XVII ICAP - 2000

The impact of atomic precision measurements in high energy physics

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Florence, June 4-9, 2000

Summary

- Why AP is relevant for HEP?
- Atomic Parity Violation
- The CPT Theorem

Why AP is relevant for HEP?

In the common sense different and very well separated scales do not interfere. Technically this is known as decoupling. It means that a generic physical observable, A, when considered at the scale $\Lambda_1 \ll \Lambda_2$ can be expressed as

$$A(\Lambda_1, \Lambda_2) \approx A(\Lambda_1) + \mathcal{O}\left(\left(\frac{\Lambda_1}{\Lambda_2}\right)^n\right)$$

Since AP and HEP typical scales are separated by $10^6 \div 10^7$, we do not expect, under normal circumstances, any interference.

Consider a combination of observables, B, such to cancel the first term

$$B = c\left(\frac{\Lambda_1}{\Lambda_2}\right)^n$$

To be able to measure B one typically needs

- The coefficient of $(\Lambda_1/\Lambda_2)^n$ is very large and one has a good experimental sensitivity.

- The experimental setup has an extremely good sensitivity.

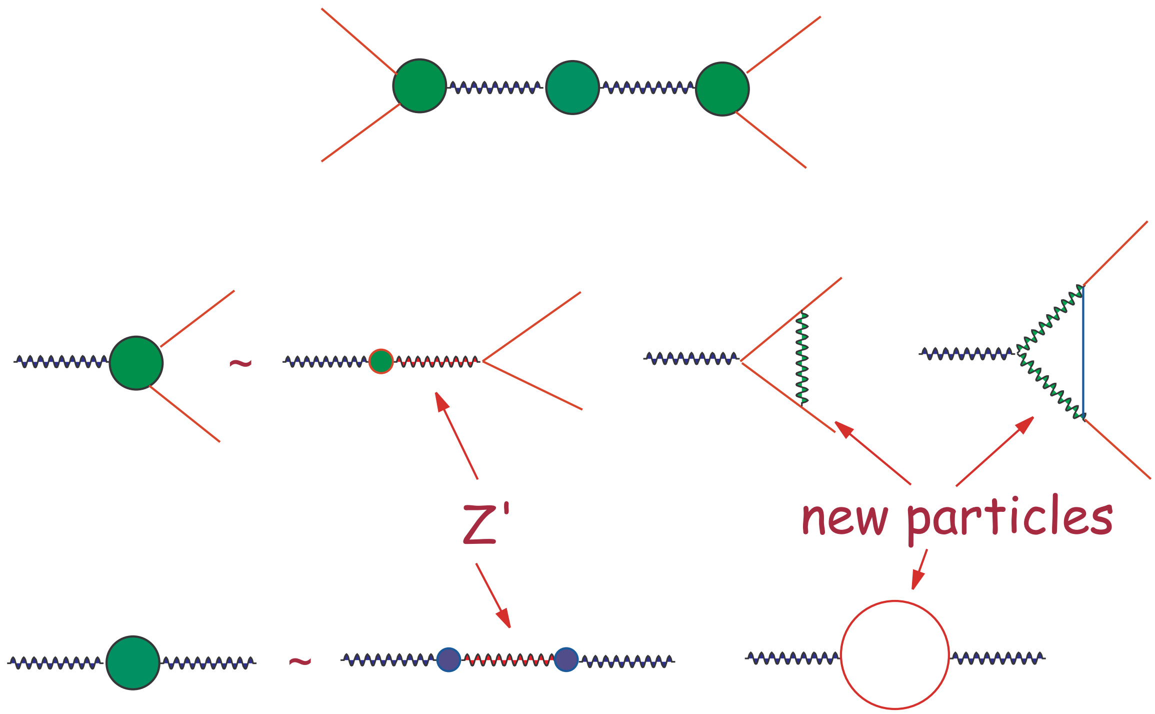
The first case arises in the experiments of Atomic Parity Violation. The second is what we need to test possible CPT violations.

Atomic Parity Violation

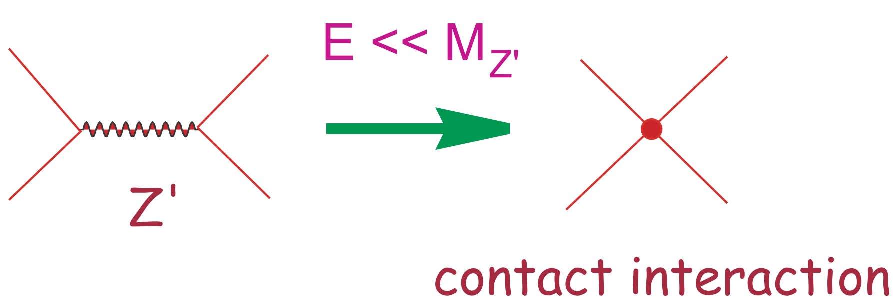
- Discussion of the latest determination of Q_W of atomic cesium (2.5σ away from the SM)

- Implications of APV for new physics
  - Composite Models, Lepto-quarks
  - Extra-dimension models
  - New Vector Bosons from E_6 and LR models

LEP physics puts heavy constraints on new physics. BUT which physics is explored at LEP?



LEP not sensitive to **NEW 4-Fermi INTERACTIONS**



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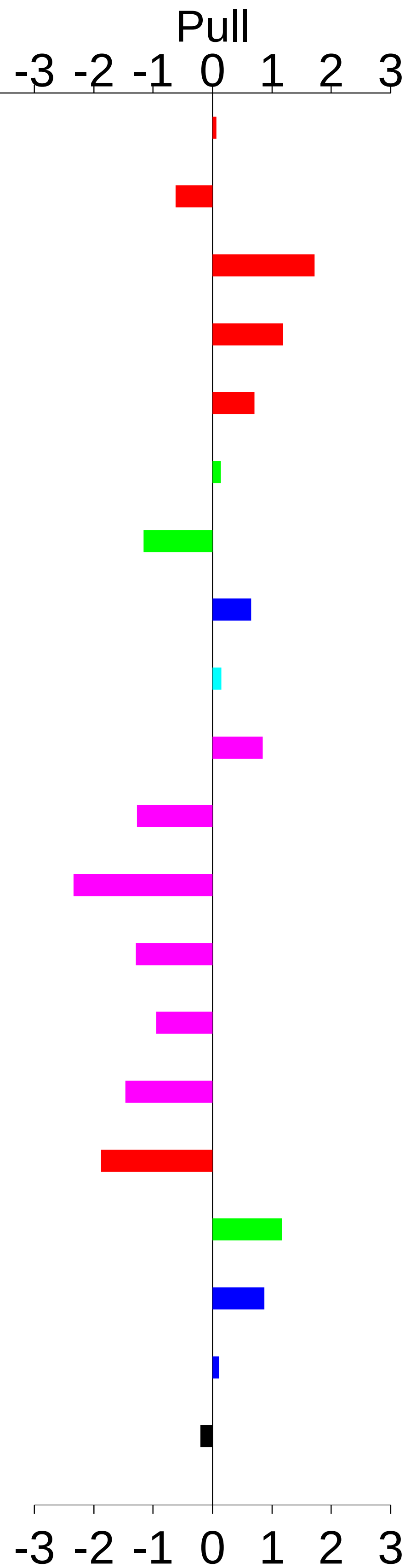
	Measurement	Pull
m_Z [GeV]	91.1871 ± 0.0021	.07
Γ_Z [GeV]	2.4944 ± 0.0024	-.62
σ^0_{hadr} [nb]	41.544 ± 0.037	1.72
R_e	20.768 ± 0.024	1.19
$A^{0,e}_{fb}$	0.01701 ± 0.00095	.70
A_e	0.1483 ± 0.0051	.13
A_τ	0.1425 ± 0.0044	-1.16
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.65
m_W [GeV]	80.401 ± 0.048	.15
R_b	0.21642 ± 0.00073	.85
R_c	0.1674 ± 0.0038	-1.27
$A^{0,b}_{fb}$	0.0988 ± 0.0020	-2.34
$A^{0,c}_{fb}$	0.0692 ± 0.0037	-1.29
A_b	0.911 ± 0.025	-.95
A_c	0.630 ± 0.026	-1.47
$\sin^2\theta^{lept}_{eff}$	0.23096 ± 0.00026	-1.87
$\sin^2\theta_W$	0.2255 ± 0.0021	1.17
m_W [GeV]	80.448 ± 0.062	.88
m_t [GeV]	174.3 ± 5.1	.11
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.20



Low-energy experiments complementary to LEP

- they probe different energy scale
- they probe a different set of model-independent electron-quark couplings

A good possibility are experiments in

Atomic Parity Violation

Measure the combinations $v_e a_q$, $a_e v_q$.
From Q_W in atomic cesium one gets

$$c_{1u,1d} = -8\, a_e\, v_{1u,1d}$$

Atomic Parity Violation

Within the SM the relevant 4-fermi PV interaction between charged leptons and quarks is given by

$$\mathcal{L}_{\rm eff}^{PV} = \frac{G_F}{\sqrt{2}}\Big[(\bar\ell\gamma_\mu\gamma_5\ell)\sum_{q=u,d} c_{1q}\bar q\gamma^\mu q +$$

$$+(\bar\ell\gamma_\mu\ell)\sum_{q=u,d} c_{2q}\bar q\gamma^\mu\gamma_5 q\Big]$$

where

$$c_{1q} = -8a_\ell v_q \qquad c_{2q} = -8v_\ell a_q$$

In terms of nucleons

$$\mathcal{L}_{\rm eff}^{PV} = -\frac{G_F}{\sqrt{2}}\Big[(\bar\ell\gamma_\mu\gamma_5\ell)\sum_{N=p,n} c_{1N}\bar N\gamma^\mu N +$$

$$+(\bar\ell\gamma_\mu\ell)\sum_{N=p,n} c_{2N}\bar N\gamma^\mu\gamma_5 N\Big]$$

where

$$c_{ip} = -2c_{iu} - c_{id}, \quad c_{in} = -c_{iu} - 2c_{id}, \quad i = 1,2$$

In the SM:

$$a_f = -1/2T_{3L}^f, \qquad v_f = 1/2(T_{3L}^f - 2s_\theta^2 Q^f)$$

$$c_{1q} = -8a_\ell v_q = -(T_3^q - 2s_\theta^2 Q^q)$$

$$c_{2q} = -8v_\ell a_q = -T_3^q(1 - 4s_\theta^2)$$

In the non-relativistic limit, for a point-like nucleus with Z protons and N neutrons

$$H_{PV} = \frac{G_F}{4\sqrt{2}m_\ell}\Big[Q_W(Z,N)\vec{\sigma}_\ell \cdot [\vec{p},\delta^3(\vec{r})]_+ +$$

$$+2(c_{2p}\vec{S}_p + c_{2n}\vec{S}_n)\cdot[\vec{p},\delta^3(\vec{r})]_+$$

$$-2i\vec{\sigma}_\ell \wedge (c_{2p}\vec{S}_p + c_{2n}\vec{S}_n)\cdot[\vec{p},\delta^3(\vec{r})]_+\Big]$$

The weak charge of the nucleus is defined as

$$Q_W(Z,N) = 2\left[c_{1p}Z + c_{1n}N\right]$$

For large values of Z, the term in Q_W is dominant (coherence effect) whereas spin terms tend to cancel (Bouchiat and Bouchiat 1974-75)

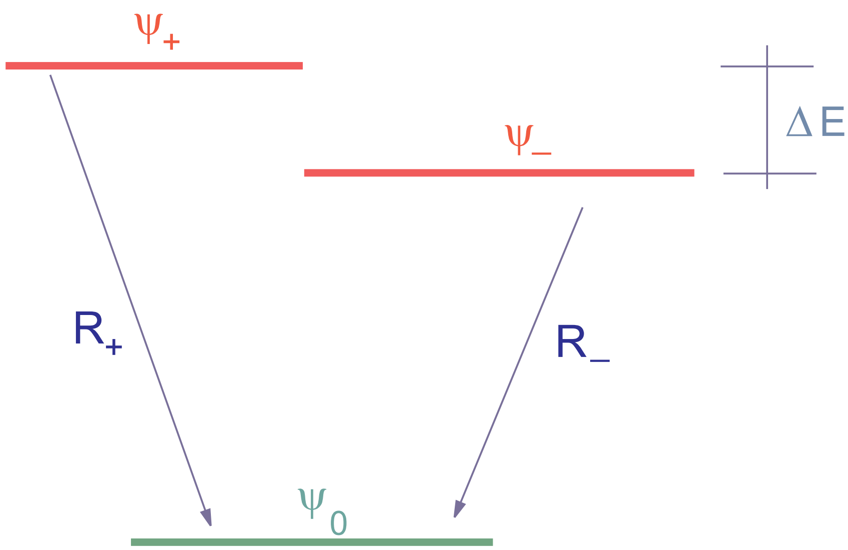
$$\langle H_{PV}\rangle \propto Z^2 Q_W(Z,N) \propto Z^3$$

One Z from the electron momentum $\vec{p}$, one Z from the wave function at the origin and one Z from Q_W

For Cesium $Z = 55$ and $Z^3 \approx 2\cdot 10^5$

In APV one looks at an optical transition between a pair of states $\psi_\pm$ mixed by H_{PV}, and ψ_0, with ψ_+ of the same nominal parity as ψ_0

$$|\psi_+\rangle \to |\psi_+\rangle + \eta|\psi_-\rangle \qquad \eta = \frac{\langle\psi_-|H_{PV}|\psi_+\rangle}{\Delta E}$$



$R_\pm$ = Decay probabilities

$$R_+ = |M_1|^2, \qquad R_- = |E_1^{pv}|^2$$

The total transition probability

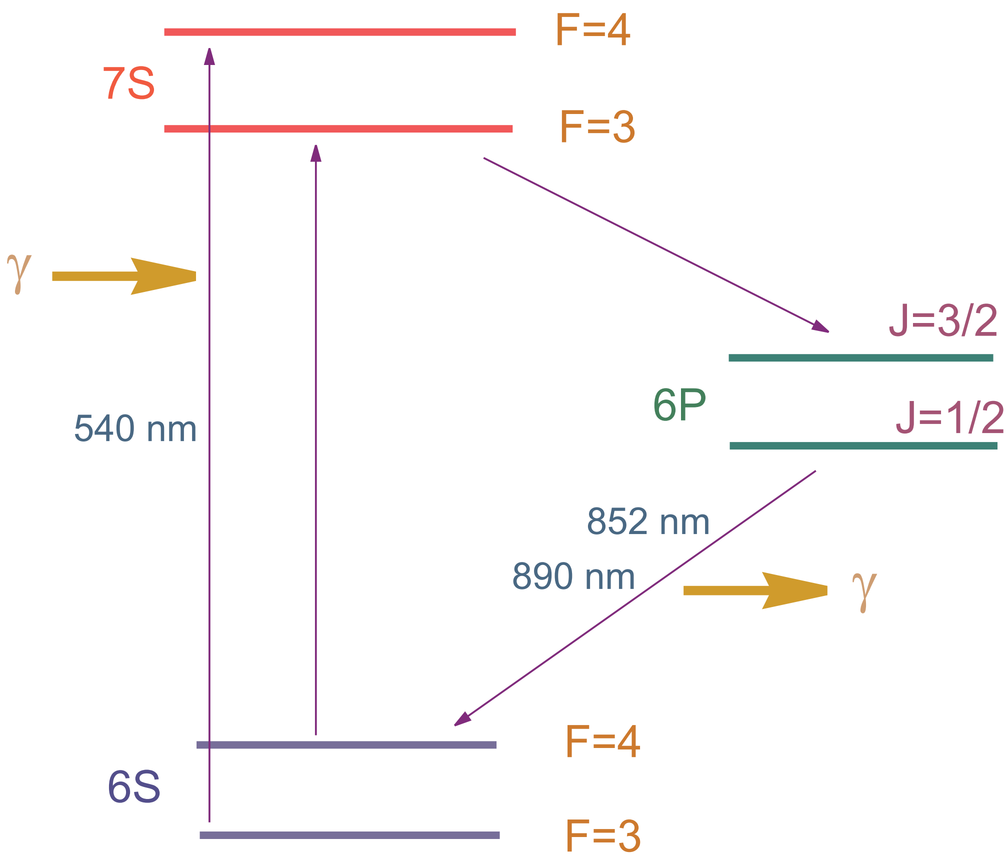
$$W \sim M_1^2 + |E_1^{pv}|^2 \pm 2Im(E_1^{pv})M_1$$

$\pm$ depending on the helicity of the absorbed (emitted) photon.
Measure the circular dichroism

$$\delta = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \sim 2Im(E_1^{pv})/M_1$$

PV effect proportional to $Im(E_1^{pv})/M_1$ (bigger if R_+ suppressed transition)

Last technique used for the $6S \to 7S$ in atomic $^{133}_{55}Cs$ giving rise to $\delta \approx 10^{-4} \div 10^{-3}$, in Paris (1982,1984) and in Boulder (1985,1988,1997).



To overcome the background the interference with a large electro-induced (Stark) transition has been used

$$\text{Experiment} \longrightarrow \langle H_{PV} \rangle \approx Q_W \, \kappa_{PV}$$

The atomic form-factor κ_{PV} must be evaluated theoretically.
High-precision measurements must be coupled with calculations of similar accuracy for a precise determination of Q_W.

Theoretical evaluation of κ_{PV}

Blundell et al., Dzuba et al.

κ_{PV} from many-body perturbative theory with Hartree-Fock potential

The error on k_{PV} estimated from the discrepancies with experimental values of PC observables (energy levels, hyperfine splittings etc.) and by requiring stability against variation of the parameters

(1992) $\Delta\kappa_{PV}/\kappa_{PV} \sim 1\%$

(Taken into account: nuclear distribution, nuclear spin-dependent effects, Z-exchange among the electrons)

In 1999 $\Delta\kappa_{PV}/\kappa_{PV}$ down to 0.4%

(Bennett and Wieman)

- new measurements of relevant quantities in cesium are in better agreement with the theoretical calculation
- a problem of the previous calculations, when applied to sodium and lithium, leading to 1% discrepancy in the lifetimes, it has now disappeared after new experiments

Calculations must be extended to higher orders in many-body perturbation theory to confirm the small theoretical error

Quantity measured	Calculation tested	Difference ($\times 10^3$) Dzuba, *et al.*	Blundell *et al.*	σ_{Expt}
6S→7S dc Stark shift	$\langle 7P \parallel D \parallel 7S \rangle$	−3.4[19]	−0.7[22]	1.0[4]
$6P_{1/2}$ lifetime	$\langle 6S \parallel D \parallel 6P_{1/2} \rangle$	−4.2[−8]	4.3[1]	1.0[43]
$6P_{3/2}$ lifetime	$\langle 6S \parallel D \parallel 6P_{3/2} \rangle$	−2.6[−41]	7.9[−31]	2.3[22]
α	$\langle 7S \parallel D \parallel 6P_{1/2} \rangle$, and $\langle 7S \parallel D \parallel 6P_{3/2} \rangle$	–	−1.4	3.2
β	same as α	–	−0.8	3.0
6S HFS	$\psi_{6S}(r=0)$	1.8	−3.1	–
7S HFS	$\psi_{7S}(r=0)$	−6.0	−3.4	0.2
$6P_{1/2}$ HFS	$\langle 1/r^3 \rangle_{6P}$	−6.1	2.6	0.2
$7P_{1/2}$ HFS	$\langle 1/r^3 \rangle_{7P}$	−7.1	−1.5	0.5

The data on APV

Experimental result in 1988 (Boulder group) and theoretical evaluation of k_{PV} in 1991 (Blundell et al., Dzuba et al.) get Q_W at a level of 2.5%

$$Q_W\left({}^{133}_{55}Cs\right) = -71.04 \pm (1.58)_{\text{exp}} \pm (0.88)_{\text{theor}}$$

In 1999, the same group (Bennett,Wieman) with uncertainty 0.6% (measure of Stark amplitude)

$$\boxed{Q_W\left({}^{133}_{55}Cs\right) = -72.06 \pm (0.28)_{\text{exp}} \pm (0.34)_{\text{theor}}}$$

to be compared with the SM result

$$Q_W^{\text{SM}}\left({}^{133}_{55}Cs\right) = -73.24 \pm 0.13 \qquad (m_H = 100\ GeV)$$

↑ from hadronic loops

$$\boxed{Q_W^{\text{exp}} - Q_W^{\text{theor}} = 1.18 \pm 0.46, \quad (2.57\ SD)}$$

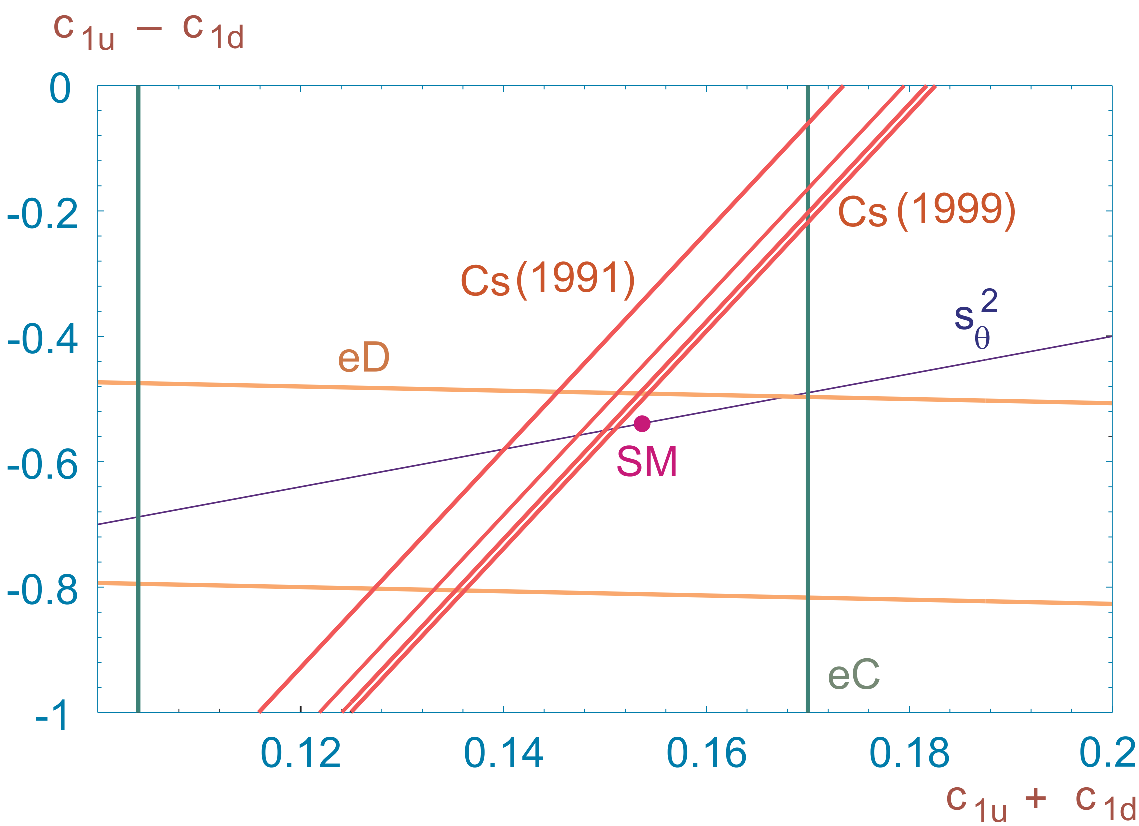
If this difference is not due to an experimental error, or a statistical fluctuation, or an error in the theoretical calculations, or some overlooked contributions in the SM, then

SM disfavored at 99%CL

For increasing m_H, Q_W decreases and the discrepancy increases

$$Q_W = -2[(2Z+N)c_{1u} + (Z+2N)c_{1d}]$$

$$\text{Cs}(1991) \longrightarrow Q_W = -71.08 \pm 1.81$$
$$\text{Cs}(1999) \longrightarrow Q_W = -72.06 \pm 0.44$$



$$\text{eD[SLAC]}(1995) \longrightarrow 2c_{1u} - c_{1d} = 0.94 \pm 0.26$$
$$\text{eC[Bates]}(1990) \longrightarrow c_{1u} + c_{1d} = 0.137 \pm 0.033$$

The new Cesium result deviates of $\approx 2.5\sigma$ from the SM prediction

Let us parameterize: (Altarelli,Barbieri,Caravaglios)

$$Q_W = -72.72 \pm 0.13 - 102\epsilon_3^{\rm rad} + \delta_N Q_W$$

For $m_t = 175\ GeV$, $m_H = 100(300)\ GeV$

$$\epsilon_3^{\rm rad} = 5.110(6.115) \times 10^{-3}$$

For instance, new physics contributing to the Z self-energy (oblique corrections) gives

$$\delta_N Q_W(\text{oblique}) = -102\,\epsilon_{3N}$$

To compensate the discrepancy on Q_W one would need

$$\epsilon_{3N} = (-11.6 \pm 4.5) \times 10^{-3}$$

LEP and SLC physics constrain strongly deviations from the SM:

$$\epsilon_3^{\rm exp} = \epsilon_3^{\rm rad} + \epsilon_{3N} = (4.19 \pm 1) \times 10^{-3}$$

then $\epsilon_{3N} \sim 10^{-3}$ (for a light Higgs)

almost an order of magnitude too small

We need new physics not constrained by LEP

Old results

Other APV experiments (Oxford and Seattle) in Tallium give

$$Q_W(Tl)^{\text{exp}} = -114.8 \pm 1.2\,(\text{exp}) \pm 3.4\,(\text{th})$$

Values at the level of 3%, compatible with the SM result

$$Q_W(Tl)^{\text{th}} = -116.7 \pm 0.1$$

New experiments

- Bouchiat group is planning a new experiment on cesium, but the planned experimental sensitivity is lower than the one in Boulder

- In Berkeley and Seattle there are plans for isotope ratio measurements

Possible experimental improvements

Figure of merit for the Boulder experiment:

$$\left|\frac{Q_W^{\text{exp}} - Q_W^{\text{theor}}}{Q_W^{\text{exp}}}\right| = 0.016 \pm 0.0038(\text{exp}) \pm 0.005(\text{th})$$

The prospective isotope ratio limits for APV studies in Seattle and Berkeley give

$$\left|\frac{\mathcal{R}^{\text{exp}} - \mathcal{R}^{\text{theor}}}{\mathcal{R}^{\text{exp}}}\right| = \ ? \ \pm 0.001(\text{exp}) \pm 0.004(\text{th})$$

where, (A_{PV} is a PV observable)

$$\mathcal{R} = \frac{A_{PV}(N') - A_{PV}(N)}{A_{PV}(N') + A_{PV}(N)}$$

and it can be expressed as

$$\mathcal{R} = \mathcal{R}_{\text{SM}}(1 + \delta_{\mathcal{R}})$$

$\delta_{\mathcal{R}}$ contains new physics and depends on the variation of the neutron density along the isotope chaine. In a simple model of constant neutron and proton density ($\Delta N = N' - N$)

$$\delta_{\mathcal{R}}(\text{radius}) = -\frac{3}{7}\frac{N'}{\Delta N}(Z\alpha)^2 \delta \left(\frac{R_{N'} - R_N}{R_p}\right)$$

For Cesium and Barium, Pollock (1992), Chen and Vogel (1994) have shown that nuclear theory is a factor two away from the required experimental sensitivity

Bounds on $\delta_N Q_W$

$$Q_W^{\text{exp}} - Q_W^{\text{SM}}(m_H) =$$
$$0.66 + 102\epsilon_3^{\text{rad}}(m_H) - \delta_N Q_W \pm 0.46$$

For a light Higgs ($m_H = 100\ GeV$)

$$\boxed{95\%\text{CL}, \quad 0.28 \le \delta_N Q_W \le 2.08}$$

The positive lower bound implies strong restrictions on new physics.
All the models leading to $\delta_N Q_W \le 0$ are excluded. For increasing m_H both bounds increase.

One possible neglected contribution to Q_W is the difference between neutron and proton spatial distributions in the nucleus. It is small ($Q_W^{n-p} \approx 0.1$) but largely model-dependent.
Even with a conservative error estimate

$$\Delta Q_W^{n-p} = \pm 0.3 \text{ (Pollock,Welliver)}$$

the deviation with respect to SM remains $\approx 2\sigma$
The following analysis is based on the result by Bennett and Wieman

Bounds on New Physics

The interesting parity violating effective lagrangian within the SM is

$$\mathcal{L}^{\mathsf{PV}}_{\mathsf{SM}} = \frac{G_F}{\sqrt{2}}\,\bar{e}\gamma_\mu\gamma_5 e \sum_{q=u,d} c_{1q}\bar{q}\gamma^\mu q$$

The typical low-energy contribution of new physics is a similar 4-fermi interaction

$$\mathcal{L}^{\mathsf{PV}}_{\mathsf{NP}} = \frac{4\pi g_{NP}}{\Lambda^2}\,\bar{e}\gamma_\mu\gamma_5 e \sum_{q=u,d} h_{1q}\bar{q}\gamma^\mu q$$

An experimental sensitivity to $(h_{1q} \approx c_{1q})$

$$\frac{\Delta Q_W}{Q_W} \approx 1\% \mapsto \Lambda \approx 17\, g_{NP}\, TeV$$

strong NP, $g^2_{NP} \approx 1 \mapsto \Lambda \approx 17\ TeV$

weak NP, $g^2_{NP} \approx \alpha \mapsto \Lambda \approx 1.5\ TeV$

Q_W probes NP at $\Lambda \gtrsim 1\ TeV$

Models of New Physics

Contact Interactions from Compositeness

A typical example operator: (Langacker)

$$\mathcal{L} = \pm \frac{g^2}{\Lambda^2} \bar{e}\, \Gamma_\mu\, e\, \bar{q}\, \Gamma^\mu\, q$$

with $\Gamma_\mu = \gamma_\mu \frac{1-\gamma_5}{2}$, Λ the compositeness scale, and g^2 the strength of the interaction (we expect $g^2 \sim 4\pi$).
Using

$$Q_W = -2\left[(2Z+N)c_{1u} + (Z+2N)c_{1d}\right]$$

The contact interactions modify the coefficients

$$c_{1u,1d} \to c_{1u,1d} + \Delta C_{1u,1d}$$

In this example: $\Delta C_{1u} = \Delta C_{1d} = \Delta C$

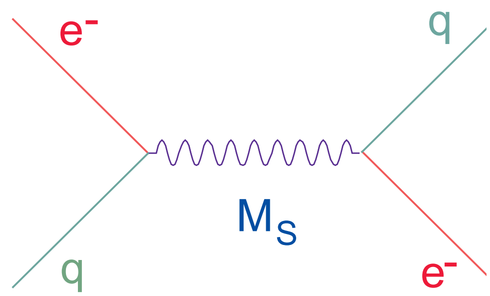
$$\Delta C = \mp \frac{\sqrt{2}\pi}{G_F \Lambda^2}$$

The negative sign in $\mathcal{L}$ is excluded, whereas for the positive sign we have the 95%CL bound

$$12.1 \leq \Lambda(TeV) \leq 32.9$$

to be compared with PDG limit, $\Lambda \geq 3.5\ TeV$

Lepto-quarks



A typical example: $SU(5)$ inspired lepto-quark (Langacker)

$$\mathcal{L} = \frac{\eta_L^2}{2M_S^2} \bar{e}_L \gamma_\mu e_L \bar{u}_L \gamma^\mu u_L + (L \to R)$$

$$c_{1u} \to c_{1u} + \Delta C, \qquad \Delta C = \mp \frac{\sqrt{2}\eta_{L,R}^2}{8 G_F M_S^2}$$

From $\pi^0 \to \ell^+ \ell^- \longrightarrow \eta_L \sim 0$ or $\eta_R \sim 0$.

If $\eta_R \neq 0$, $\Delta C > 0 \longrightarrow \delta_N Q_W < 0$ (excluded).

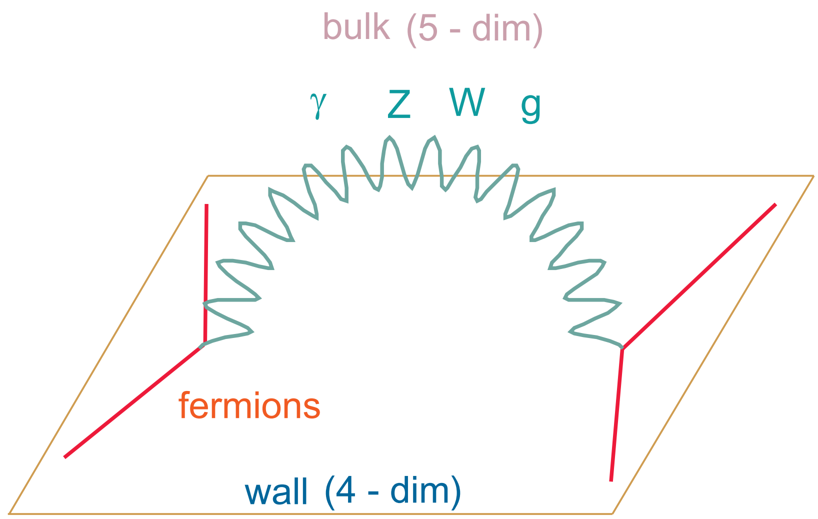
If $\eta_R = 0$ ($\eta_L \neq 0$) the 95%CL bound is

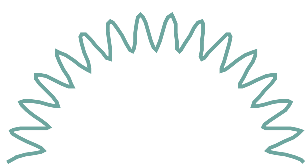
$$1.7 \leq \frac{M_S(TeV)}{\eta_L} \leq 4.5$$

or, assuming a weak interaction, $\eta_L^2 \sim 4\pi\alpha_{\rm em}$

$$0.5 \leq M_S(TeV) \leq 1.4$$

Extra-dimension Models





Theories with extra compact dimensions involve KK excitations of the SM gauge bosons with mass

$$M_{KK}^2 = \frac{n^2}{R^2}$$

and couplings to fermions

$$\text{KK couplings} = \sqrt{2}\ \text{SM couplings}$$

with R the compactification radius

The corrections to Q_W (Casalbuoni, De Curtis, Dominici, Gatto) come from

- from Z-like KK modes
- from the W-like KK modes (they give a correction to G_F)
- a change of s_θ^{eff}

The first two contributions give (for zero mixing)

$$\delta_N Q_W \sim (\sqrt{2})^2 \sum_{n=1}^{\infty} (M_Z^2 - M_W^2) \frac{R^2}{n^2} Q_W^{SM} < 0$$

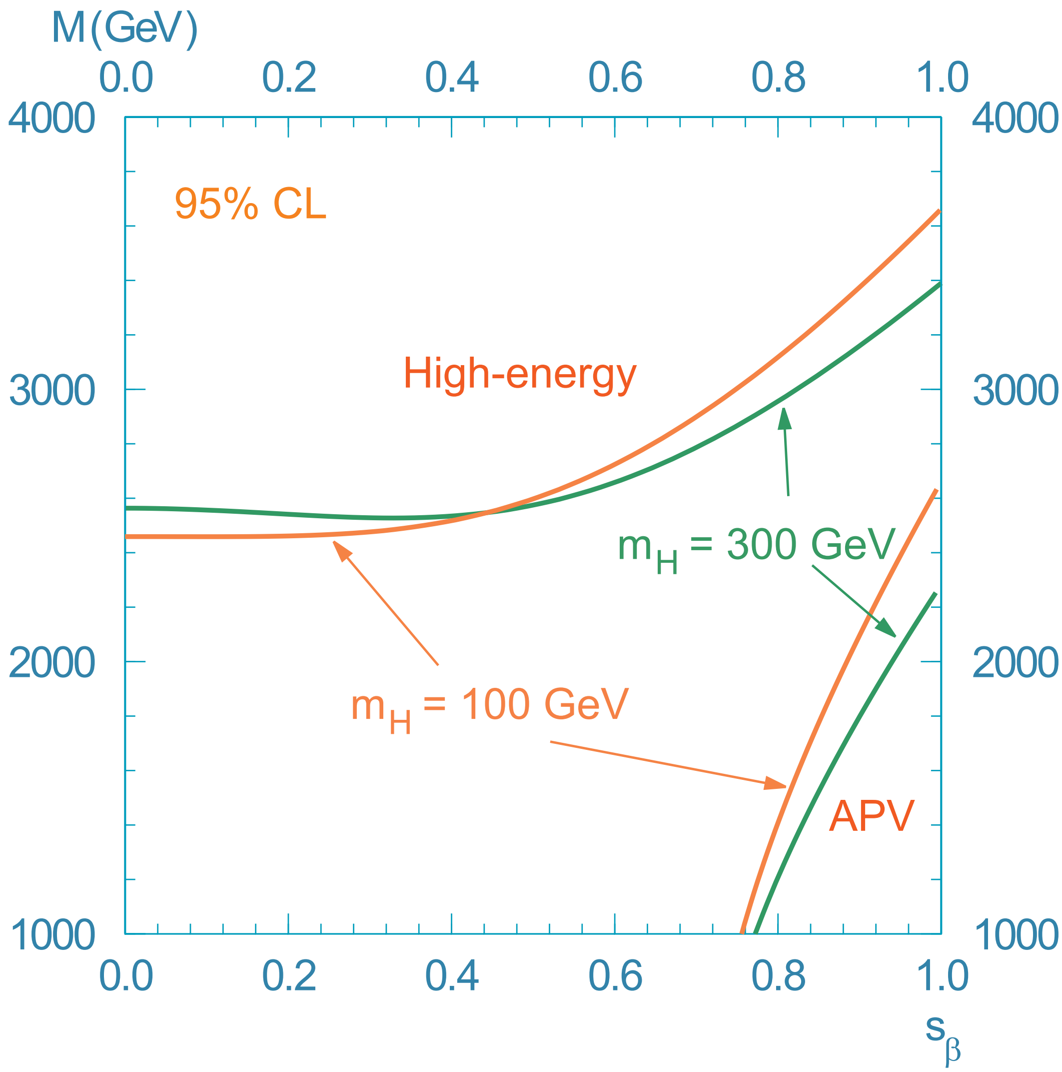
The change of s_θ^{eff} also gives $\delta_N Q_W < 0$

Extra-dimension models (with Higgs in the bulk) are disfavored at more than 99%CL for any compactification radius

The result does not substantially change in presence of mixing terms ($\tan\beta = \frac{<\phi_2>}{<\phi_1>}$ with ϕ_1=Higgs in the bulk and ϕ_2=Higgs in the 4D wall)

As long as $\sin\beta < 0.707$ the new physics contribution is $\delta_N Q_W < 0$

95%CL Bounds on the compactification scale $M = 1/R$ for given s_β from the high-energy precision measurements (ϵ parameters) and from APV data (Casalbuoni,De Curtis,Dominici,Gatto)

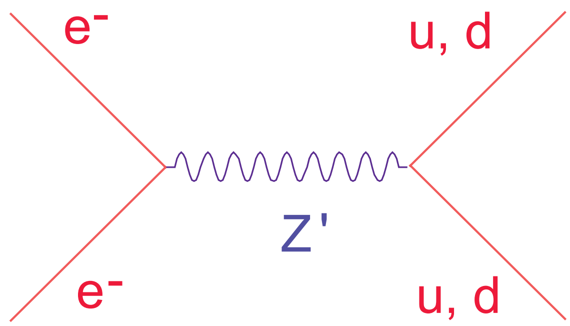


The two regions are incompatible at 95%CL

A global fit to all data is possible but the $\chi^2_{\rm min}$/d.o.f is unpleasantly large

Extra-U(1) Models from E_6

$$E_6 \to SO(10) \times U(1)_\psi \to SU(5) \times U(1)_\chi \times U(1)_\psi$$
$$\to SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$$



Models with a further massive neutral vector boson, Z' coupled to ordinary fermions

$$J^f_{Z'\mu} = J^f_{\chi\mu} \cos\theta_6 + J^f_{\psi\mu} \sin\theta_6$$
$$= \bar{f}\left[\gamma_\mu v'_f + \gamma_\mu \gamma_5 a'_f\right] f$$

The couplings v'_f, a'_f depend on the angle θ_6 (defining the embedding of $U(1)'$ in E_6)

Model	χ	ψ	η
$\theta_6(deg)$	0	90	$-\tan^{-1}\sqrt{5/3} \approx -52$

The $Z - Z'$ mixing angle θ_M is very much constrained by Z-pole observables ($\theta_M > 4$ mrad excluded for most models)

Both $Z - Z'$ mixing and the direct Z' contribution affect the NC experiments off the Z-pole

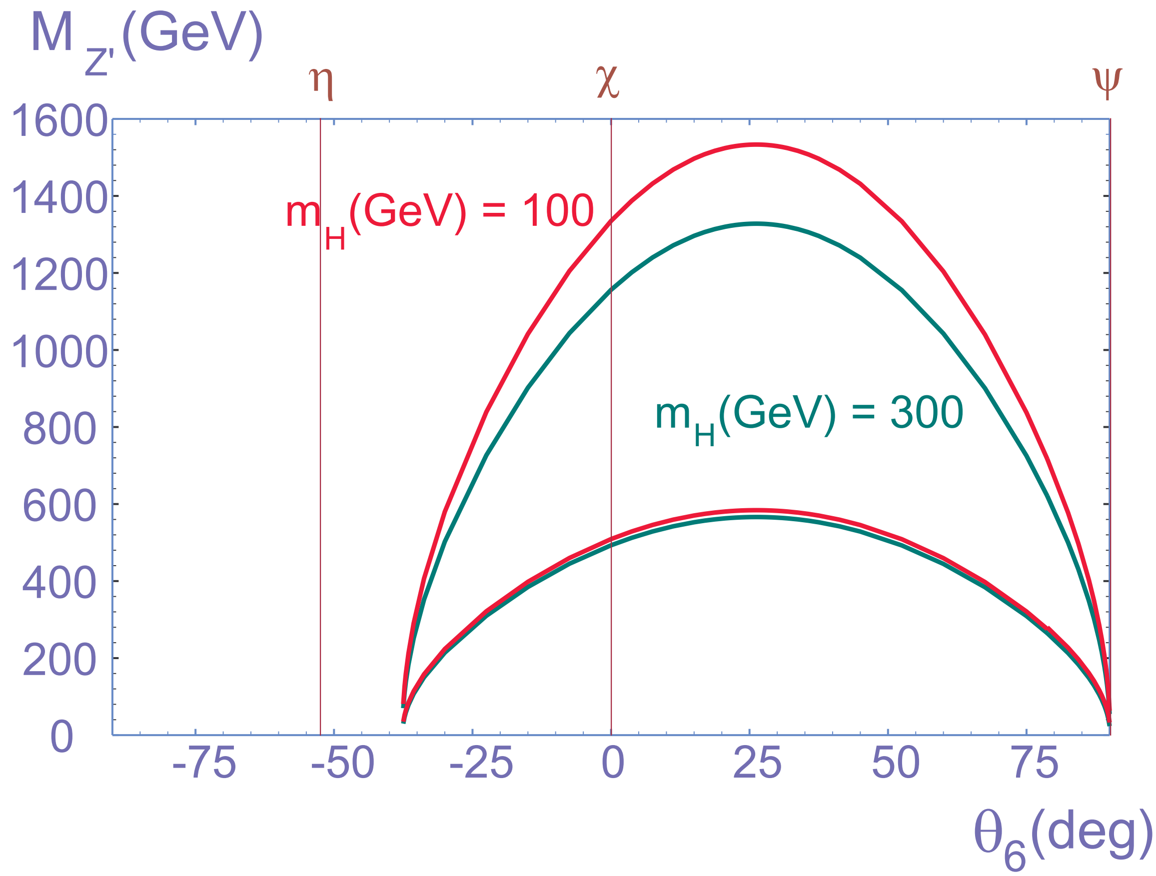
Bounds on Extra Z' from Q_W

Casalbuoni, De Curtis, Dominici, Gatto

The correction at Q_W, for $\theta_M = 0$, is

$$\delta_N Q_W = 16 a'_e \left[(2Z+N)v'_u + (Z+2N)v'_d\right] \frac{M_Z^2}{M_{Z'}^2}$$

Bounds on $\delta_N Q_W$ $\rightarrow$ 95%CL bounds on $M_{Z'}$



Lower positive bound on $\delta_N Q_W \longrightarrow$ upper bound on $M_{Z'}$.

Excluded region $\longrightarrow$ $\delta_N Q_W \leq 0$ (η and ψ models are excluded).

Direct search at Tevatron $\longrightarrow$ gives approximately $M_{Z'}(GeV) \geq 600$.

LR models

$$SO(10) \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

The further massive neutral vector boson, Z'_{LR} is coupled to the current

$$J^\mu_{Z'} = \alpha_{LR} J^\mu_{3R} - \frac{1}{2\alpha_{LR}} J^\mu_{B-L}$$

with $\alpha_{LR} = \sqrt{g_R^2/g_L^2 \cot^2\theta_W - 1}$.
In the L-R symmetric model (LR) one has $g_R = g_L$ and the fermionic couplings

$$a'_e v'_{u,d} = -a_e^{SM} v_{u,d}^{SM}$$

giving

$$\delta_N Q_W = -\frac{M_Z^2}{M_{Z'}^2} Q_W^{SM} > 0$$

The 95%CL bound is (for light Higgs)

$$\boxed{540 \leq M_{Z'}(GeV) \leq 1470}$$

The limit from Tevatron is $M_{Z'} \geq 630\ GeV$

Sequential Standard Model

A simple scaled Z' (with the same couplings to fermions as the SM) would give

$$\delta_N Q_W = \frac{M_Z^2}{M_{Z'}^2} Q_W^{SM} < 0$$

Excluded at more than 99%CL

Present bounds on Z'

Indirect bounds: recent fits (Cho,Hagiwara,Umeda; Erler,Langacker) to all the high-energy and the low-energy observables including Q_W (not the latest measurement). The best fit value for θ_M is ~ 0.

χ	ψ	η	LR	SSM
545	146	365	564	809

95%CL mass limits in GeV (Erler,Langacker (1999))

An update of this analysis (including the latest Q_W) gives a very good fit for the χ model with $M_{Z'_\chi} = 812^{+339}_{-152}$ GeV, $\sin\theta_M = (-1.12 \pm 0.80) \times 10^{-3}$, ($M_H = 145^{+103}_{-61}$ GeV, $\alpha_s = 0.1233 \pm 0.0039$)

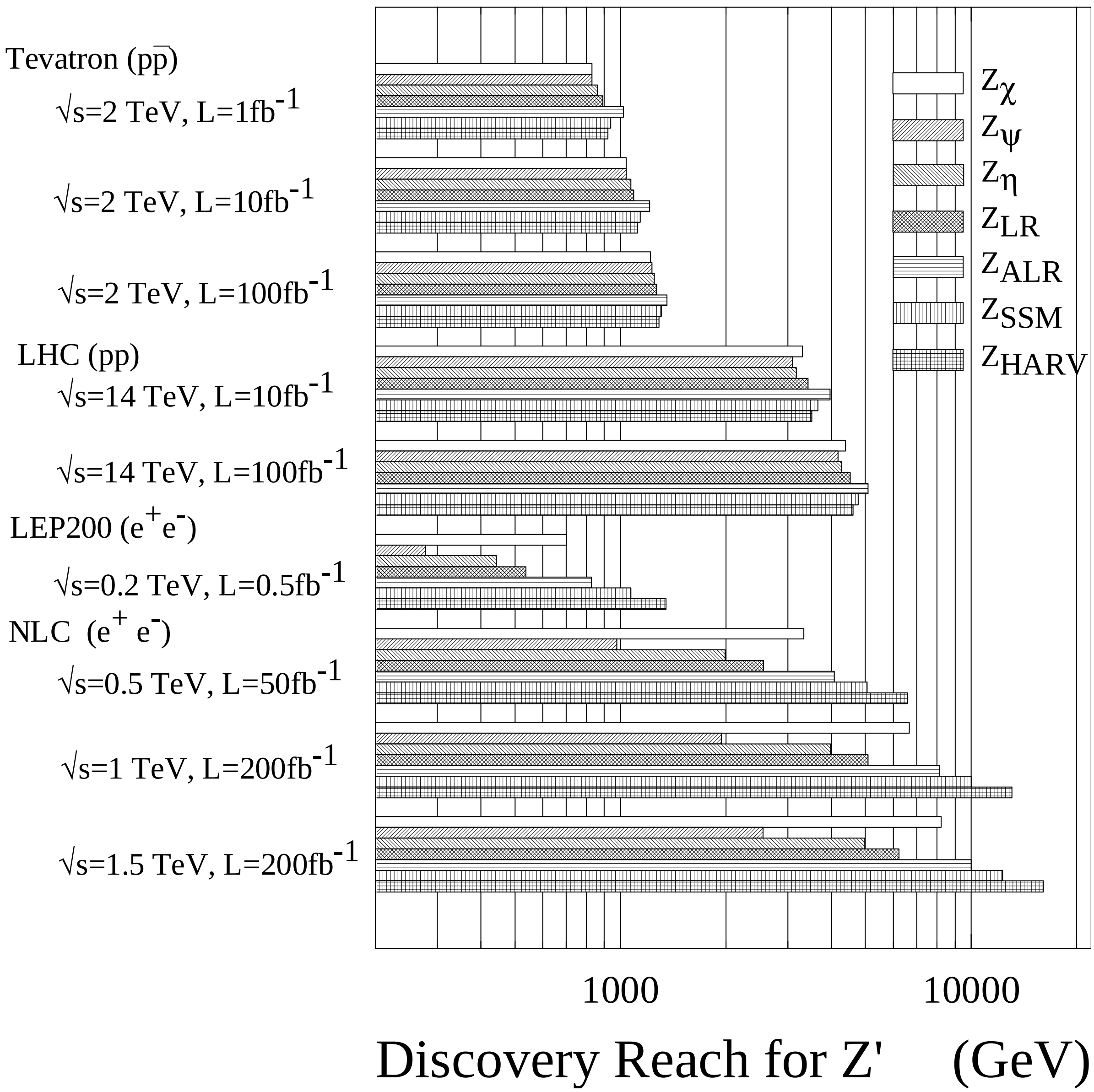
Direct search: from Tevatron, 95%CL limit on $M'_Z(GeV)$ from $\sigma(pp \to Z')B(Z' \to ll)$ (CDF coll. (1997))

χ	ψ	η	LR	SSM
595	590	620	630	690

From LEP2 (sensitive to $M_{Z'}$) and LEP1 (Γ_Z), the average of the 4 expts gives (for $\theta_M = 0$)
$M_{Z'_{\chi,\psi,\eta}} < 464, 298, 323$ GeV (E.Gross, Tampere 1999)

Discovery reach for Extra Z'

Cvetic, Godfrey; Rizzo



At hadron colliders from DY production of lepton pairs (e, μ). Bounds are relatively insensitive to the specific models.

Discovery limit at Tevatron Upgrade:

$M'_Z = 900(1000) GeV$ for $L = 1(10) fb^{-1}$

Discovery limit at LHC:

$M'_Z = 3.5(4.5) TeV$ for $L = 10(100) fb^{-1}$

Atomic Physics and CPT Violation

- CPT Violation. How to break CPT?

- An effective theory of CPT breaking

- Atomic Physics tests

CPT Violation

CTP theorem corner stone of local relativistic field theories. Look carefully at the axioms. Examples:

- The Pitagora theorem. Based upon

  $$\sum_{i=1}^{3} \alpha_i = 180^0$$

  following directly from the parallel postulate of the euclidean geometry. To give up the theorem $\mapsto$ give up the famous 5$^{\text{th}}$ postulate (Riemann and Lobachevsky geometries)

- The Goldstone theorem. To evade it give up one of the following two: i) positivity of the Hilbert space, ii) manifest covariance. Possible within gauge theories

CPT Theorem

In a field theory satisfying

1. Locality ($\Rightarrow$ microcausality)

2. Lorentz invariance

3. Analiticity of the Lorentz group representations ($\exp(i\xi\vec{K}\cdot\vec{p}/|\vec{p}|)$, $\xi \to i\pi$, $p^\mu \to -p^\mu$)

the CPT transformation is a symmetry of the theory.

Conditions 1)+2) $\mapsto$ local relativistic field theory. Condition 3) is satisfied for any finite-dimensional representation of the Lorentz group (the ones of the SM), but violated for unitary, infinite-dimensional, representations (E. Majorana, Il Nuovo Cimento 9, 335, 1932, in italian, translated in english by Fradkin, AJP 34, 314, 1966).

Historical remark

Majorana in 1932 formulated a first-order relativistic generalization of the Schrödinger wave equation very much like the Dirac equation but giving up energy-negative states. Majorana introduced a unitary infinite-dimensional representation of the Lorentz group (Lorentz group is non-compact), nowadays known as the Majorana representation. The wave equation describes an infinite tower of particles with increasing spin $1/2, 3/2, \cdots$ and, not containing antiparticles, it evades the CPT theorem. This is because, unitary, infinite dimensional representations are not analytical in the rapidity parameter. Easy way of evading the CPT theorem, **BUT** quarks and leptons appear to belong to analytical finite-dimensional representations.

Then to violate CPT give up locality and/or relativity. **BUT** locality is strictly related to microcausality.

$C\not{P}T\Rightarrow$ **GIVE UP LORENTZ INVARIANCE**

In string theory there is the possible to break spontaneously Lorentz invariance and/or CPT at a scale of the order of M_P (V.A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545, 1991; Phys. Lett. **B381**, 389, 1996; V.A. Kostelecký and S. Samuel, Phys. Rev. Lett. **63**, 224, 1989; **66**, 1811, 1991; Phys. Rev. **D39**, 683, 1989; **D40**, 1886, 1989). String theory being in $D > 4$ needs to break $SO(D,1)$ anyway.

The breaking could leak in 4-dim.

Effects accounted by writing down a local effective lagrangian with CPT and Lorentz breaking terms.

We proceed as follows

- Define the effective lagrangian breaking CPT and Lorentz invariance as an expansion in terms of derivatives over the Planck mass. For the fermionic case

$$\mathcal{L}_p = \sum_n \frac{g_n}{M_P^n} \bar{\psi} \Gamma (i\partial)^n \psi$$

where $[g_n] = 1$.

- Assume $SU(3) \otimes SU(2) \otimes U(1)$ invariance.

CPT and Lorentz breaking terms should be depressed also for $n = 0$, therefore we expect

$$g_0 = c_0 \frac{m^2}{M_P}$$

with m some low-energy mass scale. Without any other information the relevant terms are the ones with $n = 0$ and $n = 1$.

The resulting theory is renormalizable

For a single fermion interacting with the e.m. field one finds

$$\mathcal{L}_{\text{free}} = \bar{\psi}[i\hat{D} - m]\psi, \qquad D_\mu = \partial_\mu - iqA_\mu$$

$$\boxed{\mathcal{L}_p^{(n=0)} = \bar{\psi}[-a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}]\psi}$$

The $n = 1$ terms compatible with the SM are

$$\boxed{\mathcal{L}_p^{(n=1)} = \bar{\psi}[ic_{\mu\nu}\gamma^\mu D^\nu + id_{\mu\nu}\gamma_5\gamma^\mu D^\nu]\psi}$$

whereas the not compatible $n = 1$ terms are

$$\mathcal{L}_{\not{SM}}^{(n=1)} = \bar{\psi}[ie_\mu D^\mu - \frac{1}{2}f_\mu\gamma_5 D^\mu + \frac{i}{4}g_{\lambda\mu\nu}\sigma^{\lambda\mu}D^n u]\psi$$

All the tensors in the expansion are constant. In a Earth's frame they have periodical diurnal variation. Their expected order of magnitude is

$$a_\mu,\ b_\mu,\ H_{\mu\nu} \approx \mathcal{O}\left(m^2/M_p\right)$$

$$c_{\mu\nu},\ d_{\mu\nu} \approx \mathcal{O}\left((m/M_p)\right)$$

whereas for the SM not compatible terms we expect a further suppression factor.

For sake of simplicity we will consider here only the $n = 0$ terms. According to the table we have

$$a_\mu \gamma^\mu, \; b_\mu \gamma^\mu \gamma_5 \;\longmapsto\; \text{break CPT and Lorentz}$$

$$\frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} \;\longmapsto\; \text{breaks Lorentz}$$

When dealing with a single fermion a_μ is a trivial gauge background field

$$a_\mu = \partial_\mu (a \cdot x)$$

Physics does not depend on a_μ. Situation different for more fermions.
The order of magnitude of the CPT and Lorentz violating factor is expected to be

$$\frac{m}{M_P} \approx 10^{-22} \div 10^{-17} \quad \text{for } m = m_e \div v$$

$m_e =$ electron mass, $v = 250\ GeV$ the electroweak symmetry breaking scale.

	1	γ_5	γ^μ	$\gamma^\mu\gamma_5$	$\sigma^{\mu\nu}$	∂^μ
P	$+1$	-1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$	$(-1)^\mu$
T	$+1$	-1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$	$-(-1)^\mu$
C	$+1$	$+1$	-1	$+1$	-1	$+1$
CPT	$+1$	$+1$	-1	-1	$+1$	-1

Also the photon part

$$\mathcal{L}_{\text{free}}^{\text{photon}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

could be modified by CPT and Lorentz breaking terms as

$$\mathcal{L}^{\text{CPT}-\text{odd}} = \frac{1}{2}(k_{AF})^\mu \epsilon_{\mu\nu\lambda\sigma} A^\nu F^{\lambda\sigma}$$

$$\mathcal{L}^{\text{CPT}-\text{even}} = -\frac{1}{4}(k_F)_{\mu\nu\lambda\sigma} F^{\mu\nu} F^{\lambda\sigma}$$

(Here too the tensors are constant). The term $(k_{AF})_\mu$ gives rise to inconsistencies, but it can be put safely to zero. The CPT-even term gives rise to interesting phenomena such as the birifrangence of the vacuum.

CPT violating experiments

- $K - \bar{K}$ mass difference
- Neutral meson oscillations (meson factories)
- Muons
- Spin polarized solids
- Clock comparison experiments
- Consequences for baryogenesis
- QED experiments: Trapping $e^{\pm}$, p, $\bar{p}$, H^{-}, H, $\bar{H}$ (to be discussed in this talk)

$K - \bar{K}$ mass difference

This is the best high-energy result. The figure of merit is (FNAL and CERN)

$$\frac{|m_K - m_{\bar{K}}|}{m_K} \lesssim 10^{-18}$$

Neutral-meson oscillations

Time evolution through a 2×2 hamiltonian depending on two complex parameters

- ϵ_P measuring CP violation
- δ_P measuring CPT violation

$$\text{SM} \longmapsto \delta_P = 0$$

CPT violation in oscillations depend only on a_μ. We need at least two flavors (e.g. $B_d \approx \overline{b}d$). One finds (V.A. Kostelecky (1998))

$$\delta_P \approx i \sin \widehat{\phi} \exp(i\widehat{\phi}) v_\mu \Delta a^\mu / \Delta m$$

$$\Delta a_\mu = a_\mu^{q_1} - a_\mu^{q_2}, \quad q_i = \text{valence quarks}$$

$$\widehat{\phi} = \arctan\left(\frac{2\Delta m}{\Delta\gamma}\right), \quad \Delta m = m_P - m_{\bar{P}}$$

$$\Delta\gamma = \Gamma_P - \Gamma_{\bar{P}}, \quad v_\mu = \text{meson four} - \text{velocity}$$

Only two experimental results at LEP

$$\text{OPAL} \longmapsto Im\, \delta_{B_d} = -0.020 \pm 0.016 \pm 0.006$$

$$\text{DELPHI} \longmapsto Im\, \delta_{B_d} = -0.011 \pm 0.017 \pm 0.005$$

Muons

At RAL and LANL studies of the ground state Zeeman hyperfine transitions for the muonium $\mu^+ e^-$ have been performed with very high precision (about 20 ppb). The shift on the frequencies due to the CPT and Lorentz breaking are (here the parameters refer to the muon because the relevant transitions involve pure muon-spin flips)

$$\delta\nu_{12} \approx -\delta\nu_{34} \approx -\frac{b_3 + H_{12}}{\pi}$$

Since the LAB rotates with the Earth, the frequencies oscillate with a period of 23 h 56 m. Achieving a bound the variation of the frequencies at the level of 100 Hz, one gets (Bluhm et al. (1999))

$$\frac{2\pi|\delta\nu_{12}|}{m_\mu} \approx \frac{2\pi|\delta\nu_{34}|}{m_\mu} \approx \frac{2(b_3 + H_{12})}{m_\mu} \lesssim 4 \times 10^{-21}$$

Number in the right ballpark since the expected suppression factor is

$$\frac{m_\mu}{M_P} \approx 10^{-20}$$

A second way of using muons is to measure the muon anomalous magnetic moment. This can be done through the measure of the angular anomaly frequency ω_a (A recent BNL experiment, R.M. Carey et al. (1999), makes use of relativistic muons in a constant 1.45 T magnetic field)

$$\omega_a = \omega_s(\text{spin precession}) - \omega_c(\text{cyclotron}) =$$

$$= \left(g - 2 + \frac{2}{\gamma}\right) \mu_\mu B - 2\frac{\mu_\mu}{\gamma} B = (g-2)\mu_\mu B$$

The CPT and Lorentz breaking terms induce a variation of ω_a

$$\delta\omega_a^{\pm} = \pm 2\,\frac{b_3}{\gamma} + H_{12}$$

$$\Delta\omega_a = \delta\omega^{+} - \delta\omega_a^{-} = 4\frac{b_3}{\gamma}$$

Again the magnetic field rotates with the Earth. By time averaging

$$\overline{\Delta\omega_a} = \frac{4}{\gamma} b_Z \cos\chi$$

$Z =$ Earth's rotation axis, $\chi =$ colatitude

From the CERN $g-2$ experiment

$$\frac{\overline{\Delta\omega_a}}{m_\mu} \lesssim 10^{-22}$$

The BNL projection gives

$$\frac{\overline{\Delta\omega_a}}{m_\mu} \lesssim 10^{-23}$$

Spin polarized solids

- Torques on spin-polarized torsion pendulum

- Induced magnetization in a paramagnetic salt

CPT and Lorentz breaking terms manifest via a coupling to the spin of the solid (R. Bluhm, V.A. Kostelecky (2000))

$$\delta H \approx -\tilde{b}_i \sum_n \sigma_n^i$$

$$\tilde{b}_i = b_i - \frac{1}{2}\epsilon_{ijk} H_{jk}$$

From the Eöt-Wash experiment (M.G. Harris, PhD Thesis, Washington (1998))

$$|\tilde{b}_Z \approx (1.4 \pm 0.8) \times 10^{-28}\ GeV$$

The relevant figure of merit is

$$\frac{|\tilde{b}_Z|}{m_e} \approx 3 \times 10^{-25}$$

with an expected suppression factor

$$\frac{m_e}{M_P} \approx 5 \times 10^{-23}$$

Clock comparison experiments

The experiments involve all CPT and Lorentz breaking terms. Useful bounds very difficult to be extracted. The typical bound on the combinations of dimensionful parameters of the order of 10^{-27} GeV (V.W. Hughes et al. (1960), R.W.P. Drever (1961), J.D. Prestage et al. (1985), S.K. Lamoreaux et al. (1986), T.E. Chupp et al. (1989)).

Baryogenesis

CPT violating terms corresponding to non renormalizable terms of the type

$$\bar{\psi}\Gamma \cdot (i\partial)^2\psi$$

may reproduce the observed value of the baryon asymmetry (O. Bertolami et al. (1997))

QED experiments

Tests using Penning traps

- Trapping $e^{\pm}$ or p, $\bar{p}$, and measuring anomaly frequencies

- Comparison of the cyclotron frequencies of H^{-} ions and $\bar{p}$

- Trapping H and $\bar{H}$ and measuring the ground-state hyperfine levels

Confining particles in a Penning trap

- Comparison of $e^{\pm}$ gyromagnetic ratios measuring cyclotron and anomaly frequencies (R.S. Van Dick, P.B. Schwinberg and H.G. Dahmelt (1986 and 1987))

$$\left|\frac{g_- - g_+}{g_{\mathrm{av}}}\right| \lesssim 2 \times 10^{-12}$$

- Comparison of r = charge/mass ratio for p, $\bar{p}$ and $e^{\pm}$ (Gabrielse et al. (1999))

$$\left|\frac{r_p - r_{\bar{p}}}{r_{\mathrm{av}}}\right| \lesssim 9 \times 10^{-11}$$

(P.B. Schwinberg, R.S. van Dyck Jr. and H.G. Dehmelt (1981))

$$\left|\frac{r_{e^-} - r_{e^+}}{r_{\mathrm{av}}}\right| \lesssim 1.3 \times 10^{-7}$$

V.A. Kostelcky et al. (1997) argued that these figures of merit can be misleading since at the lowest order in the CPT-Lorentz breaking $g_+ = g_-$ and the charge/mass ratio does not depend on the CPT violation.

Start with the Dirac equation for an electron or a proton with CPT and Lorentz violating terms

$$\left(i\gamma^\mu D_\mu - m - b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu}\right) \psi = 0$$

In a Penning trap, radial confinement by a strong axial magnetic field, while the axial confinement due to a quadrupole electric field. The main CPT and Lorentz breaking corrections by taking A_μ as the potential for a constant magnetic field.

For the energy shifts due to CPT and Lorentz breaking use the relativistic Landau level wave functions, but for the unperturbed levels use is made of the full quantum corrections.

The reference frame is chosen with the z-axis along the magnetic field. Also remember the non relativistic Landau levels

$$E_{n,\sigma} = \left(n + \frac{1}{2} + \frac{g}{2}\sigma\right)\frac{Be}{m}, \quad \sigma = \pm\frac{1}{2}$$

Cyclotron and anomalous frequencies are extracted as follows

$$\omega_c \equiv E_{1,-1/2} - E_{0,-1/2} = \frac{Be}{m}$$

$$\omega_a \equiv E_{0,+1/2} - E_{1,-1/2} = \frac{g-2}{2}\frac{Be}{m}$$

The relevant CPT and Lorentz breaking corrections to the energy levels are

$$\delta E^{e^-}_{n,\pm 1/2} = \mp b_3 \pm H_{12}$$

$$\delta E^{e^+}_{n,\pm 1/2} = \mp b_3 \mp H_{12}$$

The frequencies for the antiparticles are defined by inverting the spin, therefore

$$\omega_c^{e^-} \approx \omega_c^{e^+} \approx \omega_c$$

$$\omega_a^{e^\mp} \approx \omega_a \mp 2b_3 + 2H_{12}$$

and

$$\Delta\omega_c \equiv \omega_c^{e^-} - \omega_c^{e^+} \approx 0$$

$$\Delta\omega_a \equiv \omega_a^{e^-} - \omega_a^{e^+} \approx -4b_3$$

The usual relation $(g-2)/2 = \omega_a/\omega_c$ does not hold here.

Accuracy in frequencies $\longmapsto$ 10^{-9}

Some useful relations

$$\delta E^{e^-}_{n,\pm 1/2} = \mp b_3 \pm H_{12}$$

$$\delta E^{e^+}_{n,\pm 1/2} = \mp b_3 \mp H_{12}$$

$$\delta \omega^{e^-}_c = \delta E^{e^-}_{1,-1/2} - \delta E^{e^-}_{0,-1/2} = 0$$

$$\delta \omega^{e^+}_c = \delta E^{e^+}_{1,+1/2} - \delta E^{e^+}_{0,+1/2} = 0$$

$$\delta \omega^{e^-}_a = \delta E^{e^-}_{0,+1/2} - \delta E^{e^-}_{1,-1/2} = -2b_3 + 2H_{12}$$

$$\delta \omega^{e^+}_a = \delta E^{e^+}_{0,-1/2} - \delta E^{e^+}_{1,+1/2} = 2b_3 + 2H_{12}$$

We have also

$$\delta E^{e^-}_{n,\sigma} - \delta E^{e^+}_{n,-\sigma} = -2b_3$$

Anomalous magnetic moments

Since in the Penning trap one measures frequencies a relevant figure of merit for CPT violation could be ($\mathcal{E} = E + \delta E$)

$$r^e_{\omega_a} = \frac{|\mathcal{E}^{e^-}_{n,\sigma} - \mathcal{E}^{e^+}_{n,-\sigma}|}{\mathcal{E}^{e^-}_{n,\sigma}} = \frac{|\delta E^{e^-}_{n,\sigma} - \delta E^{e^+}_{n,-\sigma}|}{\mathcal{E}^{e^-}_{n,\sigma}}$$

In the limit of a weak magnetic field one finds

$$\boxed{r^e_{\omega_a} = \frac{|\Delta\omega_a|}{2m_e} = \frac{2|b_3|}{m_e}}$$

A new analysis of the 1987 experiment (H.G. Dahmelt, R.Mittleman, R.S. Van Dyck Jr. and P.B. Schwinberg(1999)) gives the bound

$$\boxed{r^e_{\omega_a} \lesssim 1.2 \times 10^{-21}}$$

However, possible non-favorable situations due to the rotation of the axis of the magnetic field with the Earth increase the bound

$$\boxed{r^e_{\omega_a} \lesssim 3 \times 10^{-21} \div 2 \times 10^{-20}}$$

For the case of p, $\bar{p}$ no experiment at the moment. Assuming experimental sensitivity analogous to the electron-positron case ($\delta\omega_{a,c} \approx 2\ Hz$), the result, for the proton parameters, is

$$\boxed{r^{p}_{\omega_a} \lesssim 10^{-23}}$$

Charge to mass ratio

Ratio from the cyclotron frequency. It is not affected by CPT breaking. It is sensitive only to the Lorentz violating terms of the type

$$\bar{\psi} c_{\mu\nu} \gamma^{\mu} D^{\nu} \psi$$

Defining

$$\Delta^p_{\omega_c} = \frac{|\mathcal{E}^p_{1,-1/2} - \mathcal{E}^p_{0,-1/2}|}{\mathcal{E}^p_{0,-1/2}}$$

The figure of merit can be taken as the amplitude of the periodic fluctuations in $\Delta^p_{\omega_c}$. In the comoving Earth frame one has

$$r^p_{\omega_c,\mathrm{diurnal}} \approx \frac{|c_{11} + c_{22}|\omega_c}{m_p}$$

The experimental result for the $e^{\pm}$ is (H.G. Dahmelt, R.Mittleman, R.S. Van Dyck Jr. and P.B. Schwinberg (1981))

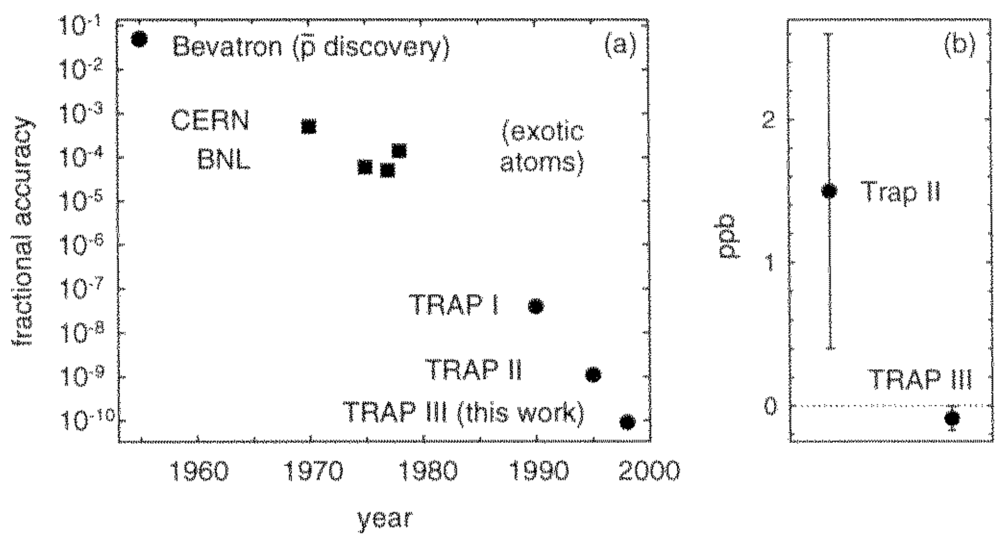
$$\boxed{r^e_{\omega_c,\mathrm{diurnal}} \lesssim 10^{-16}}$$

For p and $\bar{p}$ (G. Gabrielse et al. (1995))

$$r^{p}_{\omega_c,\text{diurnal}} \lesssim 10^{-24}$$

More recently G. Gabrielse et al. (1999) measured the charge to mass ratio for p, $\bar{p}$ with an accuracy of 9×10^{-11}

$$\frac{q}{m}(\bar{p}) \Big/ \frac{q}{m}(p) = -0.999\,999\,999\,91(9)$$



Hydrogen ions

The use of particles with different charges (p, $\bar{p}$) can offset the potentials of the trap, therefore Gabrielse et al. (1999) decided to use $\bar{p}$ and H^- ions converting the result to p taking into account the corrections from the electron-to-proton mass, the binding energy and the electron affinity. This result can be used to bound the amount of the Lorentz violation arising from the tensor $c_{\mu\nu}$ as for the p, $\bar{p}$ case. The figure of merit is

$$r_{\omega_c}^{H^-} = \frac{\Delta\omega_c^{\mathrm{H}^-}}{m_p} \lesssim 4 \times 10^{-26}$$

with

$$\Delta\omega_c^{\mathrm{H}^-} \approx (c_{00} + c_{11} + c_{22})(\omega_c - \omega_c^{H^-}) + \mathcal{O}\left(\frac{m_e}{m_p}\right)$$

(The parameters $c_{\mu\nu}$ refer to the proton)

Trapping H and $\bar{H}$

The transition $1S-2S$ for trapped H has been observed with a precision of about 10^{-12} (C.L. Ceasar et al. (1996)), however

- The dependence of the $1S-2S$ transition on the CPT and Lorentz breaking parameters is suppressed by a factor $\alpha^2/8\pi$. In fact, at the lowest order the shift is given by (including only the terms, b_μ and $H_{\mu\nu}$)

$$\delta E^H(b_3^{e^-,p}, H_{12}^{e^-,p}) = \delta E^H(b_3^{e^+,\bar{p}}, H_{12}^{e^+,\bar{p}}) =$$

$$= \frac{m_J}{|m_J|}\left(-b_3^{e^-} + H_{12}^{e^-}\right) + \frac{m_I}{|m_I|}\left(-b_3^p + H_{12}^p\right)$$

with J and I the atomic and the nuclear spin. Since the two-photon selection rule for $1S-2S$ is $\Delta F = \Delta m_F = 0$, there is no shift.

- Consider spectroscopy of H and $\bar{H}$ in a magnetic field. In the basis $|m_J, m_I\rangle$, one has the following states for $n = 1, 2$

$$|b\rangle_n = |-1/2, -1/2\rangle, \quad |d\rangle_n = |1/2, 1/2\rangle$$

$$|c\rangle_n = \sin\theta_n|-1/2, 1/2\rangle + \cos\theta_n|1/2, -1/2\rangle$$

$$|a\rangle_n = \cos\theta_n|-1/2, 1/2\rangle - \sin\theta_n|1/2, -1/2\rangle$$

$$\tan 2\theta_n = \frac{(51\ mT)}{n^3 B}$$

Transitions $|d\rangle_1 \to |d\rangle_2$ give no frequency shift (same spin configuration). Transitions$|c\rangle_1 \to |c\rangle_2$ give a shift, but they are field dependent and one has to overcome the problem of the Zeeman broadening due to the inhomegeneous trapping fields.

- Hyperfine Zeeman effect in the ground state

$$\delta E_a^H = \cos 2\theta_1 \left(b_3^e - b_3^p - H_{12}^e + H_{12}^p \right)$$

$$\delta E_b^H = \left(b_3^e + b_3^p - H_{12}^e - H_{12}^p \right)$$

$$\delta E_c^H = -\delta E_a^H, \quad \delta E_d^H = -\delta E_b^H$$

For small magnetic field $\delta E_a^H \approx \delta E_c^H \approx 0$ and in the transition $|c\rangle_1 \to |a\rangle_1$ the CPT and Lorentz violation is suppressed. The transitions $|d\rangle_1 \to |a\rangle_1$ and $|b\rangle_1 \to |a\rangle_1$ have unsuppressed contributions, but field dependent $\to$ Zeeman broadening.

- Eliminate the frequency dependence on B (at first order) by choosing a field independent transition point. With a convenient B ($\approx 0.65T$), $|c\rangle_1$ is highly polarized ($|1/2, -1/2\rangle$). Then

$$\delta\omega^{H,\bar{H}}_{c\to d} \approx 2(\mp b^p_3 + H^p_{12})$$

($d \to c$ is a proton spin-flip transition), and

$$\Delta\omega_{c\to d} = \omega^{H}_{c\to d} - \omega^{\bar{H}}_{c\to d} \approx -4b^p_3$$

The figure of merit is

$$r^H_{c\to d} = \frac{|\Delta\omega_{c\to d}|}{m_H} = 4\frac{|b^p_3|}{m_H}$$

With a resolution of about 1 mHz

$$\boxed{r^H_{c\to d} =\lesssim 5\times 10^{-27}, \quad (10^{-23} \text{ from } \omega^p_a)}$$

System	Expt.	Fig. Merit	Est. Bound	Parms.	Test
$e^- e^+$	$\Delta\omega_a$	$r^e_{\omega_a}$	10^{-21}	b^e_j	CPT
	ω_a diurnal	$r^e_{\omega_a,\mathrm{diurnal}}$	10^{-21}	d^e_{j0}, H^e_{jk}	Lorentz
	ω_c diurnal	$r^e_{\omega_c,\mathrm{diurnal}}$	10^{-18}	c^e_{jj}	Lorentz
$p\,\bar{p}$	$\Delta\omega_a$	$r^p_{\omega_a}$	10^{-23}	b^p_j	CPT
	ω_a diurnal	$r^p_{\omega_a,\mathrm{diurnal}}$	10^{-21}	d^p_{j0}, H^p_{jk}	Lorentz
	ω_c diurnal	$r^p_{\omega_c,\mathrm{diurnal}}$	10^{-24}	c^p_{jj}	Lorentz
$H^-\,\bar{p}$	$\Delta\omega_c$	$r^{H^-}_{\omega_c}$	10^{-25}	c^e_{jj}, c^p_{jj}	Lorentz
$H\,\bar{H}$	$\Delta\omega_{c\to d}$	$r^H_{c\to d}$	10^{-26}	b^p_j	CPT

Conclusions

From Atomic Physics many important consequences for high-energy physics

- APV - A first indication of New Physics beyond the SM?

- CPT - Atomic Physics tests of this fundamental symmetry at a spectacular level of sensitivity