STRING THEORY – THE EARLY YEARS
A PERSONAL PERSPECTIVE

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Introduction

Some of my material has appeared previously in three papers that I wrote in 2000 (hep-th/0007117, 0007118, 0011078). There is also some historical material in the reprint volumes I edited in 1985 and the textbooks I coauthored in 1987 and 2007.

The GGI May 2007 Meeting The Birth of String Theory featured talks by Ademollo, Veneziano, Di Vecchia, Fairlie, Ramond, Neveu, Gliozzi, Green. Their slides are on the GGI website.
OUTLINE

• 1960 – 68: The analytic S matrix (Ademollo, Veneziano)
• 1968 – 70: The dual resonance model (Veneziano, Di Vecchia, Fairlie, Neveu)
• 1971 – 73: The NSR model (Ramond, Neveu)
• 1974 – 75: Gravity and unification
• 1975 – 79: Supersymmetry and supergravity (Gliozzi)
• 1979 – 84: Superstrings and anomalies (Green)
1960 – 68: The analytic S matrix

**GOAL**: Theory of hadrons.

UC Berkeley was center of the Universe (Chew, Mandelstam, Weinberg, Glashow, . . . ) I was a graduate student there 1962 – 66 (so was David Gross).

Geoff Chew was my advisor. He inculcated the following principles, which prepared me well for my career as a string theorist.
PRINCIPLES

• Only the S matrix is physical. Field theory is misguided (except for QED).

• Unitarity and analyticity

• Analyticity in angular momentum.

  This developed into Regge Pole Theory

• Bootstrap conjecture

  This developed into Duality.
Regge Pole Theory

\[ A(s,t) \sim \beta(t) \left( \frac{s}{s_0} \right)^{\alpha'(t)} , \quad t < 0 \]

\[ \alpha' \sim 1.0 \text{ (GeV)}^{-2} \]
Duality

\[ \Sigma_X = \Sigma_Y \]

\[ \begin{align*}
X & = \begin{array}{c}
1 \quad 2 \\
3 \quad 4
\end{array} \\
Y & = \begin{array}{c}
1 \quad 2 \\
3 \quad 4
\end{array}
\end{align*} \]

Mean - mean scattering
1968 – 70: The dual resonance model

Veneziano formula:

\[ T = A(s, t) + A(s, u) + A(t, u) \]

\[ A(s, t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} \]

where

\[ \alpha(s) = \alpha(0) + \alpha' s \]

This formula gives an explicit realization of duality and Regge behavior in the narrow resonance approximation. The motivation was phenomenological, but this turned out
to be a tree amplitude in a theory!

**Virasoro formula**

\[
T = \frac{\Gamma(-\frac{1}{2} \alpha(s)) \Gamma(-\frac{1}{2} \alpha(t)) \Gamma(-\frac{1}{2} \alpha(u))}{\Gamma(-\frac{1}{2} \alpha(t) - \frac{1}{2} \alpha(u)) \Gamma(-\frac{1}{2} \alpha(s) - \frac{1}{2} \alpha(u)) \Gamma(-\frac{1}{2} \alpha(s) - \frac{1}{2} \alpha(t))}
\]

has similar virtues.

• **N-particle generalization of Veneziano formula:**

\[
A_N = \int \mu(y) \prod dy_i \prod_{i<j} (y_i - y_j)^{\alpha' k_i \cdot k_j}
\]

This has cyclic symmetry in the \(N\) external lines.
• N-particle generalization of Virasoro formula:

\[ T_N = \int \mu(z) \prod d^2 z_i \prod_{i<j} |z_i - z_j|^{\alpha'k_i \cdot k_j} \]

This has total symmetry in the \( N \) external lines.

Both of these formulas were shown to have a consistent factorization on a spectrum of single-particle states described by an infinite number of oscillators

\[ \{ a^\mu_m \} \quad \mu = 0, 1, \ldots, d - 1 \quad m = 1, 2, \ldots \]

one set of such oscillators in the Veneziano case and two sets in the Virasoro case.
Eventually these formulas were interpreted as describing the scattering of modes of a relativistic string: open strings in the first case and closed strings in the second case. Amazingly, the formulas preceded the interpretation!

Having found the factorization, it became possible to study radiative corrections (loop amplitudes). This was initiated by Kikkawa, Sakita, and Virasoro and followed up by many others. Let me describe my role in this.

I was at Princeton, where I collaborated with Gross, Neveu, and Scherk in studying one-loop amplitudes. In
particular, we found new singularities in the “nonplanar” open string loop. The world sheet is a cylinder with two external particles attached to each boundary. We discovered that the amplitude contains branch points that violate unitarity.

Soon thereafter Claude Lovelace observed that these become poles provided that

\[ \alpha(0) = 1 \quad \text{and} \quad d = 26. \]

Later, these poles were interpreted as closed string modes.
1971 – 73: The NSR model

In January 1971 Pierre Ramond used his “correspondence principle” to find a dual-resonance model analog of the Dirac equation. His proposal was that just as the momentum $p^\mu$ is the zero mode of a density $P^\mu(\sigma)$, so should the Dirac matrices $\gamma^\mu$ be the zero modes of a density $\Gamma^\mu(\sigma)$. Then he defined

$$F_n = \int_0^{2\pi} e^{-in\sigma} \Gamma \cdot P d\sigma \quad n \in \mathbb{Z}.$$ 

In particular,

$$F_0 = \gamma \cdot p + \text{oscillator terms}$$
He proposed the wave equation

\[(F_0 + m)\psi = 0,\]

which I call the Dirac–Ramond Equation.

He also observed that the Virasoro algebra generalizes to

\[\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3}m^2\delta_{m,-n}\]

\[[L_m, F_n] = (\frac{m}{2} - n)F_{m+n}\]

\[[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m^3\delta_{m,-n}\]
André Neveu and I published our dual pion model in March 1971. It has a similar structure, but the periodic density $\Gamma^\mu(\sigma)$ is replaced by an antiperiodic one $H^\mu(\sigma)$. Then the modes

$$G_r = \int_0^{2\pi} e^{-ir\sigma} H \cdot P d\sigma \quad r \in \mathbb{Z} + 1/2$$

satisfy a similar super-Virasoro algebra. We also constructed N-particle amplitudes analogous to those of the Veneziano model. Soon thereafter, Neveu, Charles Thorn, and I assembled these bosons and fermions into a unified interacting theory.
Later in 1971 Gervais and Sakita observed that the string world-sheet theory

\[ S = \int d\sigma d\tau \left( \partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right) \]

has two-dimensional supersymmetry

\[ \delta X^\mu = \bar{\epsilon} \psi^\mu \]

\[ \delta \psi^\mu = -i \rho^\alpha \epsilon \partial_\alpha X^\mu. \]

Significant 1972 developments included:

• Discovery that \( d = 10 \) is the critical dimension (JHS)
- **Light-cone gauge quantization** of strings (Goddard, Goldstone, Rebbi, Thorn)

- **Proof of no-ghost theorems** (Brower; Goddard and Thorn; JHS)

Later in 1972, thanks to Murray Gell-Mann, I moved to Caltech.

One of the first things I did at Caltech (with C. C. Wu) was to compute fermion-fermion scattering. This led me to realize that the ground-state fermion has to be massless.
Some other developments at about this time included:

- Completion and acceptance of the **standard model**.

- The Wess–Zumino work on **four-dimensional supersymmetric theories** (motivated by the search for a 4d analog of the 2d Gervais–Sakita world-sheet action).

- Grand unification

  Understandably, given these successes and string theory’s shortcomings, string theory rapidly fell out of favor.
1974 – 75: Gravity and unification

String theory, as a theory of hadrons, had problems:

- Tachyons
- $d = 10$ or $d = 26$
- Massless particles with $J \leq 2$

Several years of attempts to do better were unsuccessful. Also, the success of QCD made the effort to formulate a string theory of hadrons less pressing.
Since my training was as an elementary particle physicist, gravity was far from my mind in early 1974. Traditionally, elementary particle physicists ignored the gravitational force, which is entirely negligible under ordinary circumstances. For these reasons, we were not predisposed to interpret string theory as a physical theory of gravity.

General relativists formed a completely different community. They attended different meetings, read different journals, and had no need for serious communication with particle physicists, just as particle physicists felt they had no need for black holes and the early universe.
In 1974, when Joël Scherk was spending a half year at Caltech, we decided to interpret the massless spin 2 state as a graviton. We showed that string theory agrees with general relativity at low energies. (Yoneya also did this.)

We proposed to use string theory as a quantum theory of gravity, unified with the other forces. (It was known from previous work of Neveu and Scherk that the massless spin 1 particles give Yang–Mills interactions at low energies.) To account for Newton’s constant, this required that \( \alpha' \sim 10^{-38} \text{GeV}^{-2} \) instead of \( \alpha' \sim 1 \text{GeV}^{-2} \).
This proposal had several advantages:

- Gravity was required by the theory
- String theory has no UV divergences
- Extra dimensions could be a good thing
- Unification with forces described by Yang–Mills theories was automatic

I was very excited and decided to dedicate my life to this. To my surprise, it took 10 years to convince others (with a few exceptions) that this is good idea.
1975 – 79: Supersymmetry and supergravity

Following Wess and Zumino, the study of supersymmetric field theories become a major endeavor. A few highlights that are relevant to string theory included

- $\mathcal{N} = 1, d = 4$ supergravity: Ferrara, Freedman, Van Nieuwenhuizen; Deser, Zumino

- $\mathcal{N} = 1, d = 10$ and $\mathcal{N} = 4, d = 4$ super Yang–Mills theory: Brink, Scherk, JHS

- $\mathcal{N} = 1, d = 11$ supergravity: Cremmer, Julia, Scherk
In 1977 Gliozzi, Scherk, Olive proposed a projection of the RNS spectrum – the GSO Projection – that removed roughly half the states (including the tachyon). Then they did the following counting:

\[ f_{\text{NS}}(w) = \sum_{n=0}^{\infty} d_{\text{NS}}(n) w^n \]

\[ = \frac{1}{2\sqrt{w}} \left[ \prod_{m=1}^{\infty} \left( \frac{1 + w^{m-1/2}}{1 - w^m} \right)^8 - \prod_{m=1}^{\infty} \left( \frac{1 - w^{m-1/2}}{1 - w^m} \right)^8 \right]. \]
\[ f_R(w) = \sum_{n=0}^{\infty} d_R(n) w^n = 8 \prod_{m=1}^{\infty} \left( \frac{1 + w^m}{1 - w^m} \right)^8. \]

In 1829, Jacobi proved that
\[ f_{NS}(w) = f_R(w). \]

Thus the number of bosons and fermions matches at every mass level. This was compelling evidence for ten-dimensional spacetime supersymmetry of the GSO-projected theory.
In 1978–79, I spent a year at the Ecole Normale in Paris. There, Joël Scherk and I developed a scheme to break supersymmetry in the compactification of extra dimensions.

Following that, I spent a month at CERN. There, Michael Green and I began a long and exciting collaboration. Our first goal was to understand better why the GSO-projected RNS string theory has spacetime supersymmetry.

Michael, who worked at Queen Mary College London at the time, had several extended visits to Caltech in the
1980–85 period, and I had one to London in the fall of 1983. We also worked together several summers in Aspen. Sometimes we also collaborated with Lars Brink.

Our collaboration achieved the following:

• Formulated (and named) the type I, type IIA, and type IIB superstring theories

• Developed a new light-cone gauge formalism for the GSO projected theory in which spacetime supersymmetry of the spectrum and interactions was easily proved
• Used this formalism to compute various tree and one-loop amplitudes and elucidate their properties

• Formulated superstring field theory in the light-cone gauge for both type I and type IIB

• Formulated a covariant world-sheet theory with manifest spacetime supersymmetry (and non-manifest kappa symmetry).

• Studied gauge and gravitational anomalies in $\mathcal{N} = 1$, $d = 10$ string theory and effective field theory
Anomalies

Type I superstring theory is a well-defined 10d theory at tree level for any $SO(n)$ or $Sp(n)$ gauge group. However, in every case it is chiral (i.e., parity violating) and the $d = 10$ SYM sector is anomalous. Evaluation of a one-loop hexagon diagram yields

$$\partial \mu J^\mu \sim \varepsilon^{\mu_1 \cdots \mu_{10}} F_{\mu_1 \mu_2} \cdots F_{\mu_9 \mu_{10}}.$$
In 1983 Alvarez-Gaumé and Witten derived the formulas for gravitational anomalies in any dimension and showed that they cancel in type IIB supergravity. It appeared unlikely that type I superstring theory could be consistent for any choice of gauge group, but Michael and I were not convinced.

We worked on this problem for almost two years until the crucial breakthroughs were made in August 1984 (in Aspen). That summer I organized a workshop on “Physics in Higher Dimensions” at the Aspen Center for Physics.
We benefitted from the presence of many leading experts (Zumino, Bardeen, Friedan, Shenker, ...).

Suddenly everything fell into place, which was very exciting.

The first public announcement of our result was made at a physics cabaret held at the Hotel Jerome. Later, Mike Green gave a seminar at the ACP about our results, which were not yet written up.
I won’t belabor the details, just state the conclusion:

Anomalies can cancel for a theory with $\mathcal{N} = 1$ supersymmetry in 10d only if the YM gauge group is

$$SO(32) \quad \text{or} \quad E_8 \times E_8.$$ 

It is crucial for this result that the coupling to supergravity is included.

The $SO(32)$ case could be accommodated by type I superstring theory, but we didn’t know a superstring theory with gauge group $E_8 \times E_8$. 

Before the end of 1984 there were two other major developments:

- **The heterotic string** (Gross, Harvey, Martinec, Rohm) accommodated both of the gauge groups.

- **Calabi–Yau compactification** (Candelas, Horowitz, Strominger, Witten) of the $E_8 \times E_8$ heterotic string gave four-dimensional effective theories with many qualitatively correct features.
By the beginning of 1985, superstring theory – with the goal of unification – had become a mainstream activity.

THE END (and the beginning)

Postscript: The construction of a dual string theory description of QCD is still an actively pursued goal. We now understand that it requires a fifth dimension and large curvature.