NGB and their parameters

- Gradient expansion: parameters of the NGB's
- Masses of the NGB's
- The role of the chemical potential for scalar fields: BE condensation
- Dispersion relations for the gluons

Gradient expansion: NGB's parameters

Recall from HDET

$$L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV \cdot D_{AB} & -\Delta_{AB} \\ -\Delta^*_{AB} & i\tilde{V} \cdot D_{AB} \end{bmatrix} \chi^B$$

$$\Gamma^{AB} = \left\langle \psi^{AT} C \psi^B \right\rangle, \quad \Delta_{AB} = \frac{G}{2} V_{CDAB} \Gamma^{CD}$$

$$A = (\alpha, i) \quad \Rightarrow \quad V_{(\alpha i)(\beta j)(\gamma k)(\delta \ell)} = -(3\delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\delta})\delta_{ik}\delta_{j\ell}$$

Condensate in CFL:

$$\langle 0|\psi^{\alpha}_{iL}\psi^{\beta}_{jL}|0\rangle = -\langle 0|\psi^{\alpha}_{iR}\psi^{\beta}_{jR}|0\rangle \propto \Delta\varepsilon^{\alpha\beta I}\varepsilon_{ijI}$$

$$\Delta_{AB} = \frac{G}{2} V_{CDAB}\Gamma^{CD} = \Delta\,\varepsilon^{\alpha\beta I}\varepsilon_{ijI}$$

Changing basis:

$$\psi^{\alpha}_{i} = \frac{1}{\sqrt{2}}\sum_{A=1}^{9}(\lambda_A)^{\alpha}_{i}\psi^{A}$$

$$\langle\psi^{A}\psi^{B}\rangle = \frac{1}{2}\sum(\lambda_A)^{i}_{\alpha}(\lambda_B)^{j}_{\beta}\Delta\varepsilon^{\alpha\beta I}\varepsilon_{ijI} = \frac{\Delta}{2}\mathrm{Tr}\left[\sum_{I}(\lambda_A\varepsilon_I\lambda_B\varepsilon_I)\right]$$

$$\boxed{\Delta_{AB} = \Delta_A\delta_{AB}}$$

$$\Delta_A = \begin{cases} A=1,\cdots,8 & \Delta_A = \Delta \\ A=9 & \Delta_9 = -2\Delta \end{cases}$$

Free action

$$L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV\cdot\partial & -\Delta_A \\ -\Delta_A & i\tilde{V}\cdot\partial \end{bmatrix} \chi^A$$

Propagator

$$S_{AB} = \frac{\delta_{AB}}{V\cdot\ell\tilde{V}\cdot\ell - \Delta_A^2} \begin{bmatrix} \tilde{V}\cdot\ell & \Delta_A \\ \Delta_A & V\cdot\ell \end{bmatrix}$$

Coupling to the U(1) NGB:

$$U = e^{i\sigma/f_\sigma}, \quad V = e^{i\tau/f_\tau}$$

$$\sigma = f_\sigma \varphi, \quad \tau = f_\tau \theta$$

$$\psi_L \to e^{i(\alpha+\beta)}\psi_L, \quad \psi_R \to e^{i(\alpha-\beta)}\psi_R$$

$$U \to e^{-i\alpha}U, \quad V \to e^{-i\beta}V$$

Invariant couplings

$$UV\psi_L, \quad UV^\dagger\psi_R$$

Consider now the case of the $U(1)_B$ NGB. The invariant Lagrangian is:

$$L_D = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \begin{bmatrix} iV\cdot\partial & -U^{\dagger 2}\Delta_A \\ -U^2\Delta_A & i\tilde{V}\cdot\partial \end{bmatrix} \chi^A$$

At the lowest order in σ

$$L_\sigma = \int \frac{d\vec{v}}{4\pi} \chi^{A\dagger} \Delta_A \begin{bmatrix} 0 & \frac{2i\sigma}{f_\sigma} + \frac{2\sigma^2}{f_\sigma^2} \\ -\frac{2i\sigma}{f_\sigma} + \frac{2\sigma^2}{f_\sigma^2} & 0 \end{bmatrix} \chi^A$$

generates 3-linear and 4-linear couplings

Generating functional: $Z[\sigma] = \int D\chi D\chi^{\dagger} e^{i\int \chi^{\dagger} A(\sigma)\chi}$

$$A(\sigma) = S_0^{-1} + \frac{2i\sigma\Delta_A}{f_\sigma}\Gamma_0 + \frac{2\sigma^2\Delta_A}{f_\sigma^2}\Gamma_1$$

$$S_0^{-1} = \begin{bmatrix} V\cdot\ell & -\Delta_A \\ -\Delta_A & \tilde{V}\cdot\ell \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

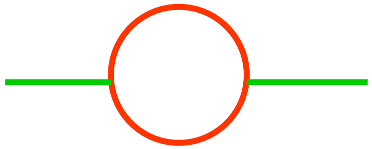
$$Z[\sigma] = (\det[A(\sigma)])^{1/2} = e^{\frac{1}{2}\mathrm{Tr}[\log A(\sigma)]}$$

$$S_{eff}[\sigma] = -\frac{i}{2}\mathrm{Tr}[\log A(\sigma)]$$

$$-i\mathrm{Tr}[\log A(\sigma)] = -i\mathrm{Tr}\left[\log\left(S^{-1}\left(1 + S\frac{2i\sigma\Delta}{f_\sigma}\Gamma_0 + S\frac{2\sigma^2\Delta}{f_\sigma^2}\Gamma_1\right)\right)\right] =$$

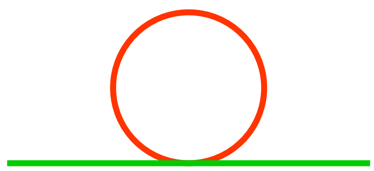
$$= -i\mathrm{Tr}\left[\log S^{-1}\right] - i\sum_{n=1}^{\infty}\frac{(-1)^{2n-1}}{n}\left(iS\frac{2i\sigma\Delta}{f_\sigma}i\Gamma_0 + iS\frac{2\sigma^2\Delta}{f_\sigma^2}i\Gamma_1\right)^n$$

At the lowest order:



$$S_{eff} = \frac{i}{4}\int dxdy\mathrm{Tr}\left[\frac{iS(y,x)2i\sigma(x)\Delta}{f_\sigma}i\Gamma_0\frac{iS(x,y)2i\sigma(y)\Delta}{f_\sigma}i\Gamma_0\right] +$$

$$+\frac{i}{2}\int dx\mathrm{Tr}\left[\frac{iS(x,x)2\sigma^2(x)\Delta}{f_\sigma^2}i\Gamma_1\right]$$



Feynman rules

- For each fermionic internal line

$$iS_{AB} = i\delta_{AB} S(p) = \frac{i\delta_{AB}}{V\cdot\ell\,\tilde{V}\cdot\ell - \Delta_A^2}\begin{bmatrix} \tilde{V}\cdot\ell & \Delta_A \\ \Delta_A & V\cdot\ell \end{bmatrix}$$

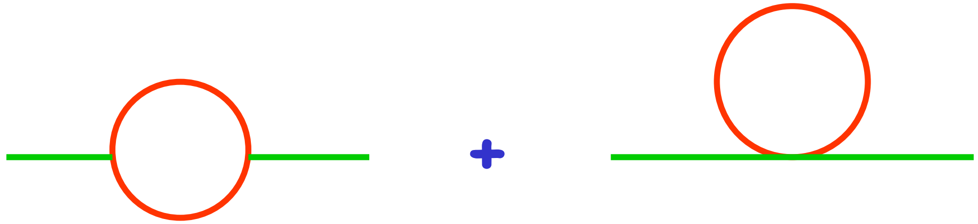
- For each vertex a term iL_{int}

- For each internal momentum not constrained by momentum conservation:

$$\frac{4\pi\mu^2}{(2\pi)^4}\int d^2\ell = \frac{\mu^2}{4\pi^3}\int_{-\delta}^{+\delta} d\ell_{\parallel}\int_{-\infty}^{+\infty} d\ell_0$$

- Factor 2x(-1) from Fermi statistics and spin. A factor 1/2 from replica trick.

- A statistical factor if needed.



$$iL_{eff} = iL_{I}(p) + iL_{II}(p) = -\frac{1}{2}\int \frac{d\vec{v}}{4\pi}\sum_{A}\frac{\mu^2\Delta_A^2}{\pi^3 f_\sigma^2}\times$$

$$\times\int d^2\ell\left[\frac{\tilde{V}\cdot(\ell+p)\sigma V\cdot\ell\sigma + V\cdot(\ell+p)\sigma\tilde{V}\cdot\ell\sigma - 2\Delta_A^2\sigma^2}{D_A(\ell+p)D_A(\ell)} - \frac{2\sigma^2}{D_A(\ell)}\right]$$

$$D_A(\ell) = V\cdot\ell\tilde{V}\cdot\ell - \Delta_A^2 + i\varepsilon$$

Goldstone theorem: $L_I(0) + L_{II}(0) = 0$

Expanding in p/Δ: →

$$L_{eff}(x) = \frac{9\mu^2}{\pi^2 f_\sigma^2}\frac{1}{2}\int\frac{d\vec{v}}{4\pi}(V\cdot\partial)\sigma(x)(\tilde{V}\cdot\partial)\sigma(x)$$

$$\int\frac{d\vec{v}}{4\pi}V^\mu\tilde{V}^\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

$$L_{eff}(x) = \frac{1}{2}\frac{9\mu^2}{\pi^2 f_\sigma^2}\left[\left(\partial_0\sigma\right)^2 - v_\sigma^2\left(\vec{\nabla}\sigma\right)^2\right]$$

$$v_\sigma^2 = \frac{1}{3}, \quad f_\sigma^2 = \frac{9\mu^2}{\pi^2}$$

CFL

For the V NGB same result in CFL, whereas in 2SC

$$v_\tau^2 = \frac{1}{3}, \quad f_\tau^2 = \frac{4\mu^2}{\pi^2}$$

2SC

With an analogous calculation:

$$L_{eff} = \frac{\mu^2(21 - 8\log 2)}{36\pi^2 F^2}\frac{1}{2}\sum_{a=1}^{8}\left[(\partial_0 \Pi^a)^2 - v^2 |\vec{\nabla}\Pi^a|^2\right]$$

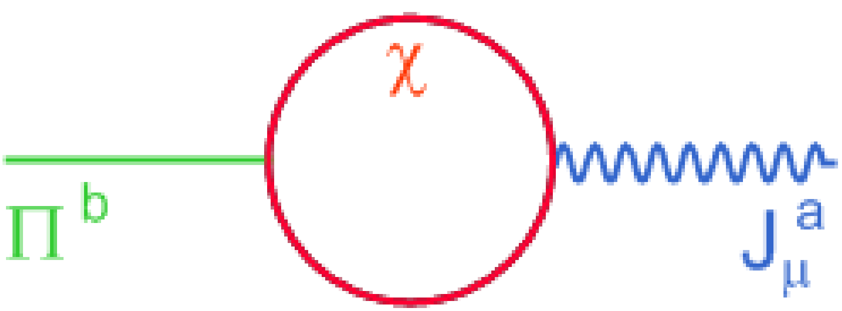
$$v^2 = \frac{1}{3}, \quad F^2 = F_T^2 = \frac{\mu^2(21 - 8\log 2)}{36\pi^2}$$

Dispersion relation for the NGB's

$$E = \pm \frac{1}{\sqrt{3}} |\vec{p}\,|$$

Different way of computing:

$$\langle 0 | J^a_\mu | \Pi^b \rangle = iF\delta^{ab}\tilde{p}_\mu , \quad \tilde{p}_\mu = \left(p^0 , \frac{1}{3}\vec{p} \right)$$



Current conservation:

$$p \cdot \tilde{p} = E^2 - \frac{1}{3} |\vec{p}\,|^2 = 0$$

Masses of the NGB's

QCD mass term: $\bar{\psi}_L M \psi_R + \text{h.c.}$

$$\psi_L \to e^{i(\alpha+\beta)} g_c \psi_L g_L^T, \quad \psi_R \to e^{i(\alpha-\beta)} g_c \psi_R g_R^T$$

$$g_c \in SU(3)_c,\ g_{L,R} \in SU(3)_{L,R},\ e^{i\alpha} \in U(1)_B,\ e^{i\beta} \in U(1)_A$$

$$M \to e^{2i\beta} g_L M g_R^\dagger$$

$$X \to g_c X g_L^T e^{-2i(\alpha+\beta)}, \quad Y \to g_c Y g_R^T e^{-2i(\alpha-\beta)}$$

$$\tilde{\Sigma} = (Y^\dagger X)^T = e^{4i\theta} \Sigma^T \qquad \tilde{\Sigma} \to e^{-4i\beta} g_L \tilde{\Sigma} g_R^\dagger$$

$$d_X = \det(X) = e^{6i(\phi+\theta)}, \quad d_Y = \det(Y) = e^{6i(\phi-\theta)}$$

$$d_X \to e^{-6i(\alpha+\beta)} d_X, \quad d_Y \to e^{-6i(\alpha-\beta)} d_Y$$

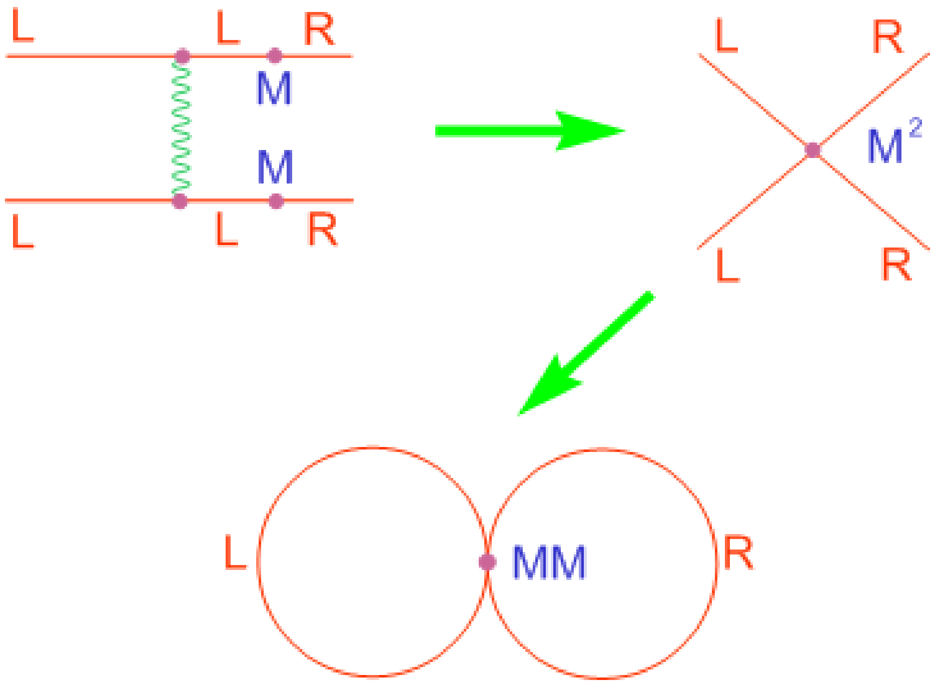
$$\det(M) \to e^{6i\beta} \det(M), \quad \det(\tilde{\Sigma}) \to e^{-12i\beta} \det(\tilde{\Sigma})$$

$$d_X d_{Y^\dagger} = \det(\tilde{\Sigma})$$

$$L_{\text{masses}} = -c\left(\det[M]\mathrm{Tr}[M^{-1}\tilde{\Sigma} + \text{h.c.}\right) - c'\left(\det(\tilde{\Sigma})\mathrm{Tr}[(M\tilde{\Sigma}^\dagger)^2]\right) -$$
$$-c''\left(\mathrm{Tr}[M\tilde{\Sigma}^\dagger]\mathrm{Tr}[M^\dagger\tilde{\Sigma}]\right)$$

Calculation of the coefficients from QCD

Mass insertion in QCD



Effective 4-fermi

$$c = \frac{3\Delta^2}{2\pi^2}, \; c' = 0, \; c'' = 0$$

Contribution to the vacuum energy

Why only c? From second order terms in M:

$$(\bar{\psi}_L M \psi_R)^2 = (\psi^{i\dagger}_{L\alpha} M^j_i \psi^\alpha_{Rj})(\psi^{k\dagger}_{L\beta} M^l_k \psi^\beta_{Rl}) \approx$$

$$\approx \varepsilon^{ikm} \varepsilon_{\alpha\beta\gamma} X^\gamma_m M^j_i M^l_k \varepsilon_{jlp} \varepsilon^{\alpha\beta\delta} Y^{p*}_\delta \approx \varepsilon^{ikm} \varepsilon_{jlp} M^j_i M^l_k \tilde{\Sigma}^p_m =$$

$$= \varepsilon^{ikm} \varepsilon_{jlp} M^j_i M^l_k M^a_m (M^{-1})^b_a \tilde{\Sigma}^p_b \approx \det(M) \mathrm{Tr}[M^{-1} \tilde{\Sigma}]$$

There is a possible other contribution to NGB masses.

Consider:

$$L_{QCD} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi - \bar{\psi}_L M \psi_R - \bar{\psi}_R M \psi_L - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

Solving for $\psi_{-,L}$ as in HDET

$$\psi_{-,L} = \frac{1}{2\mu}\left(-i\gamma_0 \not{D}_\perp \psi_{+,L} + \gamma_0 M \psi_{+,R}\right)$$

like chemical potential

$$L_D = \psi^\dagger_{+,L}(iV \cdot D)\psi_{+,L} - \frac{1}{2\mu}\psi^\dagger_{+,L}\left[(\not{D}_\perp)^2 + MM^\dagger\right]\psi_{+,L} +$$

$$+\left(L \to R, M \to M^\dagger\right) + \cdots$$

Consider fermions at finite density:

$$L = \bar{\psi} i \gamma_\nu \left(\partial^\nu - i \mu g^{\nu 0} \right) \psi$$

as a gauge field A^0

Invariant under: $\psi \to e^{i\alpha(t)} \psi, \quad \mu \to \mu + \dot{\alpha}(t)$

Define: $X_L = \frac{1}{2\mu} M M^\dagger, \quad X_R = \frac{1}{2\mu} M^\dagger M$

Invariance under:

$$\psi_{+,L} \to L(t) \psi_{+,L}, \quad \psi_{+,R} \to R(t) \psi_{+,R},$$

$$X_L \to L(t) X_L L^\dagger(t) + i L(t) \partial_0 L^\dagger(t),$$

$$X_R \to R(t) X_R R^\dagger(t) + i R(t) \partial_0 R^\dagger(t)$$

The same symmetry should hold at the level of the effective theory for the CFL phase (NGB's), implying that

$$\partial_0\Sigma \rightarrow \nabla_0\Sigma = \partial_0\Sigma + i\Sigma\left(\frac{MM^\dagger}{2\mu}\right)^T - i\left(\frac{M^\dagger M}{2\mu}\right)^T\Sigma$$

The generic term in the derivative expansion of the NGB effective lagrangian has the form

$$L_{NGB} \approx F^2\Delta^2\left(\frac{\partial_0 + iMM^\dagger/2\mu}{\Delta}\right)^n\left(\frac{\vec{\nabla}}{\Delta}\right)^m\left(\frac{M^2}{F^2}\right)^p\Sigma^q\Sigma^{\dagger r}$$

$$L_{NGB} \approx F^2\Delta^2 \left(\frac{\partial_0 + iMM^\dagger / 2\mu}{\Delta} \right)^n \left(\frac{\vec{\nabla}}{\Delta} \right)^m \left(\frac{M^2}{F^2} \right)^p \Sigma^q \Sigma^{\dagger r}$$

Compare the two contribution to quark masses:

kinetic term $$F^2\Delta^2 \frac{m^4}{\mu^2\Delta^2}\frac{1}{F^2} \approx \frac{m^4}{\mu^2}$$

mass insertion $$F^2\Delta^2 \frac{m^2}{F^2}\frac{1}{F^2} \approx \frac{\Delta^2 m^2}{F^2}$$

Same order of magnitude for $m \approx \Delta$ **since** $F \approx \mu$

The role of the chemical potential for scalar fields: Bose-Einstein condensation

- A conserved current may be coupled to the a gauge field.
- Chemical potential is coupled to a conserved charge.
- The chemical potential must enter as the fourth component of a gauge field.

Complex scalar field:

$$L = (\partial_0 + i\mu)\varphi^\dagger(\partial_0 - i\mu)\varphi - (\vec{\nabla}\varphi^\dagger)\cdot(\vec{\nabla}\varphi) - m^2\varphi^\dagger\varphi - \lambda(\varphi^\dagger\varphi)^2 =$$

$$= \partial_\mu\varphi^\dagger\partial^\mu\varphi - (m^2 - \mu^2)\varphi^\dagger\varphi - \lambda(\varphi^\dagger\varphi)^2 + i\mu(\varphi^\dagger\partial_0\varphi - \partial_0\varphi^\dagger\varphi)$$

negative mass term

breaks C

$$p^2 - (m^2 - \mu^2) + 2\mu Q p_0 = 0 \quad (Q = \pm 1)$$

Mass spectrum:

$$\Downarrow$$

$$(E + \mu Q)^2 = m^2 + |\vec{p}|^2$$

For $\mu < m$

$$m_{P,\bar{P}} = \mp\mu + m$$

At $\mu = m$, second order phase transition.
Formation of a condensate obtained from:

$$V = \left(m^2 - \mu^2\right)\varphi^\dagger\varphi + \lambda\left(\varphi^\dagger\varphi\right)^2$$

$$\left\langle \varphi^\dagger\varphi \right\rangle = \frac{\mu^2 - m^2}{2\lambda} \quad (\mu > m)$$

Charge density

$$\rho = -\frac{\partial V}{\partial \mu} = 2\mu\left\langle \varphi^\dagger\varphi \right\rangle = \frac{\mu}{\lambda}\left(\mu^2 - m^2\right)$$

Ground state = Bose-Einstein condensate

$$\langle \varphi \rangle = \frac{v}{\sqrt{2}}, \quad v^2 = \frac{\mu^2 - m^2}{\lambda} \qquad \varphi(x) = \frac{1}{\sqrt{2}}\left(v + h(x)\right)e^{i\theta(x)/v}$$

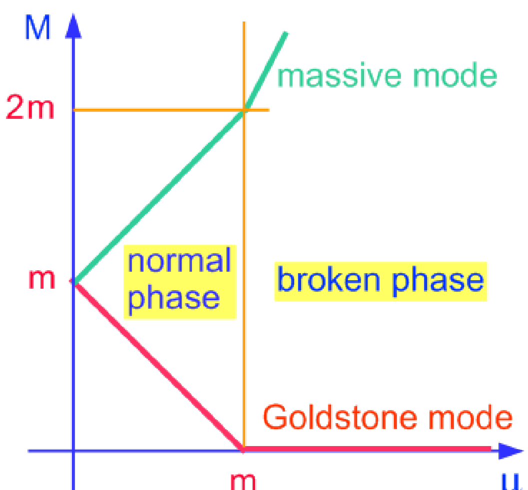
$$L_2 = \frac{1}{2}\partial_\mu\theta\partial^\mu\theta + \frac{1}{2}\partial_\mu h\partial^\mu h - \lambda v^2 h^2 - 2\mu h\partial_0\theta$$

Mass spectrum

$$\det\begin{bmatrix} p^2 - 2\lambda v^2 & 2i\mu E \\ -2i\mu E & p^2 \end{bmatrix} = 0$$

At zero momentum

$$M^2\left(M^2 - 2\lambda v^2 - 4\mu^2\right) = 0$$



$$M^2 = 0$$
$$M^2 = 6\mu^2 - 2m^2$$

At small momentum

$$E_{NGB} \approx \sqrt{\frac{\mu^2 - m^2}{3\mu^2 - m^2}}\,|\vec{p}\,|$$

$$E_{massive} \approx \sqrt{6\mu^2 - 2m^2 + \frac{9\mu^2 - m^2}{6\mu^2 - m^2}\,|\vec{p}\,|^2}$$

$$v^2_{NGB} = \frac{\mu^2 - m^2}{3\mu^2 - m^2} \xrightarrow[\mu\to\infty]{} \frac{1}{3}$$

Back to CFL. From the structure

$$m_{P,\bar{P}} = \mp\mu + m$$

$$m_{\pi^\pm} = \mp \frac{m_d^2 - m_u^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_u + m_d)m_s},$$

$$m_{K^\pm} = \mp \frac{m_s^2 - m_u^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_u + m_s)m_d},$$

$$m_{K^0,\bar{K}^0} = \mp \frac{m_s^2 - m_d^2}{2\mu} + \sqrt{\frac{2c}{F^2}(m_d + m_s)m_u}$$

$$c = \frac{3\Delta^2}{2\pi^2}$$

$$F^2 = \frac{\mu^2(21 - 8\log 2)}{36\pi^2}$$

First term from "chemical potential" like $MM^\dagger$ kinetic term, the second from mass insertions

For large values of m_s:

$$m_{\pi^\pm} = \sqrt{\frac{2c}{F^2}(m_u + m_d)m_s},$$

$$m_{K^\pm} = \mp\frac{m_s^2}{2\mu} + \sqrt{\frac{2c}{F^2}m_s m_d}, \quad m_{K^0,\bar{K}^0} = \mp\frac{m_s^2}{2\mu} + \sqrt{\frac{2c}{F^2}m_s m_u}$$

and the masses of K^+ and K^0 are pushed down.
For the critical value

$$m_s\big|_{crit} = \left(\frac{12\mu^2}{\pi^2 F^2}\right)^{1/3}\sqrt[3]{m_{u,d}\Delta^2} \approx 3.03\sqrt[3]{m_{u,d}\Delta^2},$$

masses vanish

$$m_s\big|_{crit} \approx 40 - 110\ \mathrm{MeV}$$

For larger values of m_s these modes become unstable. Signal of condensation. Look for a kaon condensate of the type:

$$\Sigma = e^{i\alpha\lambda_4} = 1 + (\cos\alpha - 1)\lambda_4^2 + i\lambda_4 \sin\alpha$$

(In the CFL vacuum, $\Sigma = 1$) and substitute inside the effective lagrangian

$$V(\alpha) = F^2 \left(-\frac{1}{2}\left(\frac{m_s^2}{2\mu}\right)\sin^2\alpha + \frac{2cm_s m}{F^2}(1 - \cos\alpha) \right)$$

negative contribution from the "chemical potential"

positive contribution from mass insertion

Defining

$$\mu_{eff} = \frac{m_s^2}{2\mu}, \quad \left(m_K^0\right)^2 = \frac{2cm_s m}{F^2}$$

$$V(\alpha) = F^2\left(-\frac{1}{2}\mu_{eff}^2 \sin^2\alpha + (m_K^0)^2(1-\cos\alpha)\right)$$

with solution

$$\cos\alpha = \frac{\left(m_K^0\right)^2}{\mu_{eff}^2}, \quad \mu_{eff} > m_K^0$$

and hypercharge density

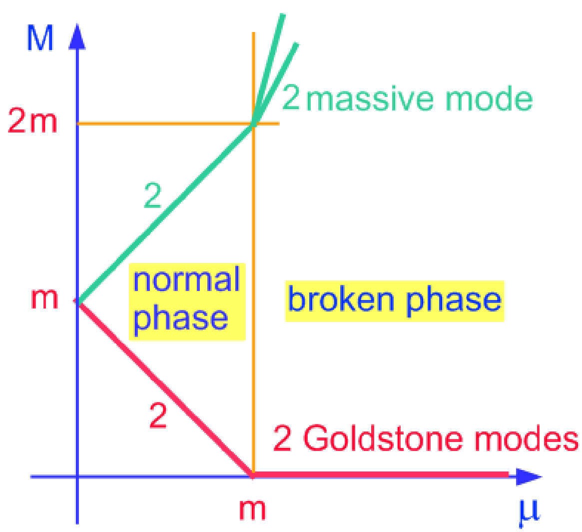
$$n_Y = -\frac{\partial V}{\partial \mu_{eff}} = \mu_{eff} F^2\left(1-\frac{\left(m_K^0\right)^4}{\mu_{eff}^4}\right)$$

Mass terms break original $SU(3)_{c+L+R}$ to $SU(2)_I xU(1)_Y$. Kaon condensation breaks this to $U(1)$

$$Q = \frac{1}{2}\left(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8\right), \quad [Q, \Sigma] = 0$$

$$\Sigma \xrightarrow[SU(2)_I \otimes U(1)_Y]{} (\pi^\pm, \pi^0) \oplus \underbrace{(K^+, K^0)} \oplus (\bar{K}^0, K^-) \oplus (\eta)$$

breaking through the doublet as in the SM



Only 2 NGB's from K^0, K^+ instead of expected 3 (see Chada & Nielsen 1976)

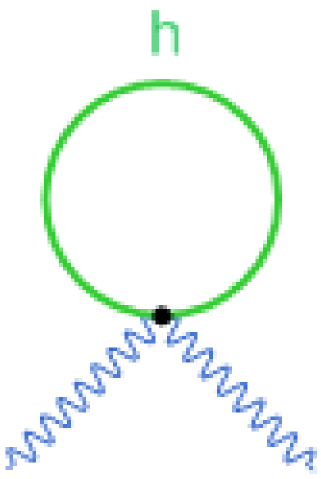
Chada and Nielsen theorem: The number of NGB's depends on their dispersion relation

- If E is linear in k, one NGB for any broken symmetry
- If E is quadratic in k, one NGB for any two broken generators

In relativistic case always of type I, in the non-relativistic case both possibilities arise, for instance in the ferromagnet there is a NGB of type II, whereas for the antiferromagnet there are to NGB's of type I

Dispersion relations for the gluons

The bare Meissner mass



The heavy field contribution comes from the term

$$-\psi^{\dagger}_{h+}\frac{(\not{D}_{\perp})^2}{2\mu+i\tilde{V}\cdot D}\psi_{h+} = -P^{\mu\nu}\psi^{\dagger}_{h+}\frac{D_{\mu}D_{\nu}}{2\mu+i\tilde{V}\cdot D}\psi_{h+}$$

$$P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}\left[V^{\mu}\tilde{V}^{\nu} + V^{\nu}\tilde{V}^{\mu}\right]$$

Notice that the first quantized hamiltonian is:

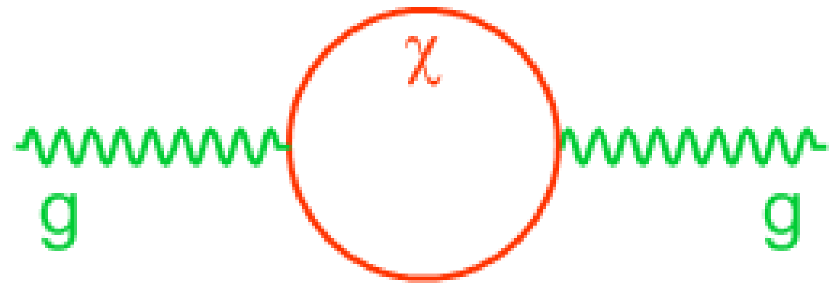
$$H = \left|\vec{p} - g\vec{A}\right| + eA_0 \approx |\vec{p}| + gA_0 - g\vec{v}\cdot\vec{A} + \frac{g^2}{2|\vec{p}|}\left(|\vec{A}|^2 - (\vec{v}\cdot\vec{A})^2\right)$$

Since the zero momentum propagator is the density one gets

$$g^2 \times \underset{\text{spin}}{2} \times N_f \times \int_{|\vec{p}|<\mu} \frac{d^3\vec{p}}{(2\pi)^3}\frac{1}{2|\vec{p}|}\mathrm{Tr}\left[\vec{A}^2 - \frac{(\vec{p}\cdot\vec{A})^2}{|\vec{p}|^2}\right]$$

$$N_f \frac{g^2\mu^2}{6\pi^2}\frac{1}{2}\sum_a \vec{A}^a\cdot\vec{A}^a = \frac{1}{2}m_{BM}^2\sum_a \vec{A}^a\cdot\vec{A}^a, \quad m_{BM}^2 = N_f\frac{g^2\mu^2}{6\pi^2}$$

Gluons self-energy



Vertices from $igA_\mu^a J_a^\mu$

Consider first 2SC for the unbroken gluons:

$$\Pi_{ab}^{00}(p) = \delta_{ab}\frac{g^2\mu^2}{18\pi^2\Delta^2}|\vec{p}|^2,$$

$$\Pi_{ab}^{kl}(p) = \Pi_{ab}^{kl,self}(p) + \Pi_{ab}^{kl}(p) =$$

$$= \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{3\pi^2}\left(1+\frac{p_0^2}{6\Delta^2}\right) - \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{3\pi^2} = \delta_{ab}\delta^{kl}\frac{g^2\mu^2}{18\pi^2\Delta^2}p_0^2,$$

from m_{BM}^2

$$\Pi_{ab}^{0k}(p) = \delta_{ab}\frac{g^2\mu^2}{18\pi^2\Delta^2}p^0p^k$$

- Bare Meissner mass cancels out the constant contribution from the s.e.

- All the components of the vacuum polarization have the same wave function renormalization

$$L = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{1}{2}\Pi_{ab}^{\mu\nu}A_\mu^a A_\nu^b = \frac{1}{2}\left(E_i^a E_i^a - B_i^a B_i^a\right) + \frac{k}{2}E_i^a E_i^a$$

$$k = \frac{g^2\mu^2}{18\pi^2\Delta^2}$$

Dielectric constant $\varepsilon = k+1$, and magnetic permeability $\lambda = 1$

$$v = \frac{1}{\sqrt{\varepsilon\lambda}} \approx \frac{\Delta}{g\mu}$$

Broken gluons

a	$\Pi^{00}(0)$	$-\Pi^{ij}(0)$
1-3	0	0
4-7	$3m_g^2/2$	$m_g^2/2$
8	$3m_g^2$	$m_g^2/3$

$$m_g^2 = \frac{g^2\mu^2}{3\pi^2}$$

But physical masses depend on the wave function renormalization

$$\approx \frac{g^2\mu^2}{\Delta^2}$$

Rest mass defined as the energy at zero momentum:

$$m_R = \sqrt{2}\Delta, \quad a = 4,5,6,7$$

$$m_R = \frac{g\mu}{\pi}, \qquad a = 8$$

The expansion in p/Δ cannot be trusted, but numerically

$$m_R \approx 0.9\Delta, \quad a = 4,5,6,7$$

In the CFL case one finds:

$$m_D^2 = \frac{g^2\mu^2}{36\pi^2}(21-8\log 2) = g^2F^2$$

$$m_M^2 = \frac{g^2\mu^2}{\pi^2}\left(-\frac{11}{36}-\frac{2}{27}\log 2+\underbrace{\frac{1}{2}}\right) = \frac{m_D^2}{3}$$

from bare Meissner mass

Recall that from the effective lagrangian we got:

$$m_D^2 = \alpha_T g^2 F_T^2, \quad m_M^2 = \alpha_S v^2 g^2 F_T^2$$

implying $\alpha_S = \alpha_T = 1$ and fixing all the parameters.

We find: $$m_R \approx \frac{m_D}{\sqrt{3\alpha_1}}, \quad \alpha_1 = \frac{g^2\mu^2}{216\pi^2\Delta^2}\left(7 + \frac{16}{3}\log 2\right)$$

$$\Downarrow$$

$$m_R \approx 1.70\Delta$$

Numerically $$\boxed{m_R \approx 1.36\Delta}$$

LOFF phase

- Different quark masses
- LOFF phase
- Phonons

Different quark masses

We have seen that for one massless flavors and a massive one (m_s), the condensate may be disrupted for

$$\mu < \frac{m_s^2}{2\Delta}$$

The radii of the Fermi spheres are:

$$p_{F_1} = \sqrt{\mu^2 - m_s^2} \approx \mu - \frac{m_s^2}{2\mu}, \quad p_{F_2} = \mu$$

As if the two quarks had different chemical potential ($m_s^2/2\mu$)

Simulate the problem with two massless quarks with different chemical potentials:

$$\mu_u = \mu + \delta\mu, \quad \mu_d = \mu - \delta\mu$$

$$\mu = \frac{\mu_u + \mu_d}{2}, \quad \delta\mu = \frac{\mu_u - \mu_d}{2}$$

Can be described by an interaction hamiltonian

$$\boxed{H_I = -\delta\mu \psi^\dagger \sigma_3 \psi}$$

Lot of attention in normal SC.

- LOFF: ferromagnetic alloy with paramagnetic impurities.

- The impurities produce a constant exchange field acting upon the electron spins giving rise to an effective difference in the chemical potentials of the opposite spins.

- Very difficult experimentally but claims of observations in heavy fermion superconductors (Gloos & al 1993) and in quasi-two dimensional layered organic superconductors (Nam & al. 1999, Manalo & Klein 2000)

H_I changes the inverse propagator

$$S_0^{-1} = \begin{bmatrix} V\cdot\ell + \delta\mu\,\sigma_3 & -\Delta \\ -\Delta^* & \tilde{V}\cdot\ell + \delta\mu\,\sigma_3 \end{bmatrix}$$

and the gap equation (for spin up and down fermions):

$$\Delta = ig\int \frac{d\vec{v}}{4\pi}\frac{\mu^2}{\pi}\int \frac{d^2\ell}{(2\pi)^2}\frac{\Delta}{(\ell_0-\delta\mu)^2-\ell_\parallel^2-\Delta^2}$$

This has two solutions:

$$a):\ \Delta = \Delta_0, \quad b):\ \Delta^2 = 2\delta\mu\,\Delta_0 - \Delta_0^2$$

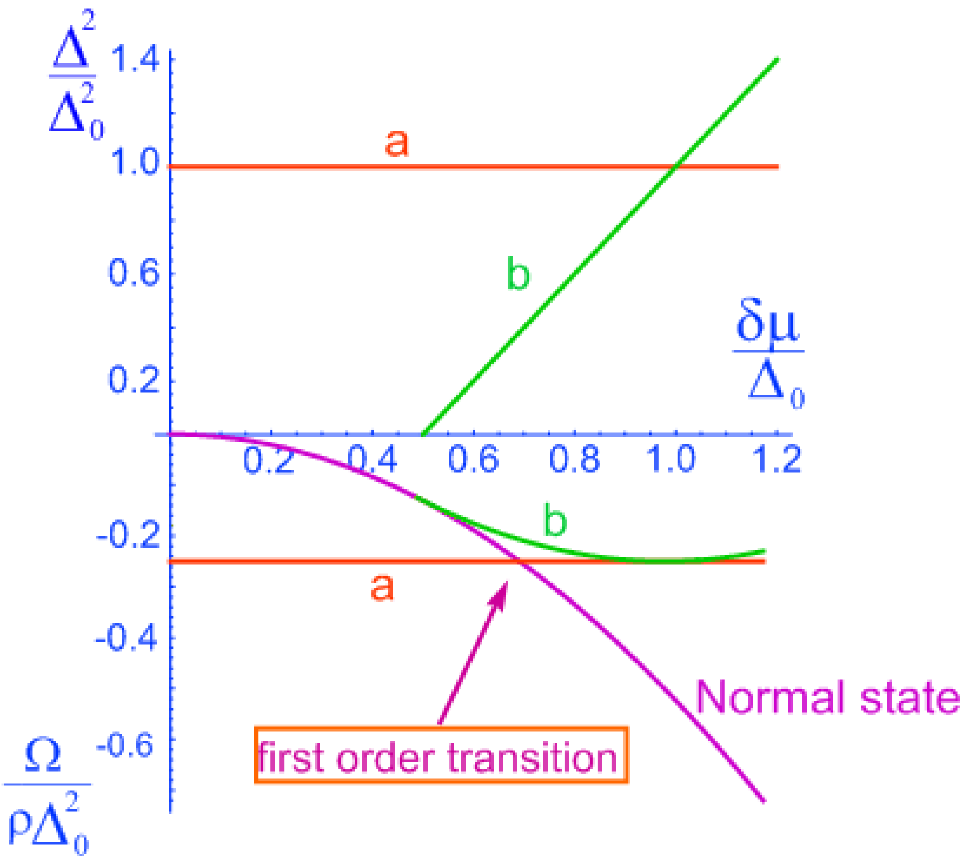
Grand potential:

$$\left(\frac{d\Delta_0}{\Delta_0} = \frac{2}{\rho}\frac{dg}{g^2}\right)$$

$$\frac{\partial\Omega}{\partial g} = \left\langle \frac{\partial H}{\partial g} \right\rangle \Rightarrow \Omega = -\int \frac{dg}{g^2}|\Delta|^2$$

$$\Downarrow$$

$$\Omega_\Delta - \Omega_0 = -\frac{\rho}{2}\int_{\Delta_0(\Delta=0)}^{\Delta_0} \Delta^2 \frac{d\Delta_0}{\Delta_0}$$



Also:

$$\Omega_0(\delta\mu) - \Omega_0(0) = -\frac{\rho}{2}\delta\mu^2$$

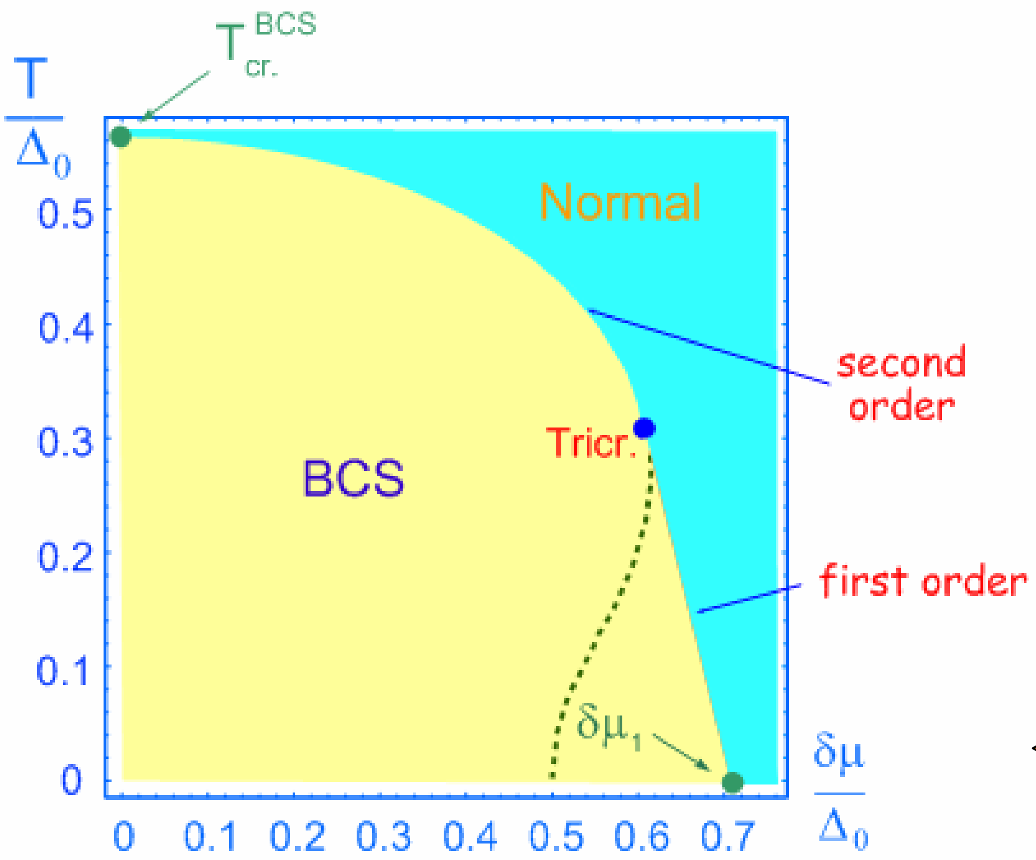
Favored solution

$$\boxed{\Delta = \Delta_0}$$

Also: $$\Omega_\Delta - \Omega_0(\delta\mu) = -\frac{\rho}{4}\left(-2\delta\mu^2 + \Delta_0^2\right)$$

First order transition to the normal state at

$$\delta\mu = \delta\mu_1 = \frac{\Delta_0}{\sqrt{2}}$$



For constant Δ, Ginzburg-Landau expanding up to Δ^6

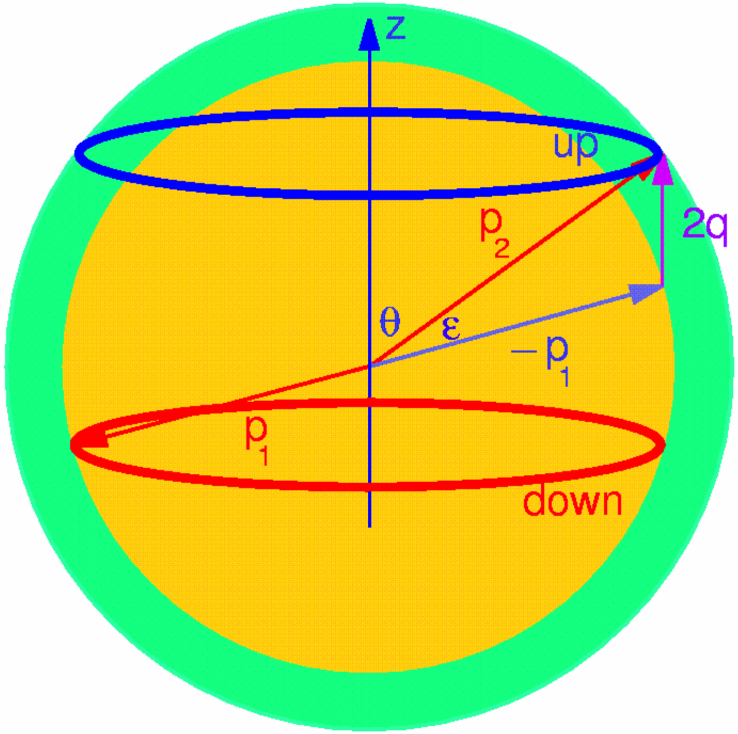
LOFF phase

In 1964 Larkin, Ovchinnikov and Fulde, Ferrel, argued the possibility that close to the first order-line a new phase could take place.

According LOFF possible condensation with **non zero total momentum of the pair**

$$\vec{p}_1 = \vec{k} + \vec{q} \quad \vec{p}_2 = -\vec{k} + \vec{q} \rightarrow \langle \psi(x)\psi(x) \rangle = \Delta e^{2i\vec{q}\cdot\vec{x}}$$

More generally $\longrightarrow$ $\langle \psi(x)\psi(x) \rangle = \Delta \sum_m c_m e^{2i\vec{q}_m\cdot\vec{x}}$



Non zero total momentum

$$\vec{p}_1 + \vec{p}_2 = 2\vec{q}$$

$|\vec{q}|$ fixed variationally

$\vec{q}/|\vec{q}|$ chosen spontaneously

$$E(\vec{p}) - \mu \to E(\pm\vec{k} + \vec{q}) - \mu \mp \delta\mu \approx \xi \mp \bar{\mu}$$

$$\bar{\mu} = \delta\mu - \vec{v}_F \cdot \vec{q}$$

$$n_u \neq n_d$$

Gap equation: $$1 = \frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{1 - n_u - n_d}{\varepsilon(\vec{p},\Delta)}$$

$$n_{u,d} = \frac{1}{e^{(\varepsilon(\vec{p},\Delta)\pm\overline{\mu})/T} + 1}$$

For $T \to 0$

blocking region $\varepsilon < |\overline{\mu}|$

$$1 = \frac{g}{2}\int \frac{d^3p}{(2\pi)^3}\frac{1}{\varepsilon(\vec{p},\Delta)}(1 - \theta(-\varepsilon - \overline{\mu}) - \theta(-\varepsilon + \overline{\mu}))$$

The blocking region reduces the gap:

$$\Delta_{LOFF} << \Delta_{BCS}$$

Possibility of a crystalline structure (Larkin & Ovchinnikov 1964, Bowers & Rajagopal 2002)

$$\langle \psi(x)\psi(x) \rangle = \sum_{|\vec{q}_i| = 1.2\delta\mu} \Delta_{\vec{q}_i} e^{2i\vec{q}_i \cdot \vec{x}}$$

$|\vec{q}_i| = 1.2\delta\mu$ → see later

The q_i's define the crystal pointing at its vertices.

The LOFF phase has been studied via a Ginzburg-Landau expansion of the grand potential

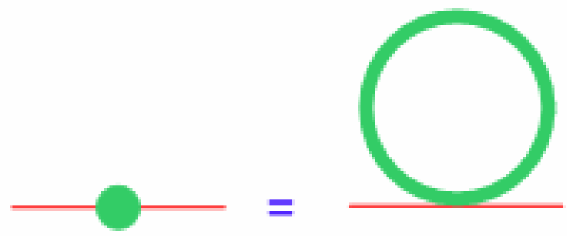
$$\Omega = \alpha\Delta^2 + \frac{\beta}{2}\Delta^4 + \frac{\gamma}{3}\Delta^6 + \cdots$$

(for regular crystalline structures all the Δ_q are equal)

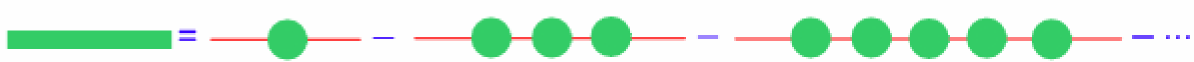
The coefficients can be determined microscopically for the different structures (Bowers and Rajagopal (2002))

General strategy

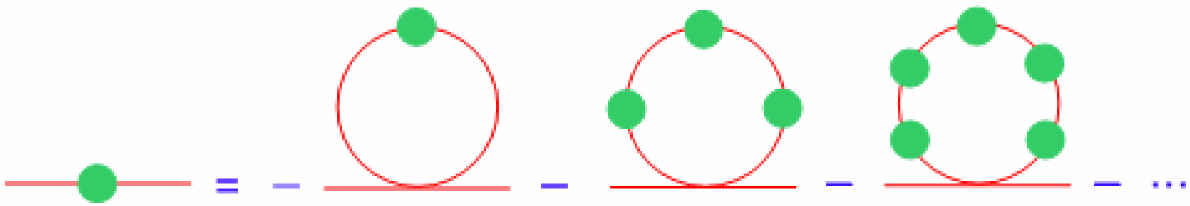
- Gap equation



- Propagator expansion



- Insert in the gap equation

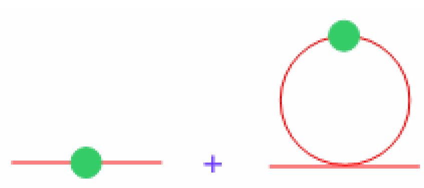


We get the equation

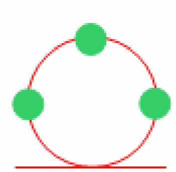
$$\alpha\Delta + \beta\Delta^3 + \gamma\Delta^5 + \cdots = 0$$

Which is the same as $\dfrac{\partial\Omega}{\partial\Delta} = 0$ with

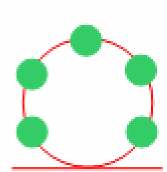
$$\alpha\Delta =$$



$$\beta\Delta^3 =$$

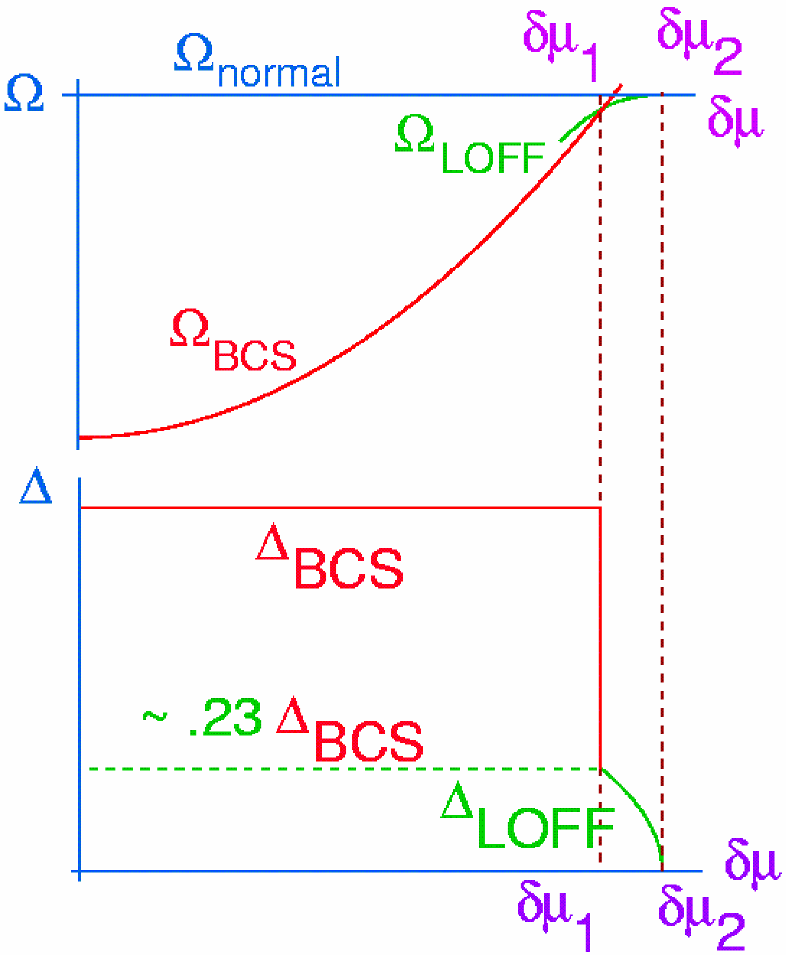


$$\gamma\Delta^5 =$$



The first coefficient has **universal structure**, independent on the crystal. From its analysis one draws the following results

LOFF and BCS



$$\Omega_{BCS} - \Omega_{normal} = -\frac{\rho}{4}(\Delta^2_{BCS} - 2\delta\mu^2)$$

$$\Omega_{LOFF} - \Omega_{normal} = -0.44\rho(\delta\mu - \delta\mu_2)^2$$

$$\Delta_{LOFF} \approx 1.15\sqrt{(\delta\mu_2 - \delta\mu)}$$

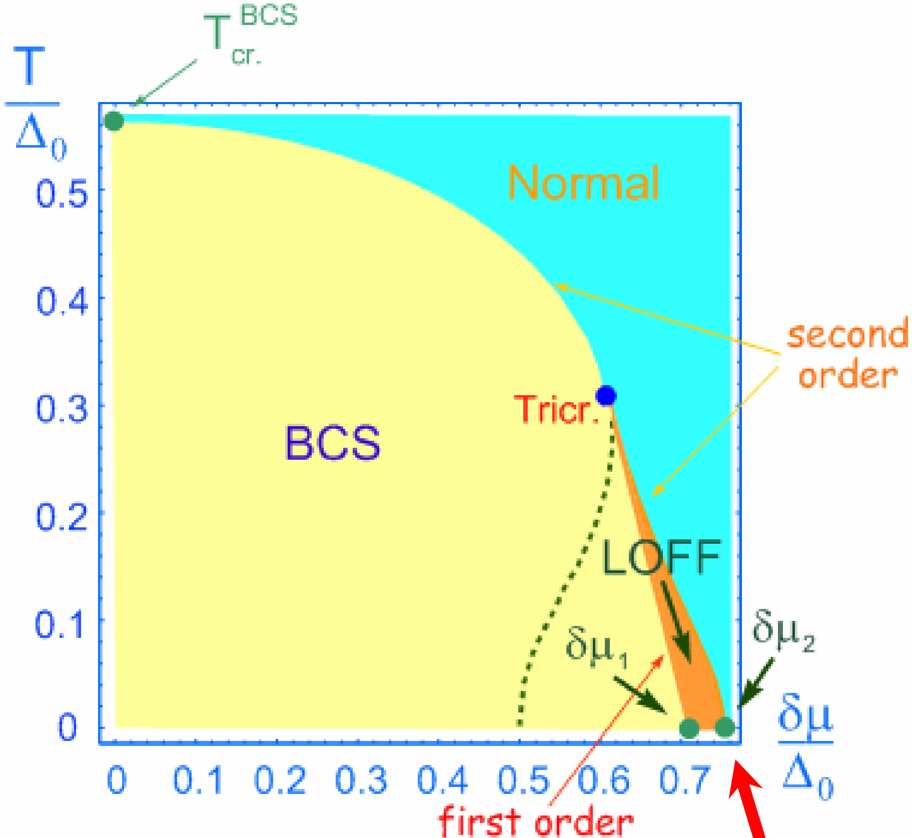
$$\delta\mu_1 = \Delta_{BCS} / \sqrt{2}$$

$$\delta\mu_2 \approx 0.754\Delta_{BCS}$$

Small window. Opens up in QCD? (Leibovich, Rajagopal & Shuster 2001; Giannakis, Liu & Ren 2002)

Results of Leibovich, Rajagopal & Shuster (2001)

μ(MeV)	$\delta\mu_{2}/\Delta_{BCS}$	$(\delta\mu_2 - \delta\mu_1)/\Delta_{BCS}$
LOFF	0.754	0.047
400	1.24	0.53
1000	3.63	2.92



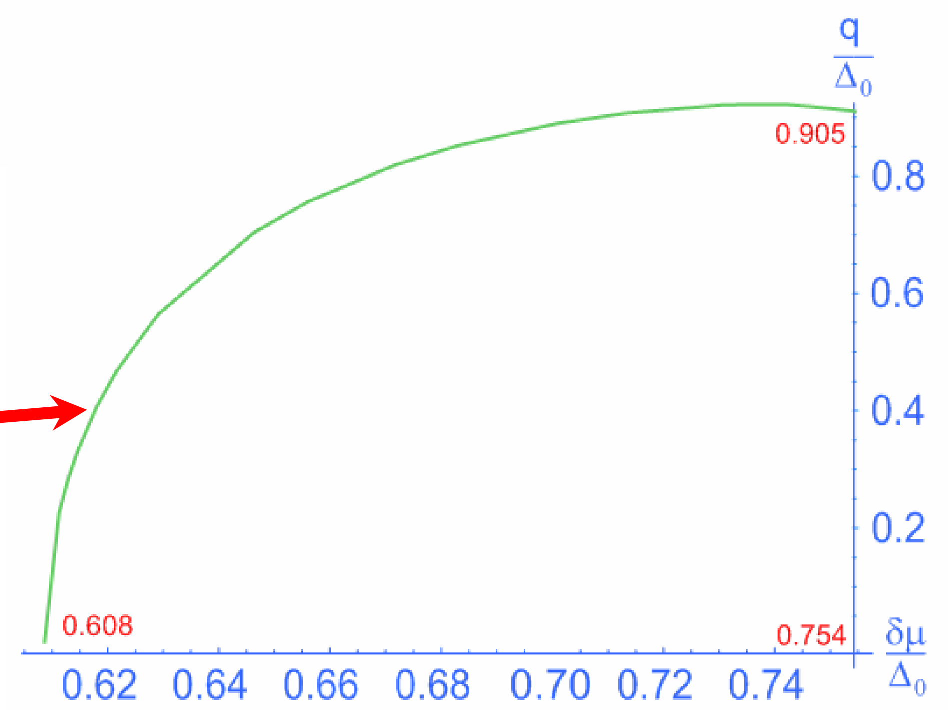
Single plane wave

Critical line from

$$\frac{\partial\Omega}{\partial\Delta} = 0, \qquad \frac{\partial\Omega}{\partial q} = 0$$

Along the critical line

$$(\text{at } T = 0,\ q = 1.2\delta\mu_2)$$



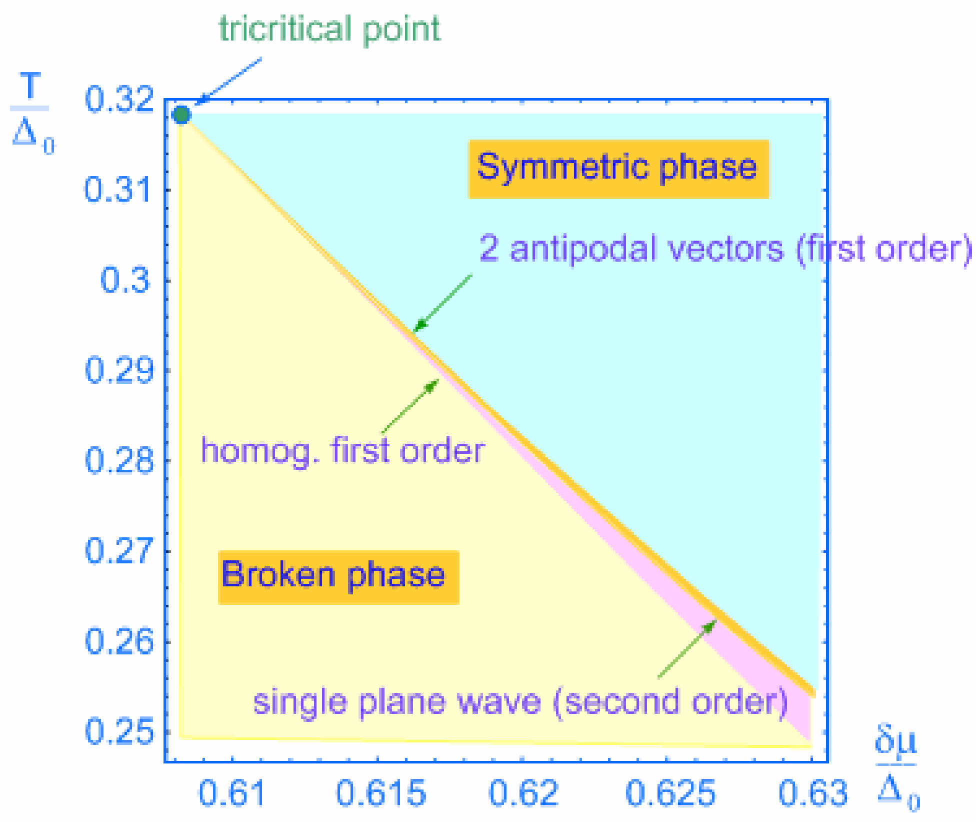
Structure	P	$\mathcal{G}$(Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty h}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	–	–
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramid	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	–	–
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	–	–
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramid	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	–	–
cube	8	$O_h(44)$	-110.757	-459.242	–	–
square antiprism	8	$D_{4d}(4\bar{4})$	-57.363	-6.866	–	–
hexagonal bipyramid	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-6}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

Bowers and Rajagopal (2002)

Preferred structure: face-centered cube

Tricritical point

General study by Combescot and Mora (2002).
Favored structure 2 antipodal vectors

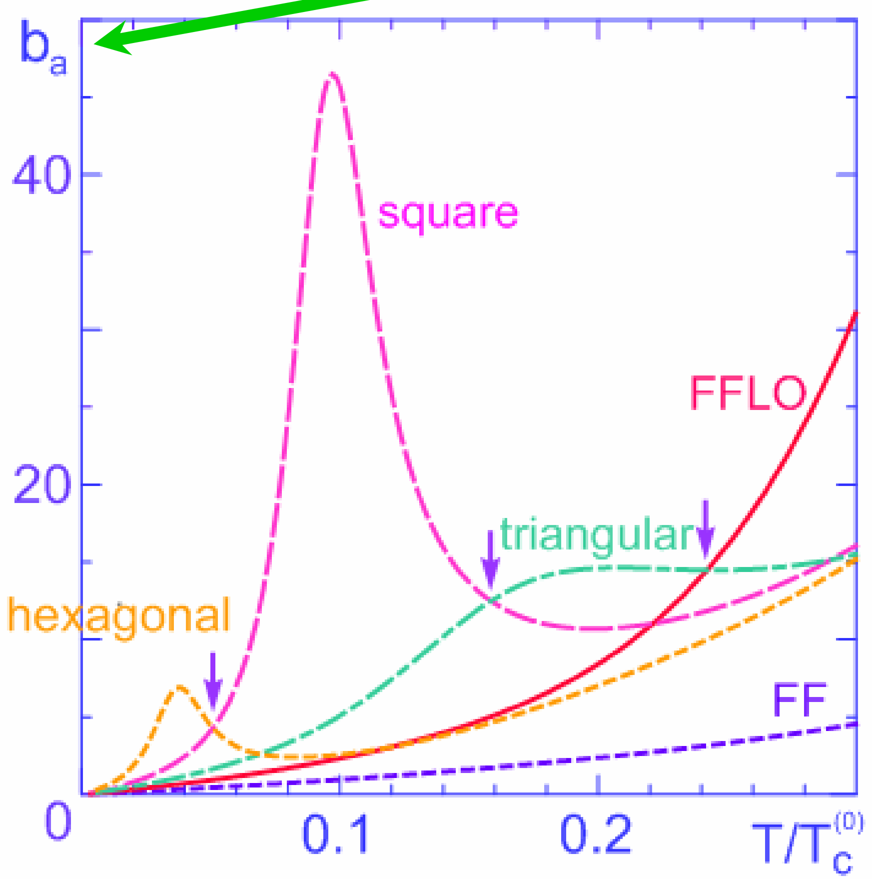


- At T = 0 the antipodal vector leads to a second order phase transition. Another tricritical point. (Matsuo et al. 1998, Casalbuoni and Tonini 2003)
- Change of crystalline structure from tricritical to zero temperature?

Two-dimensional case (Shimahara 98, Mora & Combescot 03)

Analysis close to the critical line

$$\Omega_a - \Omega_N = -\frac{1}{2}\rho b_a \left(\frac{T_c - T}{T_c}\right)^2$$



$$\Delta_a(\vec{r}) = \Delta_{FF} e^{i\vec{q}\cdot\vec{r}}$$

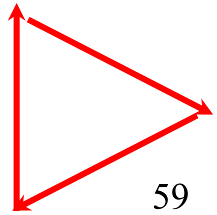
$$\Delta_a(\vec{r}) = 2\Delta_{FFLO}\cos(\vec{q}\cdot\vec{r})$$

$$\Delta_a(\vec{r}) = 2\Delta_{sq}[\cos(qx) + \cos(qy)]$$

$$\Delta_a(\vec{r}) = \Delta_{tr}[e^{i\vec{q}_1\cdot\vec{r}} + e^{i\vec{q}_2\cdot\vec{r}} + e^{i\vec{q}_3\cdot\vec{r}}]$$

$$\Delta_a(\vec{r}) = 2\Delta_{hex}[\cos(\vec{q}_1\cdot\vec{r}) + \cos(\vec{q}_2\cdot\vec{r}) + \cos(\vec{q}_3\cdot\vec{r})]$$

$$\vec{q}_1 + \vec{q}_2 + \vec{q}_3 = 0$$



Phonons

In the LOFF phase translations and rotations are broken

↓

phonons

Phonon field through the phase of the condensate (R.C., Gatto, Mannarelli & Nardulli 2002):

$$\langle \psi(x)\psi(x) \rangle = \Delta e^{2i\vec{q}\cdot\vec{x}} \to \Delta e^{i\Phi(x)} \qquad \langle \Phi(x) \rangle = 2\vec{q}\cdot\vec{x}$$

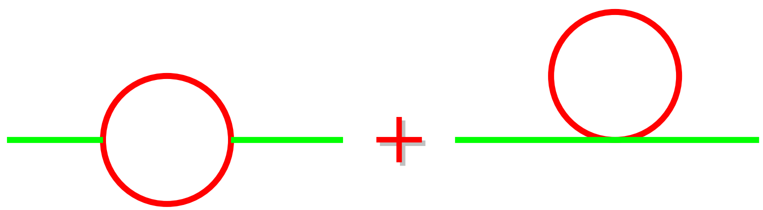
introducing $$\frac{1}{f}\phi(x) = \Phi(x) - 2\vec{q}\cdot\vec{x}$$ →

$$L_{phonon} = \left[\frac{1}{2}\dot{\phi}^2 - v_\perp^2 \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - v_\parallel^2 \frac{\partial^2 \phi}{\partial z^2} \right]$$

Coupling phonons to fermions (quasi-particles) trough the gap term

$$\Delta(x)\psi^T C\psi \to \Delta e^{i\Phi(x)}\psi^T C\psi$$

It is possible to evaluate the parameters of L_{phonon} (R.C., Gatto, Mannarelli & Nardulli 2002)



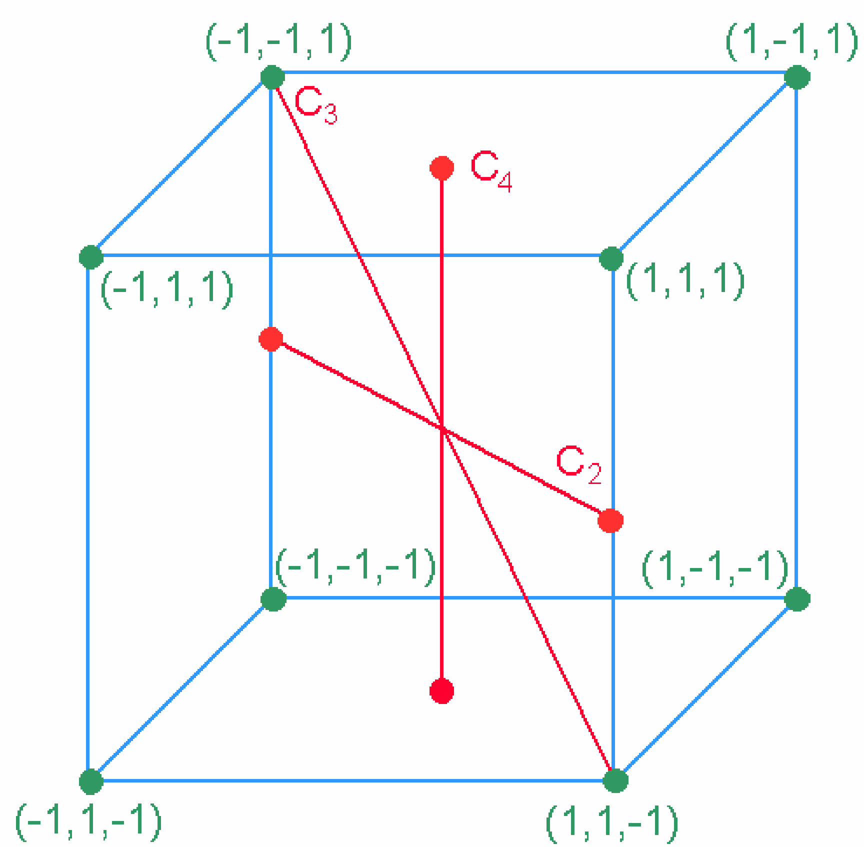
$$v_\perp^2 = \frac{1}{2}\left(1 - \left(\frac{\delta\mu}{|\vec{q}|}\right)^2\right) \approx 0.153 \qquad v_\parallel^2 = \left(\frac{\delta\mu}{|\vec{q}|}\right)^2 \approx 0.694$$

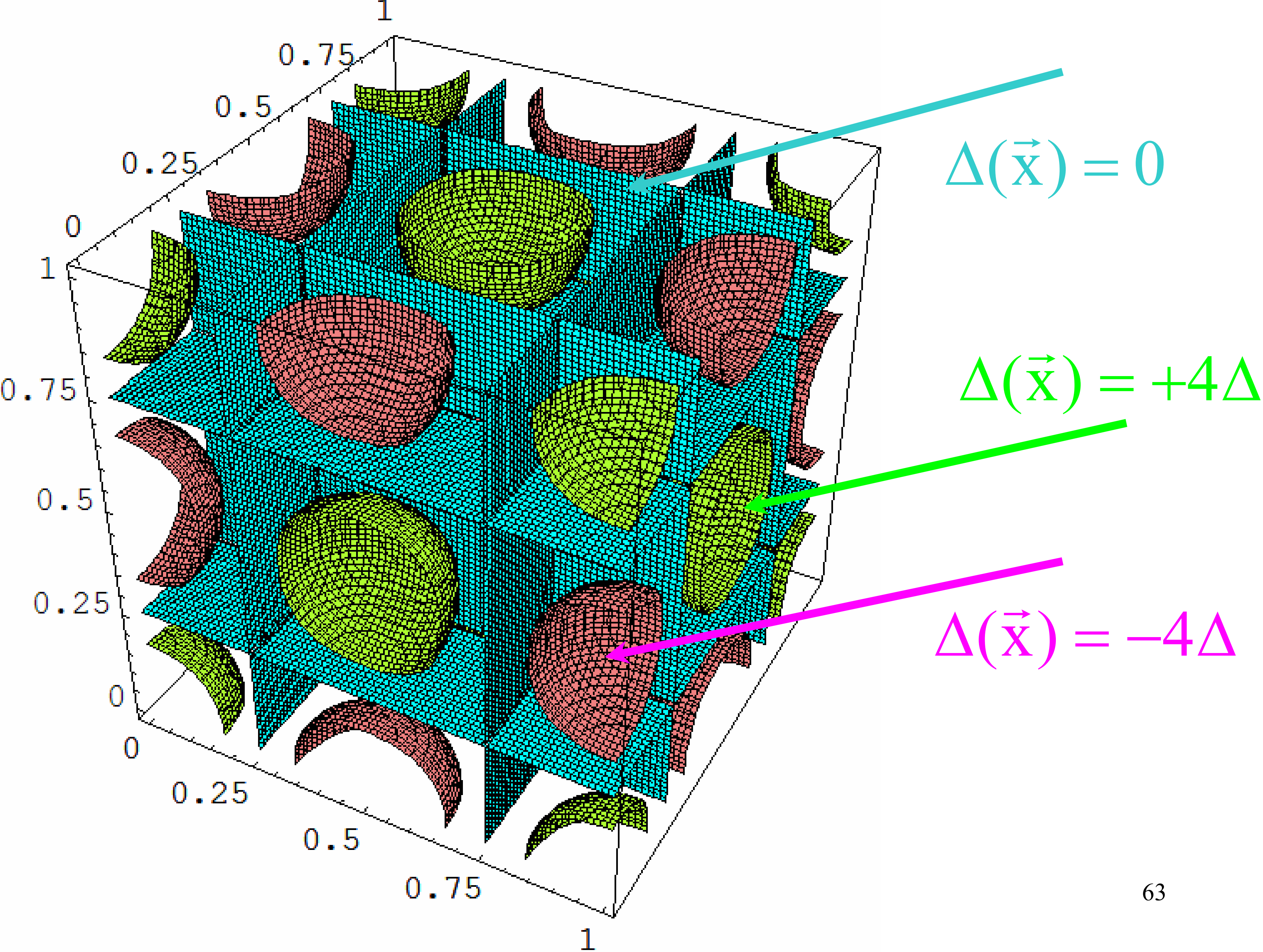
Cubic structure

$$\Delta(x)=\Delta\sum_{k=1}^{8}e^{2i\vec{q}_k\cdot\vec{x}}=\Delta\sum_{i=1,2,3;\varepsilon_i=\pm}e^{2i|\vec{q}|\varepsilon_i x_i}\Rightarrow\Delta\sum_{i=1,2,3;\varepsilon_i=\pm}e^{i\varepsilon_i\Phi^{(i)}(x)}$$

$$\left\langle\Phi^{(i)}(x)\right\rangle=2|\vec{q}|x_i$$

$$\frac{1}{f}\varphi^{(i)}(x)=\Phi^{(i)}(x)-2|\vec{q}|x_i$$





1
0.75
0.5
0.25
0
1
0.75
0.5
0.25
0
0
0.25
0.5
0.75
1
$\Delta(\vec{x}) = 0$
$\Delta(\vec{x}) = +4\Delta$
$\Delta(\vec{x}) = -4\Delta$

$\Phi^{(i)}(x)$ **transforms under the group** O_h **of the cube. Its e.v. ~** x^i **breaks** $O(3)xO_h \sim O_h^{diag}$

$$L_{phonon} = \frac{1}{2}\sum_{i=1,2,3}\left(\frac{\partial\phi^{(i)}}{\partial t}\right)^2 - \frac{a}{2}\sum_{i=1,2,3}|\vec{\nabla}\phi^{(i)}|^2$$

$$-\frac{b}{2}\sum_{i=1,2,3}\left(\partial_i\phi^{(i)}\right)^2 - c\sum_{i<j=1,2,3}\left(\partial_i\phi^{(i)}\partial_j\phi^{(j)}\right)$$

Coupling phonons to fermions (quasi-particles) trough the gap term

$$\Delta(x)\psi^T C\psi \to \Delta \sum_{i=1,2,3;\varepsilon_i=\pm} e^{i\varepsilon_i\Phi^{(i)}(x)}\psi^T C\psi$$

we get for the coefficients

$$a = \frac{1}{12} \qquad b = 0 \qquad c = \frac{1}{12}\left(3\left(\frac{\delta\mu}{|\vec{q}\,|}\right)^2 - 1\right)$$

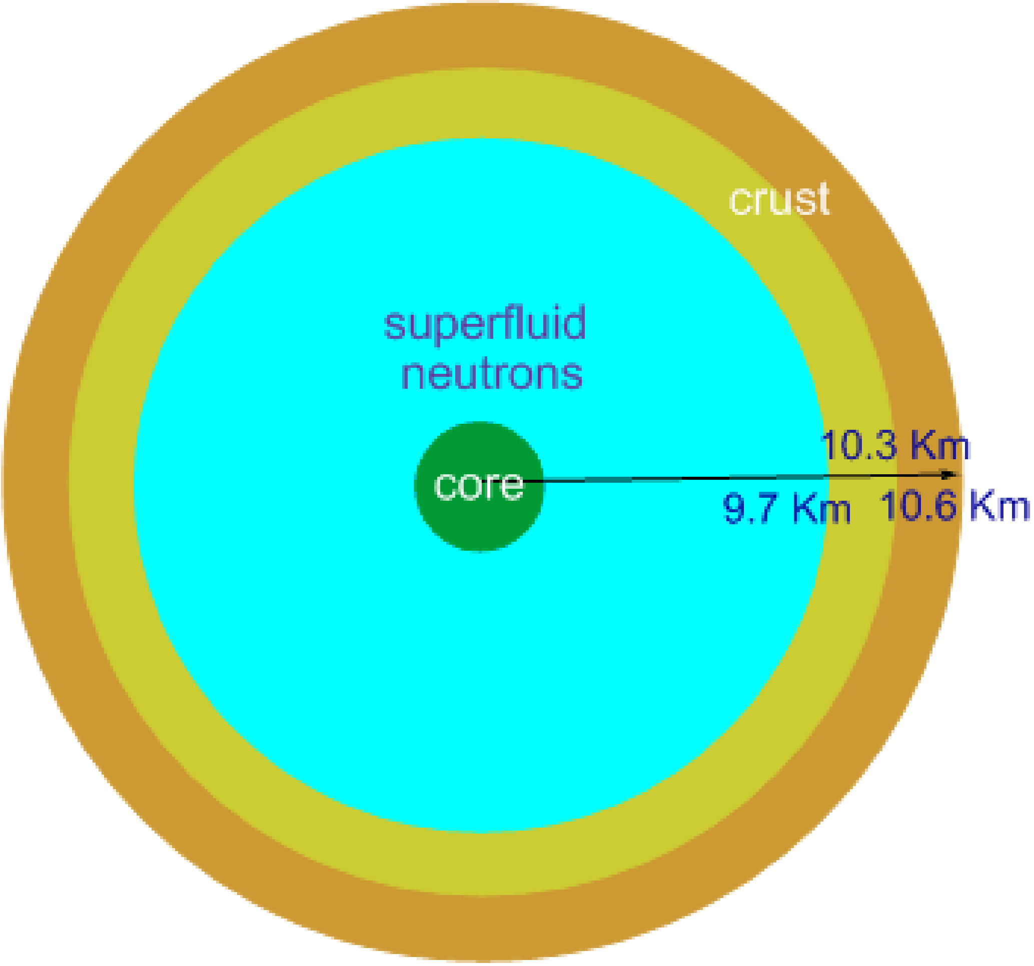
One can evaluate the effective lagrangian for the gluons in tha anisotropic medium. For the cube one finds

Isotropic propagation

This because the second order invariant for the cube and for the rotation group are the same!

Compact stellar objects

Compact stellar objects



High density core of a compact star, a good lab for testing QCD at high density.

Some features of a compact star

For simplicity consider a gas of free massless fermions.

Grand potential:

$$\Omega = 2N_f V \int \frac{d^3\vec{p}}{(2\pi)^3}(\varepsilon_{\vec{p}} - \mu)\theta(\mu - \varepsilon_{\vec{p}}) = -VN_f \frac{\mu^4}{12\pi^2}$$

Density:

$$\rho = -\frac{\partial \Omega}{\partial \mu} = VN_f \frac{\mu^3}{3\pi^2}$$

Eq. of state:

$$P = -\Omega = VN_f \frac{\mu^4}{12\pi^2} \quad \Rightarrow \quad P = K\rho^{4/3}$$

For a non-relativistic fermion:

$$P = VN_f \frac{8}{15} \frac{m^{3/2}}{\sqrt{2}\pi^2} \mu^{5/2}, \quad \rho = VN_f \frac{4}{3} \frac{m^{3/2}}{\sqrt{2}\pi^2} \mu^{3/2}$$

$$\Downarrow$$

$$P = K\rho^{5/3}$$

More generally assumed $P = K\rho^{\gamma}$

For high densities inverse beta decay becomes important $e^- p \rightarrow n\nu$

At the equilibrium $\mu_e + \mu_p = \mu_n$

$$\mu_e + \mu_p = \mu_n \quad \rightarrow \quad \rho_e^{1/3} + \rho_p^{1/3} = \rho_n^{1/3}$$

From charge neutrality $\rho_e = \rho_p$

$$\boxed{\frac{\rho_p}{\rho_n} = \frac{1}{8}}$$

Neutron star

Radius of a neutron star (Landau 1932)

N fermions in a box of volume V. Number density

$$n \approx \frac{N}{R^3}$$

Position uncertainty $\approx n^{-1/3} \simeq \frac{R}{N^{1/3}}$

Uncertainty principle

$$p_F \approx \hbar n^{1/3} \approx \hbar \frac{N^{1/3}}{R} \quad \rightarrow \quad E_F \approx \hbar c \frac{N^{1/3}}{R}$$

Gravitational energy per baryon

$$E_G \approx -\frac{GNm_B^2}{R}$$

$$E = E_G + E_F \approx \frac{\hbar c N^{1/3}}{R} - \frac{G N m_B^2}{R}$$

$E > 0$ otherwise not bounded. This condition gives

$$\frac{\hbar c N^{1/3}}{R} - \frac{G N m_B^2}{R} > 0 \quad \to \quad N \le N_{max} = \left(\frac{\hbar c}{G m_B^2} \right)^{3/2} \approx 2 \times 10^{57}$$

Maximum mass $M_{max} = N_{max} n_B \approx 1.5 M_\odot$

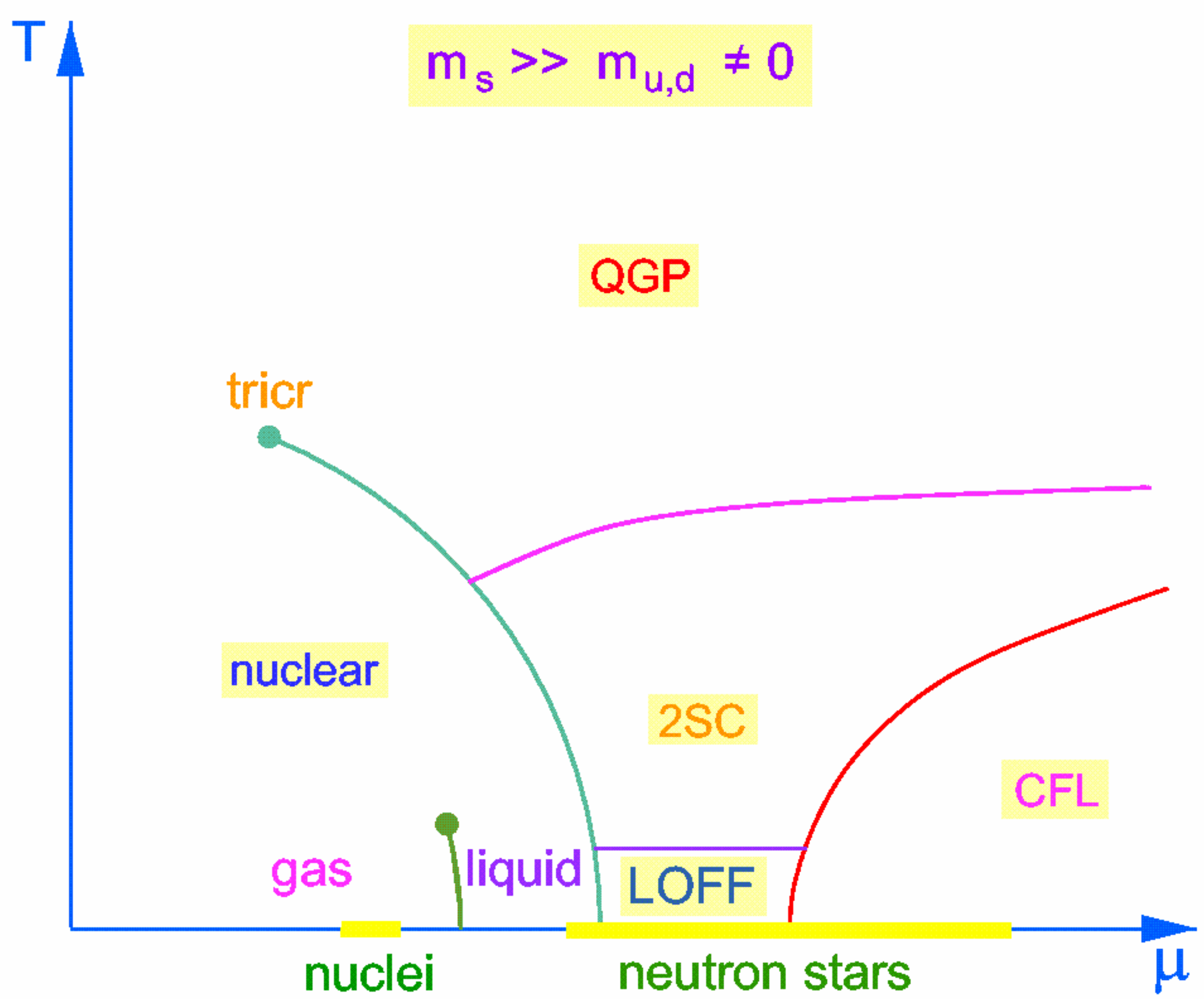
Chandrasekhar limit $M_{max} \approx 1.4 M_\odot$

$$E_F \approx mc^2 \approx \frac{\hbar c}{R} N_{max}^{1/3} \approx \frac{\hbar c}{R}\left(\frac{\hbar c}{G m_B^2}\right)^{1/2}$$

$$R \approx \frac{\hbar}{mc}\left(\frac{\hbar c}{G m_B^2}\right)^{1/2} = \begin{cases} 5\times 10^8 \text{ cm } (m = m_e) \\ 3\times 10^5 \text{ cm } (m = m_n) \end{cases}$$

Typical neutron star density $\rho \le 10^{15} \text{g}/\text{cm}^3$

Neutron stars are a good laboratory to test hadronic matter at high density and zero temperature



In neutron stars CS can be studied at T = 0

$(T_{ns} \sim 10^5\,K)$

$$\frac{T_{ns}}{\Delta_{BCS}} \approx 10^{-6} \div 10^{-7} \qquad 20 \le \Delta_{BCS}(\mathrm{MeV}) \le 100$$

$$(1\mathrm{MeV} \approx 10^{10}\,K)$$

Consider the LOFF state. From $\delta\mu \sim 0.75\,\Delta_{BCS}$

$$14 \le \delta\mu(\mathrm{MeV}) \le 70$$

Orders of magnitude from a crude model: 3 free quarks

$$M_u = M_d = 0,\ M_s \ne 0$$

Weak equilibrium:

$$\mu_u = \mu - \frac{2}{3}\mu_e, \quad p_F^u = \mu_u$$

$$\mu_d = \mu + \frac{1}{3}\mu_e, \quad p_F^d = \mu_d$$

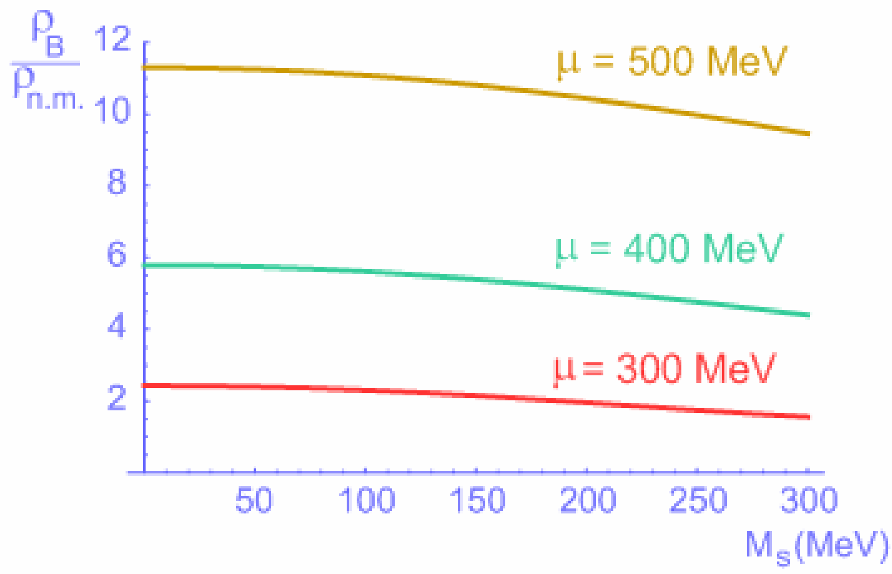
$$\mu_s = \mu + \frac{1}{3}\mu_e, \quad p_F^s = \sqrt{\mu_s^2 - M_s^2}$$

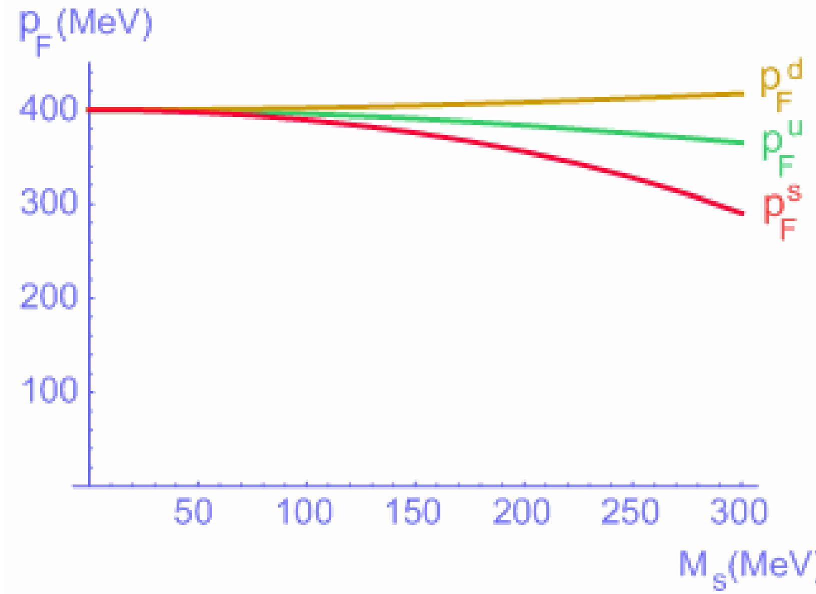
$$\sum_{i=u,d,s} \mu_i N_i + \mu_e N_e = \mu N_q - \mu_e Q \qquad N_q = \sum_{i=u,d,s} N_i$$

Electrical neutrality:

$$Q = \frac{\partial \Omega}{\partial \mu_e} = 0$$

$$\rho_B = -\frac{1}{3}\frac{\partial \Omega}{\partial \mu} = \frac{1}{3\pi^2}\sum_{i=u,d,s}(p_F^i)^2$$





- $\rho_{n.m.}$ **is the saturation nuclear density** $\sim .15\text{x}10^{15}$ g/cm
- **At the core of the neutron star** $\rho_B \sim 10^{15}$ g/cm

$$\frac{\rho_B}{\rho_{n.m.}} \approx 5 \div 6$$

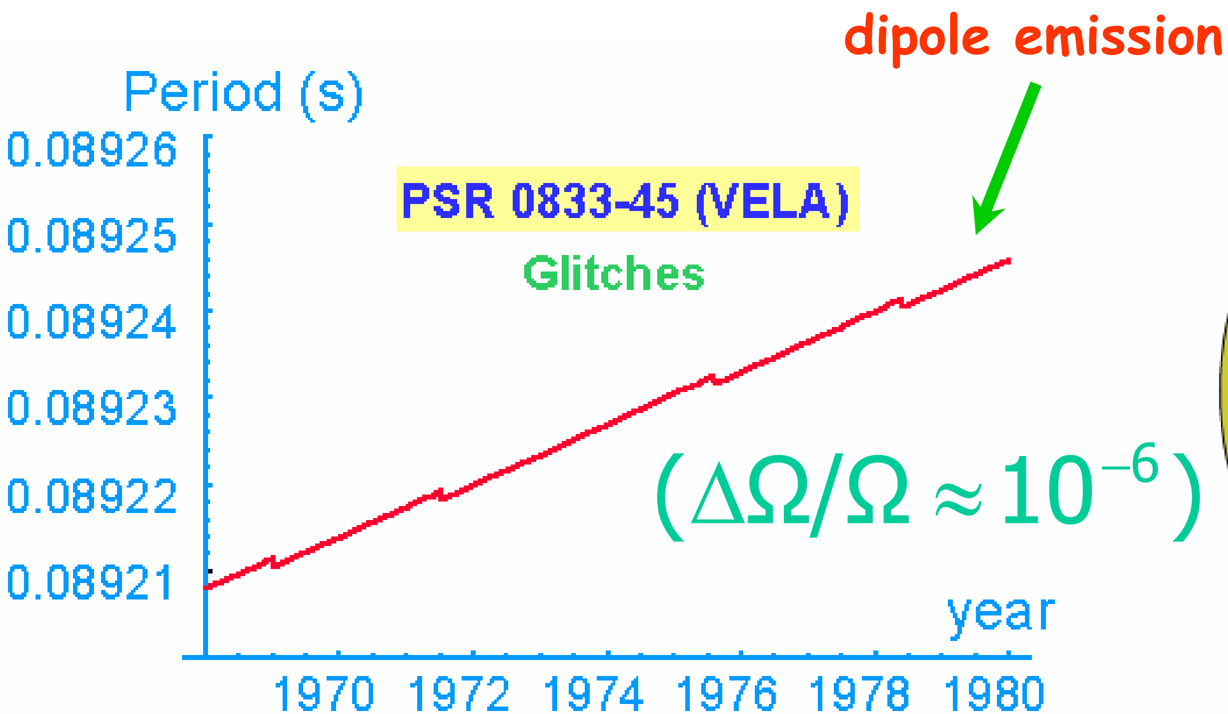
Choosing $\mu \sim 400$ MeV

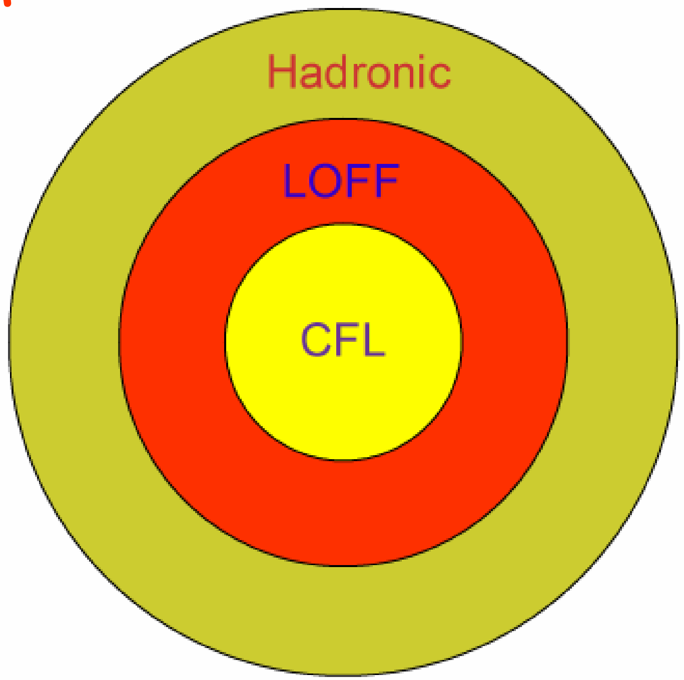
$M_s = 200$	$\delta p_F = 25$
$M_s = 300$	$\delta p_F = 50$

Right ballpark (14 - 70 MeV)

Glitches: discontinuity in the period of the pulsars

- Standard explanation: metallic crust + neutron superfluide inside
- LOFF region inside the star providing the crystalline structure + superfluid CFL phase





Conclusions

- SC almost 100 years old, but still actual
- Important technological applications
- Source of inspiration for other physical theories (SM as an example)
- Deep implications in QCD at very high density: very rich phase structure
- Possible applications for compact stellar objects
- Unvaluable theoretical laboratory