Deconstructed Higgsless Models

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Outline of the talk

(Based on papers by R.C., De Curtis, Dominici, 2004 and R.C., De Curtis, Dolce, Dominici, 2005)

- Introduction to higher-dimensional gauge theories
- Breaking the EW symmetry without Higgs and the linear moose
- EW corrections in the linear moose
- Unitarity bounds
- Delocalizing fermions
- Topological relations
- A planar moose
- The continuum limit
- Summary and conclusions

Higher Dimensional Gauge Theories

- New interest in higher dimensional theories out of the possibility of sub-millimeter extra dimensions due to softening of gravitational interactions in a subspace (Arkani-Hamed, Dimopoulos, Dvali, 1998).

- This leads to a relation between the Planck scale in D-dim, M_D, and the one in 4-dim, M_P

$$\frac{1}{M_P^2}\frac{1}{r^2} = \frac{1}{M_D^{2+d}}\frac{1}{r^2 R^d}, \quad r \gg R$$

$$M_P^2 = R^d M_D^{2+d}$$

$$\frac{M_D^2}{M_P^2} \ll 1, \quad \text{if} \quad R \gg M_D^{-1}$$

One can choose M_D as low as 1 TeV

- Other bonus of higher-dim theories is the possibility of a **geometrical Higgs mechanism** in a pure gauge theory.
- Example: consider an abelian gauge theory in 4+1 dim

$$L = -\frac{1}{2g_5^2} F_{AB}F^{AB} = -\frac{1}{2g_5^2} F_{\mu\nu}F^{\mu\nu} - \frac{1}{g_5^2} F_{\mu 5}F^{\mu 5}$$

$$F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu$$

through the gauge transformation: $A_B \Rightarrow A_B - (\partial_5)^{-1}(\partial_B A_5)$

we get $A_5 = 0,\ F_{\mu 5} = -\partial_5 A_\mu,\ (A_\mu(x, x_5) \approx \sum_n e^{inx_5/R} A_\mu^n(x))$

With a compactified 5 dim on a circle S^2 of length $2\pi R$, the non zero eigenmodes $A_\mu{}^n$ acquire a mass:

$$M_n = \frac{n}{R}$$ absorbing the mode $A_5{}^n$

The zero mode remains massless and a GB is present

- **Massless modes can be eliminated compactifying on an orbifold, that is**

$$S^2 / Z, \quad Z : x_5 \to -x_5$$

- **This allows to define fields as eigenstates of parity:**

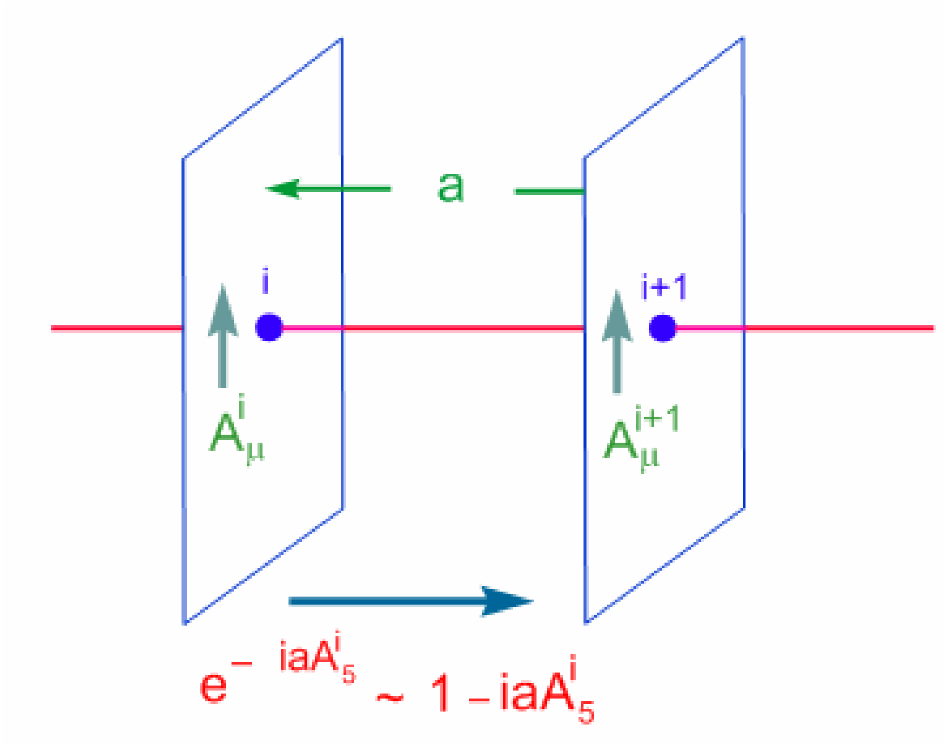
$$A_B(x_\mu, x_5) = \pm A_B(x_\mu, -x_5)$$

- **Various possibilities, for instance by choosing:**

$$A_B \quad \text{odd} \quad \Rightarrow \text{no zero mode} \Rightarrow \text{only massive gauge bosons in the spectrum}$$

Very nice geometrical structure through discretization of the extra-dim (Hill, Pokorski, Wang, 2001). One gets a deconstructed gauge theory

- **A gauge field is nothing but a connection: a way of relating the phases of the fields at nearby points. Once we discretize the space the connection is naturally substituted by a link variable realizing the parallel transport between two lattice sites**



A generalized σ - model where the Higgs mechanism is realized in a standard way in terms of a Σ – field (chiral field)

$$\Sigma \approx 1 - iaA_5 \approx e^{-iaA_5}$$
$$\Sigma\Sigma^\dagger = 1$$

- **More exactly:**

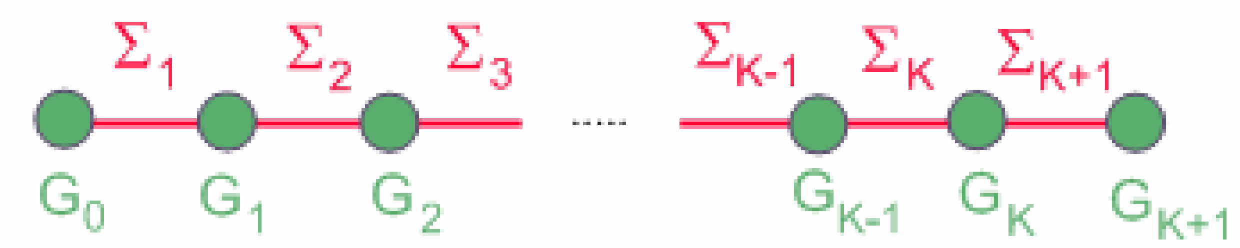
$$\Sigma_i = 1 - iaA_5^{i-1}, \quad \Sigma_i \xrightarrow[U_i \in G_i]{} U_{i-1}\Sigma_i U_i^\dagger$$

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - iA_\mu^{i-1}\Sigma_i + i\Sigma_i A_\mu^i = -iaF_{\mu 5}^{i-1}$$

$$F_{\mu 5}^i = \partial_\mu A_5^i - \partial_5 A_\mu^i - i[A_\mu^i, A_5^i]$$

$$S = \int d^4x \frac{a}{g_5^2}\left(-\frac{1}{2}\sum_i \mathrm{Tr}\left[F_{\mu\nu}^i F^{\mu\nu i}\right] + \frac{1}{a^2}\mathrm{Tr}\left[(D_\mu\Sigma_i)(D_\mu\Sigma_i)^\dagger\right]\right)$$

- **Sintetically described by a moose diagram (Georgi, 1986 –Arkani-Hamed, Cohen, Georgi, 2001)**



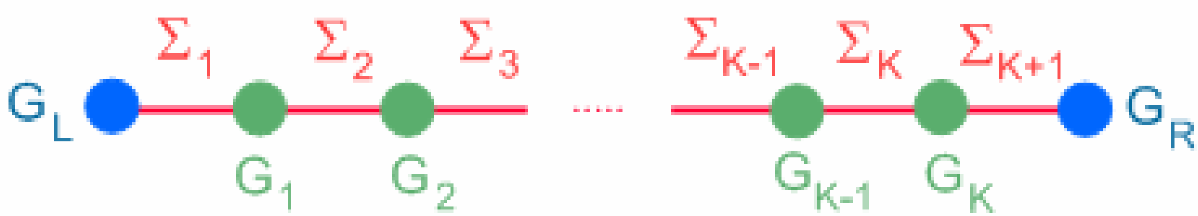
Breaking the EW Symmetry without Higgs Fields

- Abstracting from the previous construction one can study more general moose geometries.
- General structure given by many copies of the gauge group G intertwined by link variables Σ.
- Condition to be satisfied in order to get a Higgsless SM before gauging the EW group is the presence of **3 GB** and **all the moose gauge fields massive**.
- Simplest example: $G_i = SU(2)$. Each Σ_i describes three scalar fields. Therefore, in a connected moose diagram, any site (3 gauge fields) may absorb one link (3 GB's) giving rise to a massive vector field. We need:

# of links = # of sites +1

- **Example:**

- **The model has two global symmetries related to the beginning and to the end of the moose, that we will denote explicitly by G_L and G_R.**



$$\Sigma_1 \to U_L \Sigma_1 U_1^\dagger$$

$$\Sigma_{K+1} \to U_K \Sigma_1 U_R^\dagger$$

- **This $SU(2)_L$ x $SU(2)_R$ global symmetry can be gauged to the standard SU(2)xU(1) leaving us with the usual 3 massive gauge bosons, W and Z, the massless photon and 3K massive vectors. Prototypes of this theory are the BESS model, K = 1, (R.C, De Curtis, Dominici, Gatto, 1985) and its generalizations (R.C, De Curtis, Dominici, Gatto, Feruglio, 1989). Notice that $SU(2)_L$ x $SU(2)_R$ is a custodial symmetry.**

Electro-weak corrections for the linear moose

$$S_{moose} = \int d^4x \left(-\sum_{i=1}^{K} \frac{1}{2g_i^2} Tr\left[F^i_{\mu\nu}F^{\mu\nu i}\right] + \sum_{i=1}^{K+1} f_i^2 Tr\left[(D_\mu\Sigma_i)(D_\mu\Sigma_i)^\dagger\right]\right)$$

- **If the vector fields are heavy enough one can derive a low-energy effective theory for the SM fields, once we gauge**

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2) \otimes U(1)$$

- At the leading order in $(M_W/M_V)^2$ we get

$$M_W^2 = \frac{v^2}{4} g^2, \; M_Z^2 = \frac{M_W^2}{c_\theta^2}, \; e = g s_\theta = g' c_\theta$$

$$\frac{4}{v^2} \equiv \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}$$

- Only oblique corrections, captured by the epsilon parameters

(Peskin, Takeuchi, 1990/92, Altarelli, Barbieri 1991, Altarelli, Barbieri, Caravaglios, 1998):

$$\text{Custodial symmetry} \Rightarrow \varepsilon_1 = \varepsilon_2 = 0$$

$$\varepsilon_3 = \left(\frac{g^2 S}{16\pi} \right) = \frac{g^2}{4\pi} \int_0^\infty \frac{ds}{s^2} \mathrm{Im}\left(\Pi_{VV}(s) - \Pi_{AA}(s) \right)$$

$$\Pi_{VV(AA)} = \left\langle J_{V(A)} J_{V(A)} \right\rangle$$

$$J_{V(A)}\Big|_{\text{vectors}} = f_1^2 g_1 A_\mu^1 + (-) f_{K+1}^2 g_K A_\mu^K$$

Vector contributions to the vector currents associated to the global $SU(2)_L$x $SU(2)_R$

- From vector mesons saturation one gets immediately

$$\varepsilon_3 = \frac{g^2}{4}\sum_n \left(\frac{g_{nV}^2}{m_n^4} - \frac{g_{nA}^2}{m_n^4}\right) = g^2 g_1 g_K (M_2^{-2})_{1K} = g^2 \sum_{i=1}^{K} \frac{(1-y_i)y_i}{g_i^2}$$

$$y_i = \sum_{j=1}^{i} x_i, \quad x_i = \frac{f^2}{f_i^2}, \quad \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2} \Rightarrow \sum_{i=1}^{K+1} x_i = 1$$

- Since $\underline{0 \le y_i \le 1 \quad \Rightarrow \quad \varepsilon_3 \ge 0}$ (follows also from positivity of M_2^{-1})

- Example: $f_i = f_c, \quad g_i = g_c \quad \Rightarrow \quad \varepsilon_3 = \frac{1}{6}\frac{g^2}{g_c^2}\frac{K(K+2)}{K}$

$$(\varepsilon_3)_{exp} \approx 10^{-3} \quad \Rightarrow \quad (K=1), \; g_c \ge 22.3g$$

- For g_c from 2g to 5g, ε_3 natural value is of order 10^{-1}-10^{-2} not compatible with the data

- **Possible solutions:**

1) Cutting a link

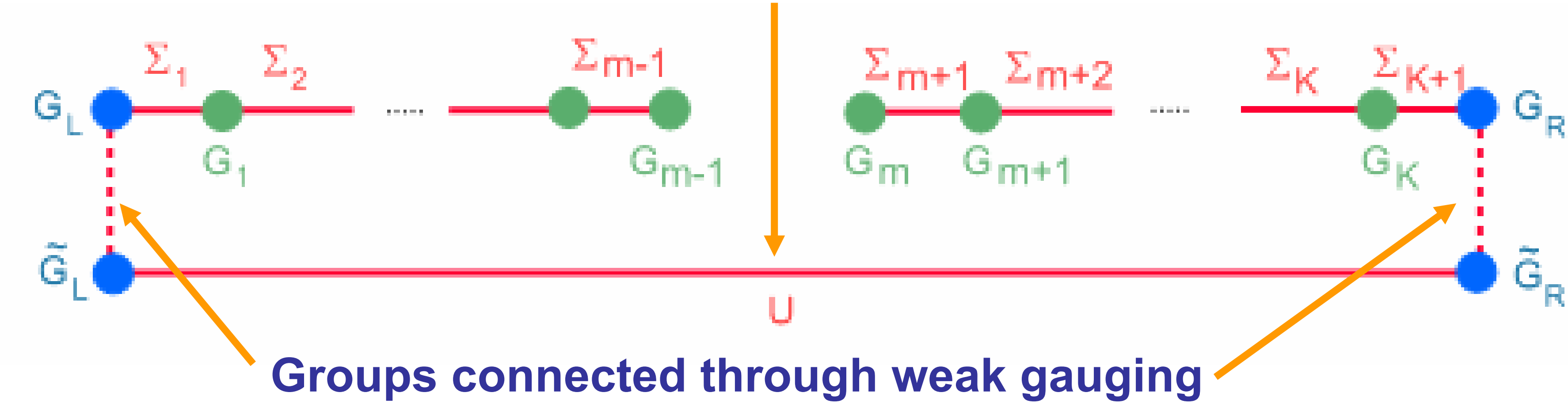
$$M_2 = \begin{bmatrix} g_1^2(f_1^2+f_2^2) & -g_1g_2f_2^2 & & & & \\ -g_1g_2f_2^2 & g_2^2(f_2^2+f_3^2) & -g_2g_3f_3^2 & & & \\ & -g_2g_3f_3^2 & g_3^2(f_3^2+f_4^2) & & & \\ & & & \ddots & & \\ & & & & g_{K-1}^2(f_{K-1}^2+f_K^2) & -g_{K-1}g_Kf_K^2 \\ & & & & -g_{K-1}g_Kf_K^2 & g_K^2(f_K^2+f_{K+1}^2) \end{bmatrix}$$

with $f_i = 0$, M_2 becomes block diagonal and $(M_2^{-2})_{1k} = 0$

$$\varepsilon_3 = 0$$

But we loose a link

Add a Wilson line: $U = \Sigma_1 \Sigma_2 \cdots \Sigma_K \Sigma_{K+1}$

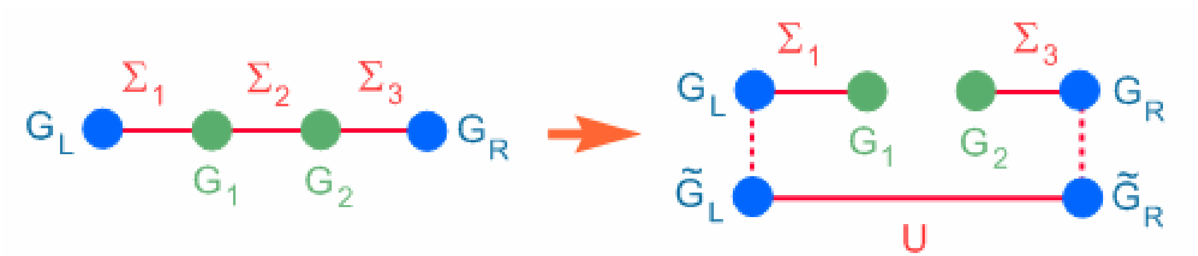


The theory has an enhanced symmetry $\left[SU(2)_L \otimes SU(2)_R\right]^2$

ensuring $\varepsilon_3 = 0$ (Inami , Lim, Yamada, 1992)

Particular example: D-BESS model

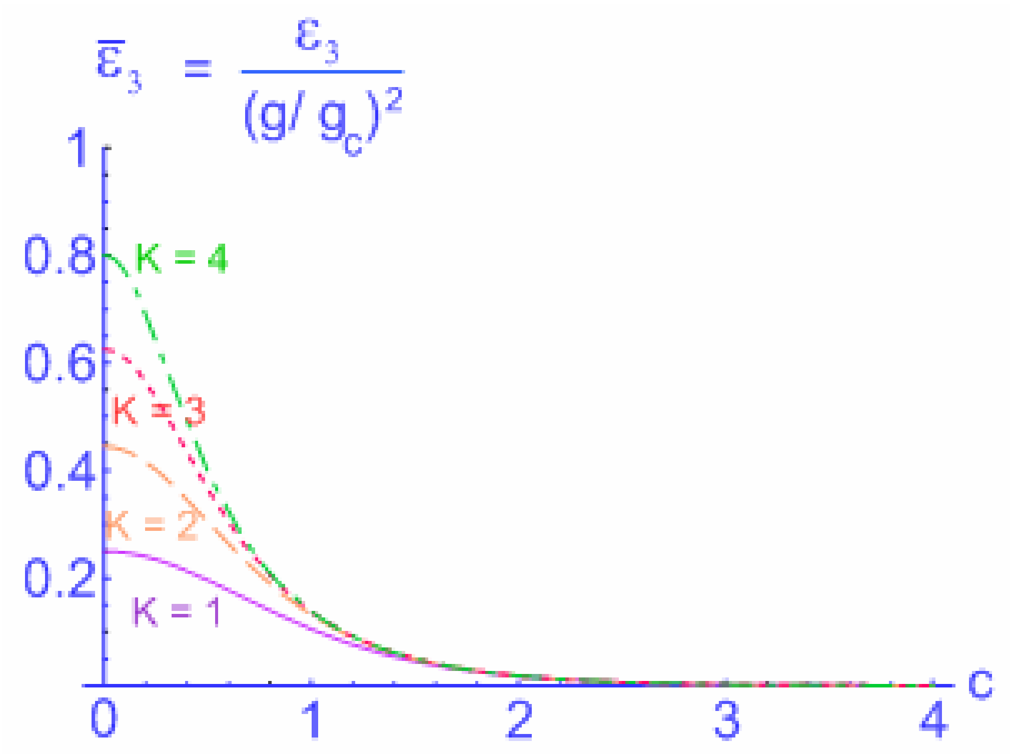
(**R.C. et al. 1995,1996**)



2) Suppressing a link

For instance assume an exponential behavior (no need of extra links)

$$f_i = \bar{f}\, e^{c(i-1)}, \quad g_i = g_c$$



$$\varepsilon_3 \xrightarrow[c\to\infty]{} \frac{g^2}{g_c^2} e^{-2c}$$

gain of about 10^{-2}

already for $c \approx 2$

But lowering the link couplings may give unitarity problems

Unitarity bounds for the linear moose

- The worst high-energy behavior comes from the scattering of longitudinal vector bosons. At energies higher than the vector mass the amplitude can be evaluated using the equivalence theorem, that is using the amplitude for the corresponding GB's.

- To do that one has to choose the unitary gauge with respect to the vector bosons

$$\Sigma_i = e^{i f \vec{\pi}\cdot\vec{\tau}/2f_i^2}$$

- The effective lagrangian for the pion and the heavy vector can be easily evaluated. The resulting 4-pion amplitude is given by

$$A_{\pi^+\pi^-\to\pi^+\pi^-} = -\frac{f^2}{4}\sum_{i=1}^{K+1}\frac{u}{f_i^6} + \frac{f^2}{4}\sum_{i,j=1}^{K} L_{ij}\left((u-t)(s-M_2)^{-1}_{ij} + (u-s)(t-M_2)^{-1}_{ij}\right)$$

$$L_{ij} = g_i g_j\left(\frac{1}{f_i^2}+\frac{1}{f_{i+1}^2}\right)\left(\frac{1}{f_j^2}+\frac{1}{f_{j+1}^2}\right)$$

- In the low-energy limit, $m_W \ll E \ll m_v$, we get the LET:

$$A_{\pi^+\pi^-\to\pi^+\pi^-} = -\frac{f^2}{4}\left(\sum_{i=1}^{K+1}\frac{1}{f_i^2}\right)^3 u = -\frac{u}{4f^2} = -\frac{u}{v^2}$$

- In the high-energy limit

$$A_{\pi^+\pi^-\to\pi^+\pi^-} = -\frac{f^2}{4}\sum_{i=1}^{K+1}\frac{1}{f_i^6}u$$

Best unitarity limit

$$\mathbf{f_i = f_c \to A = -\frac{u}{(K+1)^2 v^2}}$$

$$\mathbf{\Lambda_{moose} = (K+1)\Lambda_{HSM} \approx 1.2(K+1)TeV}$$

- By taking into account all the vector bosons the eigenchannel amplitudes, using the equivalence theorem, are given by

$$\left(\Sigma_i = e^{i\vec{\pi}_i\cdot\vec{\tau}/2f_i}\right) \qquad A_{\pi_i^+\pi_i^-\to\pi_i^+\pi_i^-} \to -\frac{u}{4f_i^2}$$

- The unitarity limit is determined by the smallest link coupling. By taking all equal (see also Chivukula, He, 2002)

Unitarity limit

$$f_i = f_c \to A \to -\frac{u}{(K+1)v^2}$$

$$\Lambda_{moose} = (K+1)^{1/2}\Lambda_{HSM} \approx 1.2\,(K+1)^{1/2}\,\mathrm{TeV}$$

$$M_V^{max} << \Lambda_{moose},\ \text{but roughly } M_V^{max} \approx K M_W$$

$$\Downarrow$$

$$K M_W << 1.2(K+1)^{1/2}\,\mathrm{TeV} \Rightarrow K << 14$$

$$\Downarrow$$

$$K \approx 3-5 \Rightarrow \Lambda_{moose} \approx 2.4-3\,\mathrm{TeV}$$

→ Hardly compatible with electro-weak experimental constraints

Delocalizing fermions

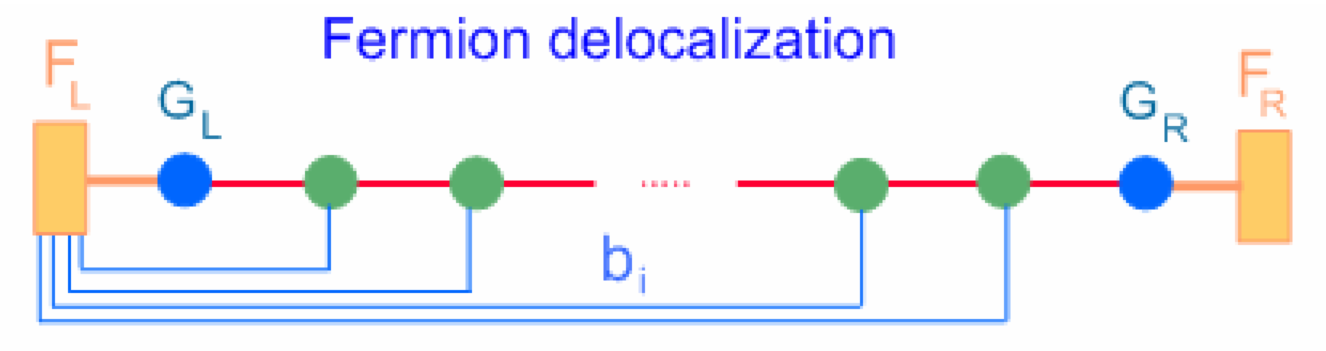
- Left- and right-handed fermions, $\psi_{L\,(R)}$, are coupled to the ends of the moose, but they can coupled to any site by using a Wilson line (see also Chivukula, Simmons, He, Kurachi, 2005)

$$\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L\,,\ \chi_L^i \to U_i \chi_L^i$$

$$\Downarrow$$

$$b_i \bar{\chi}_L^i \gamma^\mu \left(\partial_\mu + i g_i A_\mu^i + \frac{i}{2} g'(B-L) Y_\mu \right) \chi_L^i$$

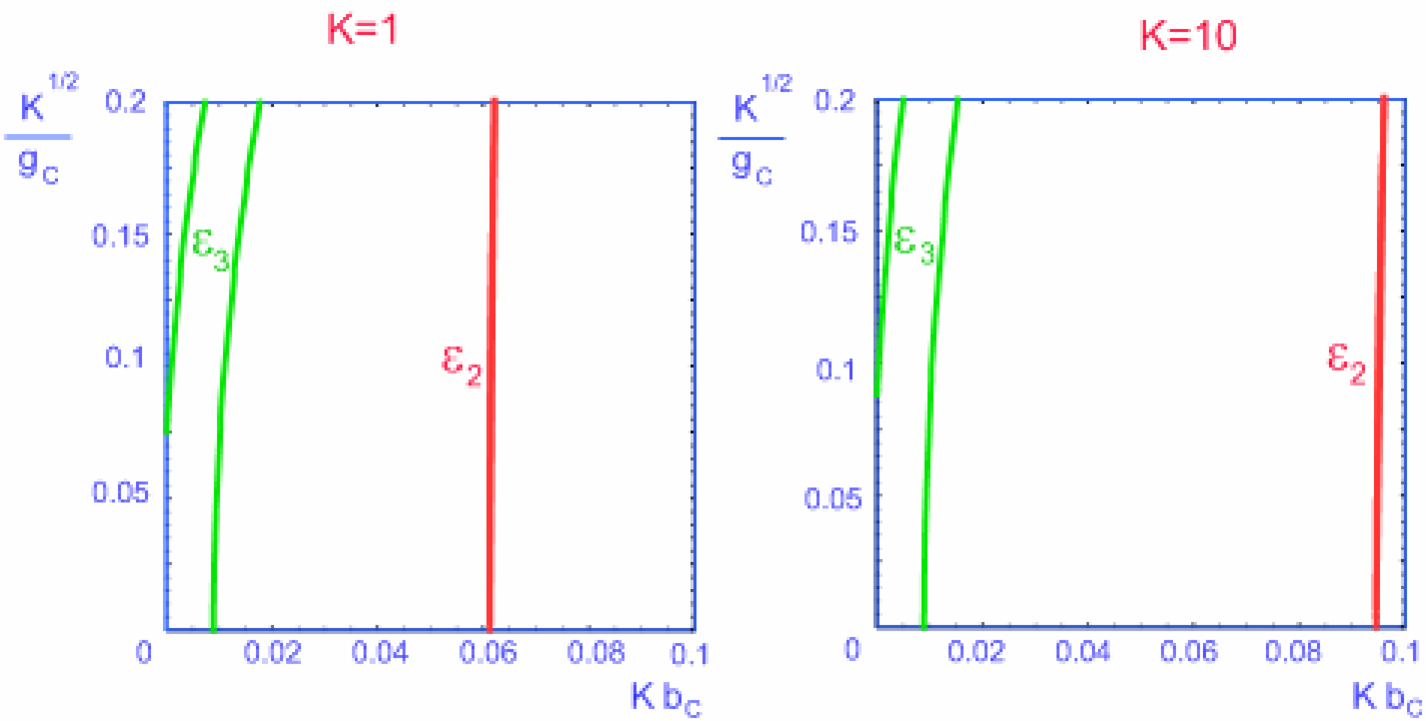
(We avoid delocalization of the right-handed fermions. Small terms since they could contribute to right-handed currents constrained by the K_L-K_S mass difference)



$$\varepsilon_1 \approx 0, \quad \varepsilon_2 \approx 0, \quad \varepsilon_3 \approx \sum_{i=1}^{K} y_i \left(\frac{g^2}{g_i^2}(1 - y_i) - b_i \right)$$

- **Possibility of agreement with EW data with some fine tuning**
- **Two cases:**

$$f_i = f_c, \quad g_i = g_c, \quad b_i = b_c$$



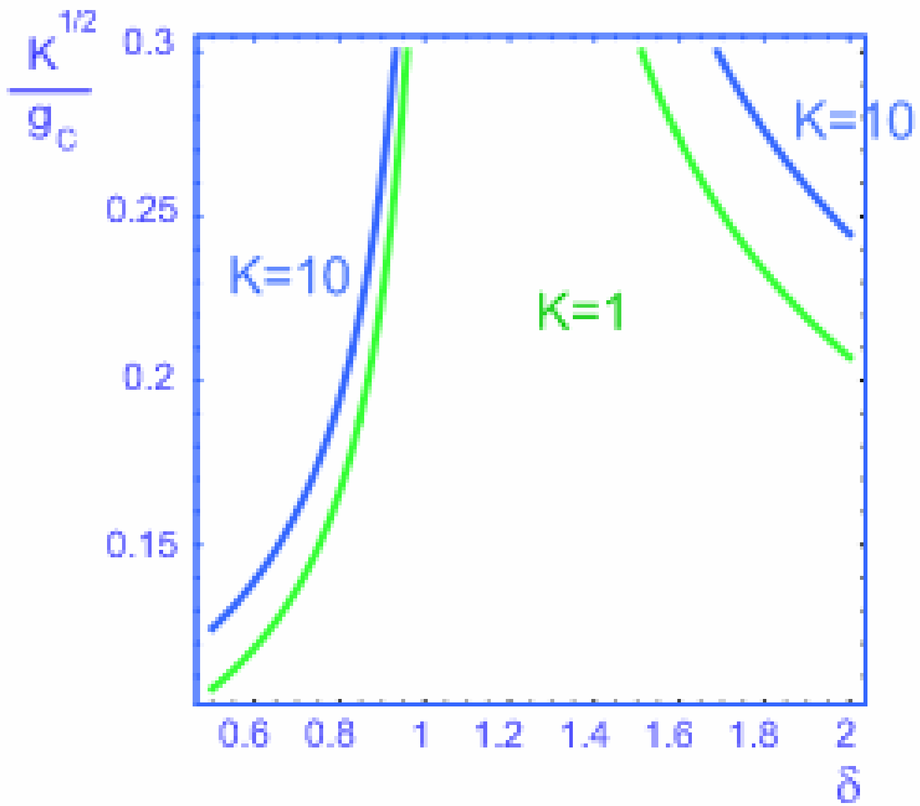
(95% CL, with rad. corrs. as in the SM with 1 TeV Higgs and mass 178 GeV for the top)

Local cancellation

$$b_i = \delta \frac{g^2}{g_i^2}(1 - y_i), \quad \text{with} \quad g_i = g_c, f_i = f_c$$

$$\Downarrow$$

$$b_i = \delta \frac{g^2}{g_i^2}\left(1 - \frac{i}{K+1}\right)$$



(95% CL, with rad. corrs. as in the SM with 1 TeV Higgs and mass 178 GeV for the top)

Topological relations

- A moose diagram looks as a planar Feynman diagram via the identification **links = legs, sites = vertices**

E = # external links

I = # of internal links

V_ℓ = # of sites with ℓ links

L = # of loops

S = # of not eaten up GB multiplets

$$L = I - \left(\sum_\ell V_\ell - 1\right), \quad S = I + E - \sum_\ell V_\ell$$

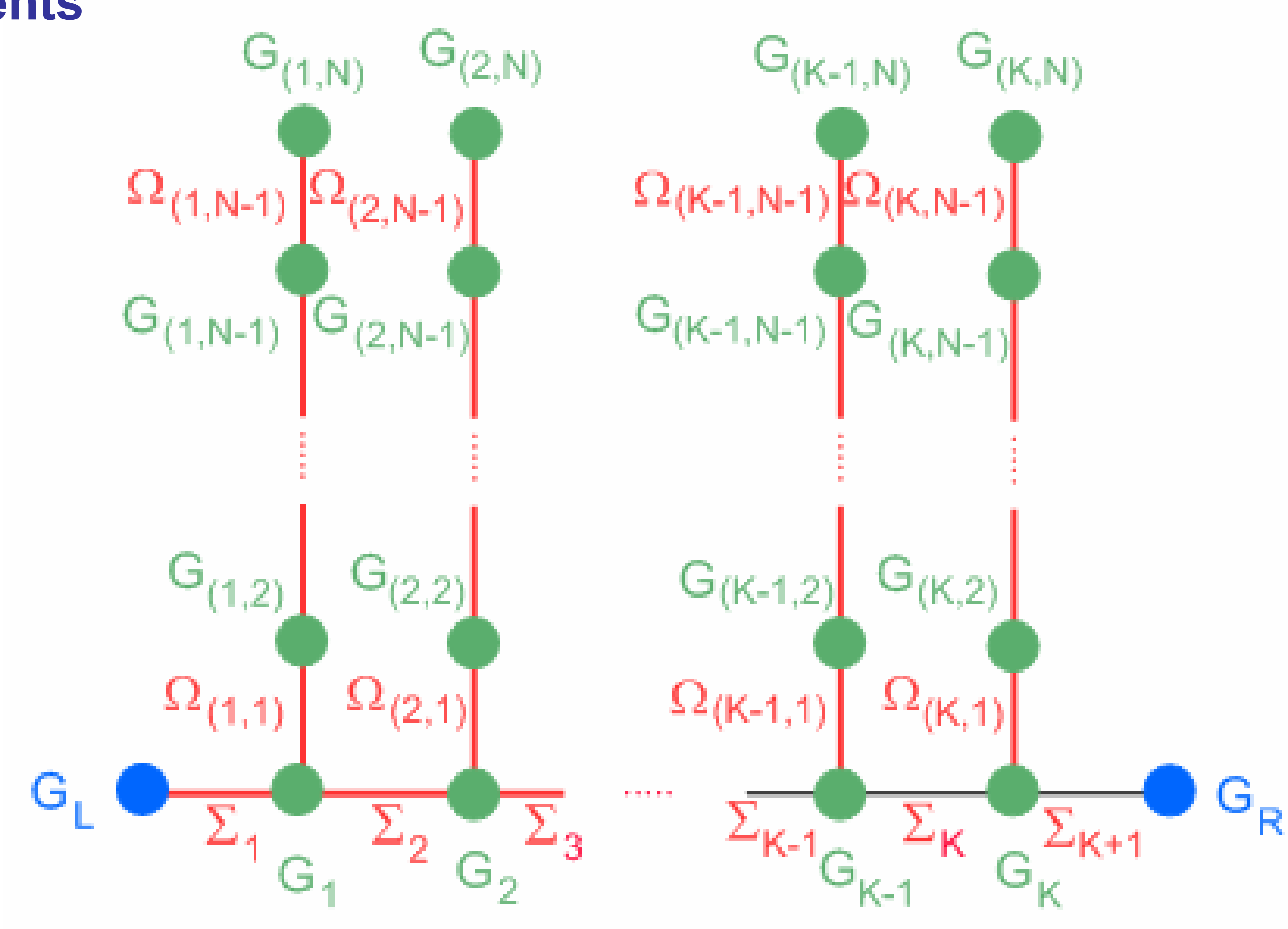
$$L = S - (E - 1)$$

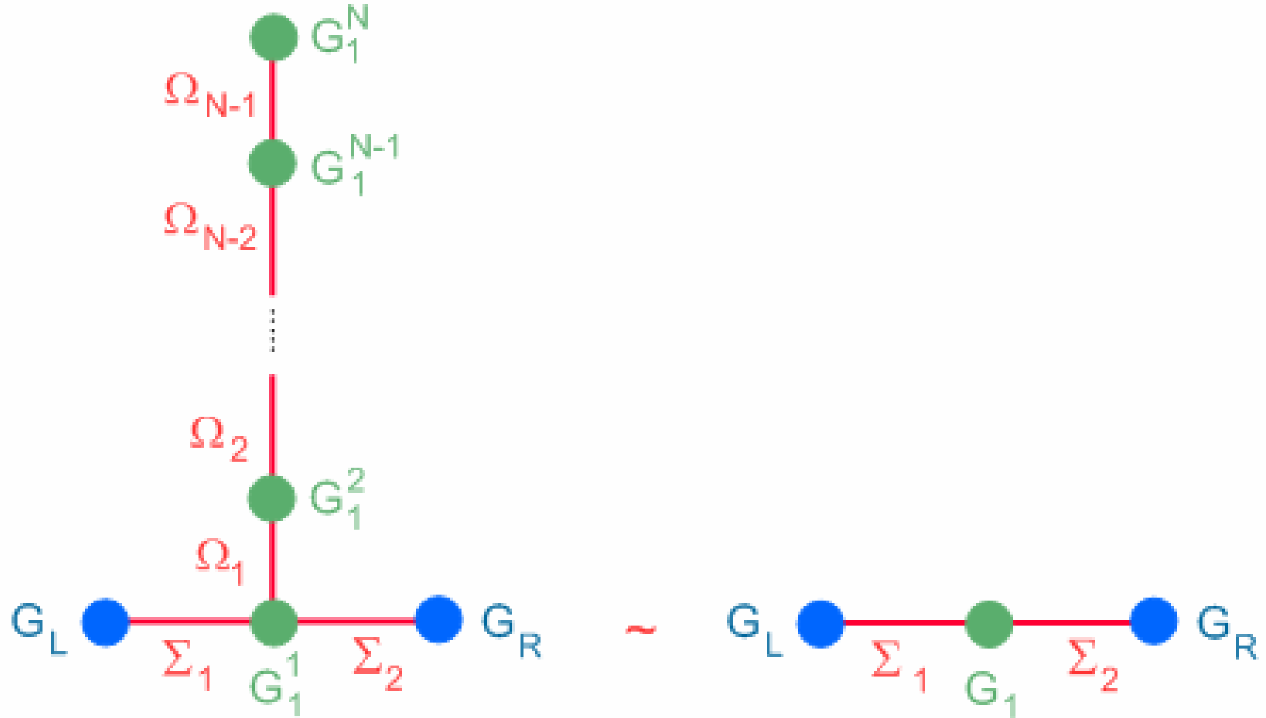
- Requiring E = 2 to insure the $SU(2)_L \times SU(2)_R$ global symmetry and S = 1 in order to get 3 GB's implies **no loops**

$$L = 0$$

A planar moose

- The most general SU(2) planar moose compatible with our requirements





- **These two moose are equivalent through the substitution:**

$$\frac{1}{g_1^2} \rightarrow \sum_{j=1}^{N} \frac{1}{g_{(1j)}^2}$$

- **In general the planar moose is equivalent to a linear one with an analogous substitution**

$$\boxed{\frac{1}{g_i^2} \rightarrow \sum_{j=1}^{N} \frac{1}{g_{(ij)}^2}}$$

The continuum limit

- The continuum limit is defined by

$$K \to \infty, \quad a \to 0, \quad Ka \to \pi R$$

$$\lim_{a\to 0} a g_i^2 = g_5^2, \quad \lim_{a\to 0} a f_i^2 = f^2(y), \quad \lim_{a\to 0} \frac{b_i}{a} = b(y)$$

- The link couplings and a variable gauge coupling can be simulated in the continuum by a non-flat 5-dim metrics. For a flat metrics, corresponding to equal f's and g's, one gets

$$\varepsilon_3 = \frac{1}{6}\frac{g^2}{g_c^2}\frac{K(K+2)}{K} \to \frac{1}{6}\frac{g^2}{g_5^2}\pi R$$

- For a Randall-Sundrum metrics, corresponding to exponential f's

$$f_i = \bar{f} e^{c(i-1)} \xrightarrow[c=ka]{} f(y) = \bar{f} e^{ky}$$

$$\varepsilon_3 = \frac{1}{4k} \frac{g^2}{g_5^2} \frac{e^{4k\pi R} - 4k\pi R e^{2k\pi R} - 1}{(1 - e^{2k\pi R})^2} \xrightarrow[k\pi R >> 1]{} \frac{1}{4k} \frac{g^2}{g_5^2}$$

- By choosing kR ~ 10 and $g_5^2 = \pi R g_4^2$, we get

$$\varepsilon_3 \approx \frac{1}{40\pi} \frac{g^2}{g_4^2} \approx 8 \cdot 10^{-3} \frac{g^2}{g_4^2}$$

could be consistent with EW data

- For the planar moose:

$$\varepsilon_3 = \frac{1}{6} \frac{g^2}{g_6^2} \pi^2 R R' \quad \text{(flat case);} \quad \varepsilon_3 \to \frac{1}{4} \frac{g^2}{g_6^2} \frac{\pi R'}{k} \quad \text{(RS case)}$$

No suppression from the vertical links

Summary and Conclusions

- Higher dimensional gauge theories naturally suggest the possibility of Higgsless theories.
- Study in 4-dim through discrete extra-dim leads to moose theories.
- Simplest case: the linear moose.
- Difficulties in EW corrections similar to TC models.
- EW corrections and unitarity bounds push in different directions.
- Possibility of easing the theory delocalizing the fermions and using some fine tuning.