longrightarrow L \sim \frac{1}{2}\left[\frac{dX}{d\tau}\cdot\frac{dX}{d\tau} - \frac{dX}{d\sigma}\cdot\frac{dX}{d\sigma}\right]$$

- Being the Lagrangian conformal invariant the generators of the conformal group were also constructed.
- But in this formulation this symmetry was just a "global" symmetry that did not imply the vanishing of the classical generator:

$$L_n = 0$$

- A non-linear string Lagrangian was also proposed that was invariant under arbitrary reparametrizations of the world-sheet coordinates σ and τ:

$$S = -cT \int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma \sqrt{(\dot{X} \cdot X')^2 - \dot{X}^2 X'^2}$$

  [Nambu and Goto, 1970]
- But it took three years to show that the spectrum and the critical dimension ($d = 26$) followed from it.
  [Goddard, Goldstone, Rebbi and Thorn, 1973]
- Immediately after also the scattering amplitudes of the DRM were derived from string theory [Ademollo et al. + Mandelstam, 1974].
- In particular, the Fubini-Veneziano operator is the open string coordinate:

$$Q(z) \to X(e^{i\tau}, \sigma = 0) \quad ; \quad z = e^{i\tau}$$

Conclusions

- It took 4 years (1969-1973) to understand the perturbative properties of the DRM (physical spectrum and scattering amplitudes at tree, one-loop and multiloop level).
- Only the integration measure in multiloop diagrams was determined later.
- Actually at one-loop level it was determined in 1973 using the Brink-Olive projection operator.
- In this period (1969-1973) the fact that the underlying theory may be a string theory played a very minor role.
- But some problems were left unsolved, namely the presence of a tachyon and the 26 dimensions....