Prospective Study

on

Muon Colliders

Strong Electroweak Symmetry Breaking at Muon Colliders

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Geneva, November 12, 1998

Analysis of a narrow resonance

Uncertainties on the energy spread (σ_E) of the lepton beams may induce relatively large errors in the determination of the parameters of a resonance of width $\Gamma \approx \sigma_E$. Assume for the distribution energy of the beam a gaussian peaked at the mass of the resonance (M) (the beam energy is assumed to be known exactly) and characterized by

$$\sigma_M(GeV) = 0.007\, R(\%)\, M(GeV)$$

where R is the energy resolution of the machine. After convoluting with a Breit-Wigner cross-section corresponding to the production of a vector resonance, V, (this is the case we will consider in the following)

$$\sigma(E) = 12\pi \frac{\Gamma(V \to \ell^+\ell^-)\Gamma}{(E^2 - M^2)^2 + M^2\Gamma^2}$$

we get (at the peak)

$$\sigma_c \approx \frac{3\pi\sqrt{2\pi}\Gamma(V \to \mu^+\mu^-)}{M_V^2 \sigma_{\sqrt{s}}} \left(1 + \frac{\pi}{8}\left[\frac{\Gamma}{\sigma_{\sqrt{s}}}\right]^2\right)^{-1/2}$$

In the limits

$$\sigma_M \ll \Gamma, \quad \sigma_c \to \sigma^{\text{peak}}$$

$$\sigma_M \gg \Gamma, \quad \sigma_c \to \underbrace{\frac{1}{2}\sqrt{\frac{\pi}{2}}}_{\approx 0.6}\left(\frac{\Gamma}{\sigma_M}\right)\sigma^{\text{peak}}$$

showing that σ_c is insensitive to σ_M for a very narrow beam, whereas in the opposite case it depends strongly on σ_M. For a more detailed study of the sensitivity let us work in the narrow resonance approximation ($\Gamma \ll M$)

$$\sigma = \frac{3\pi}{M^2} Br(V \to \ell^+\ell^-) \frac{\gamma^2}{x^2 + \gamma^2/4}$$

$$x = \frac{E - M}{\sigma_M}, \qquad \gamma = \frac{\Gamma}{\sigma_M}$$

The resulting convolution can be written as

$$M^2\sigma_c = \frac{3\pi}{\sqrt{2\pi}} Br\,\gamma^2 \int dx\, \frac{\exp(-(x-\bar{x})^2/2)}{x^2+\gamma^2/4} =$$

$$= Br\, f(\gamma, \bar{x})$$

with: $\bar{x} = \frac{E-M}{\sigma_M}$, $Br = Br(V \to \ell^+\ell^-)$. If σ_c is measured at some point $E = M + \eta\Gamma$, we get

$$M^2\sigma_c = Br\, g(\gamma)$$

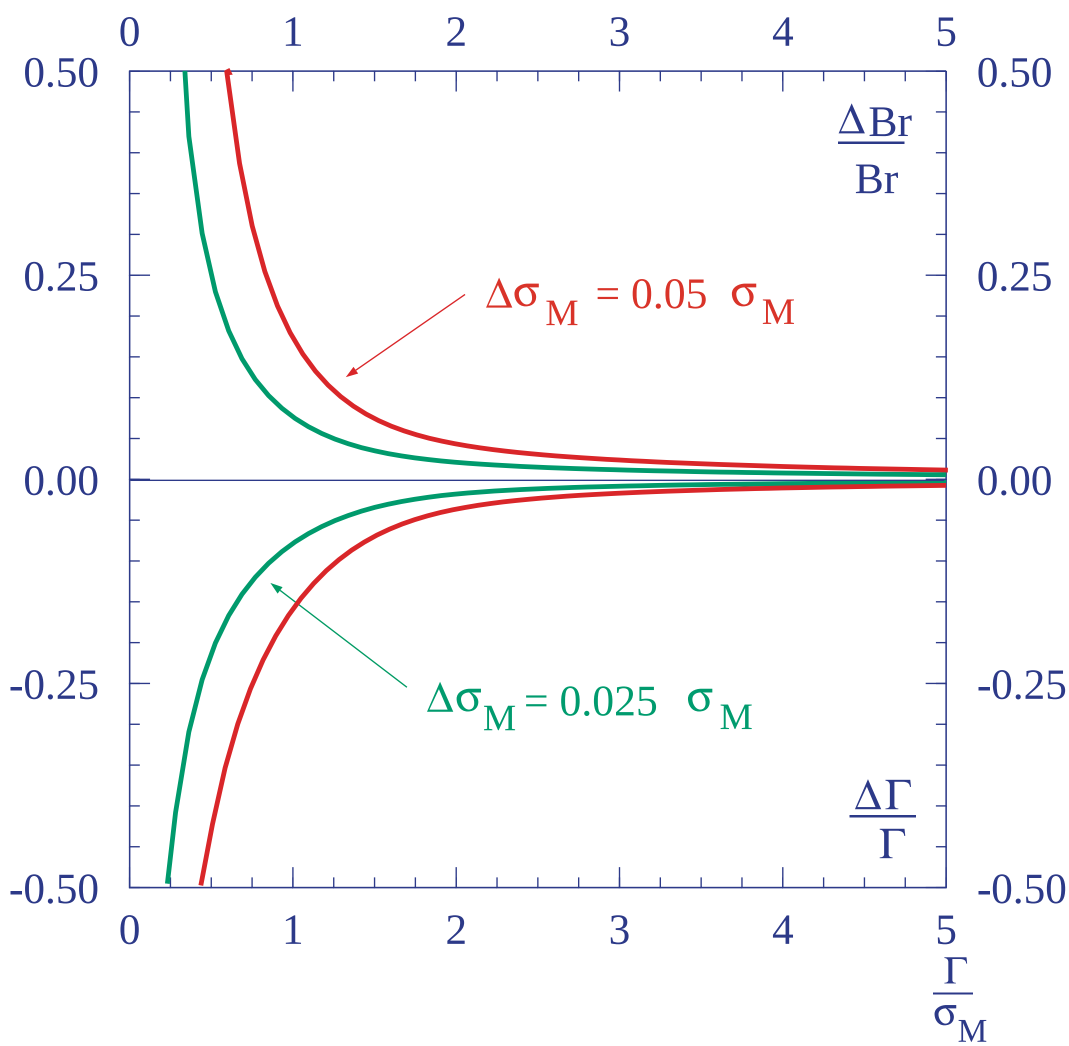
One can determine (Br, Γ) by deconvoluting σ_c, but an error on σ_M induces errors on the two parameters. Assuming that the measured value of σ_c is given by $\sigma_c(Br, \Gamma, \sigma_M)$, the induced error can be obtained by

$$\sigma_c(Br + \delta Br, \Gamma + \delta\Gamma, \sigma_M + \delta\sigma_M) = \sigma_c(Br, \Gamma, \sigma_M)$$

or

$$\left(1 + \frac{\delta Br}{Br}\right) g(\gamma + \delta\gamma) = g(\gamma)$$

For a given $\delta\sigma_M$ one can use the previous equation at two different energies (for instance at $E = M$ and $E = M + 2\Gamma$) to determine $\delta\Gamma/\Gamma$ and $\delta Br/Br$. From the previous approximate scaling law we see that the errors depend only on $\gamma = \Gamma/\sigma_M$ and not on Br.



For a given σ_M, the requirement of measuring the widths with a given error, gives a lower bound for Γ/σ_M. Then, for any model this lower bound can be translated into an observability region in the parameter space. We will consider two models of Strong ElectroWeak Symmetry Breaking (SEWS):

• BESS. The model describes an isotriplet of massive Yang-Mills vector resonances, $\vec{V}$. The parameters are the mass, the gauge coupling and the direct coupling to the fermions, respectively:

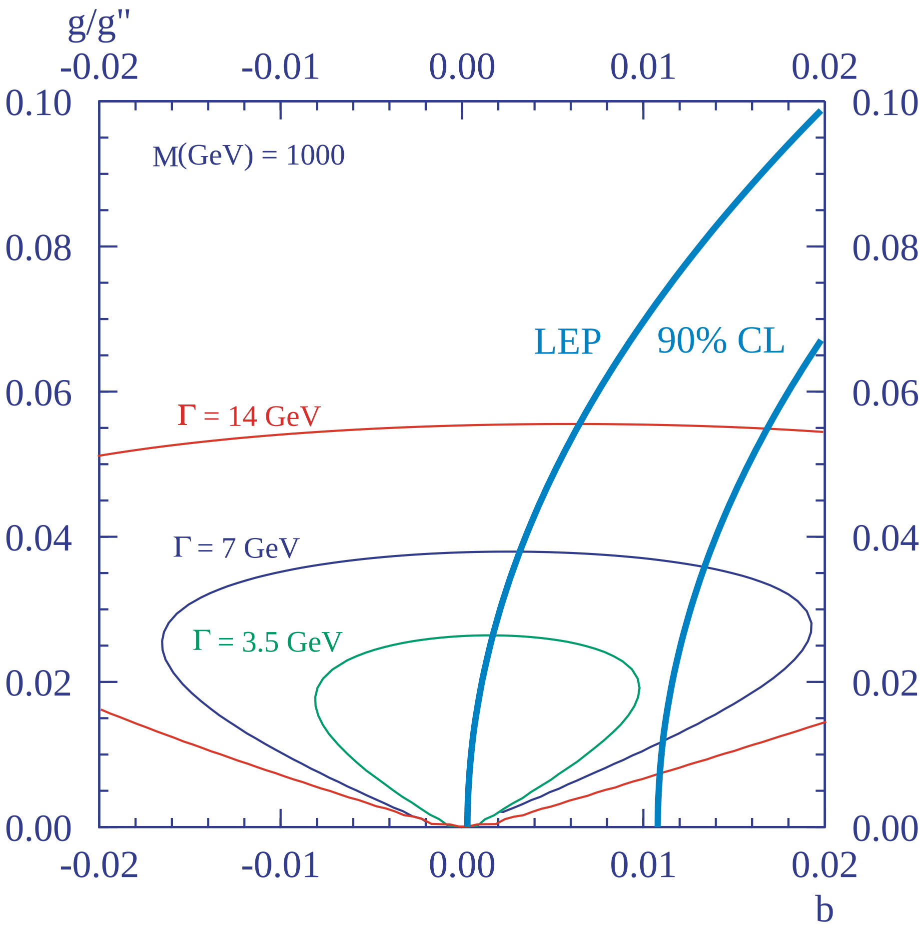
$$(M, g'', b)$$

• Degenerate BESS. The model describes two isotriplets of almost degenerate vector resonances $(\vec{R}, \vec{L})$. The parameters are the mass and the common gauge coupling:

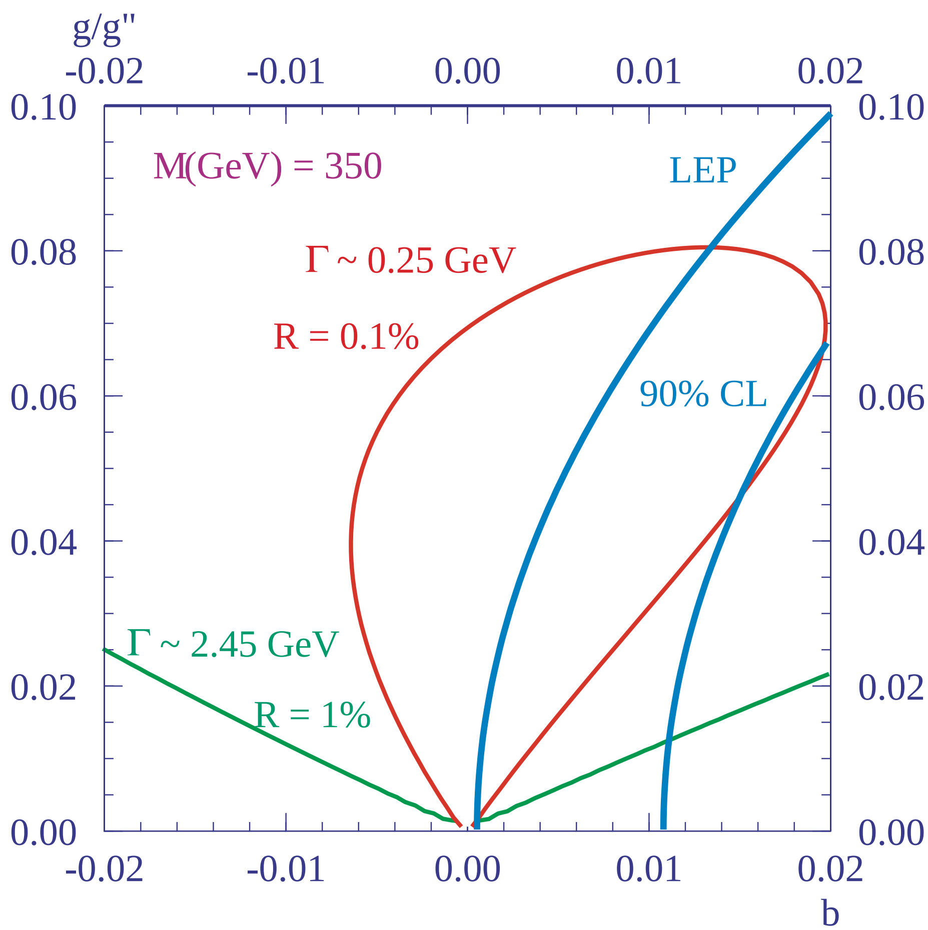
$$(M, g'')$$

BESS model

Here the vector resonances do not decouple for large mass, therefore the precision experimental data do not put any practical bound on the mass M, but only on $(g/g'', b)$. The main decay mode is $V \to WW$, and the widths are complicated functions of the parameters



Consider $M = 350\ GeV$. Such a resonance should be visible at the Tevatron upgrade, LHC and NLC. By taking $\delta\sigma_M = 5\%$ and requiring to measure Γ better than 20%, we have from our universal figure $\Gamma \gtrsim \sigma_M$

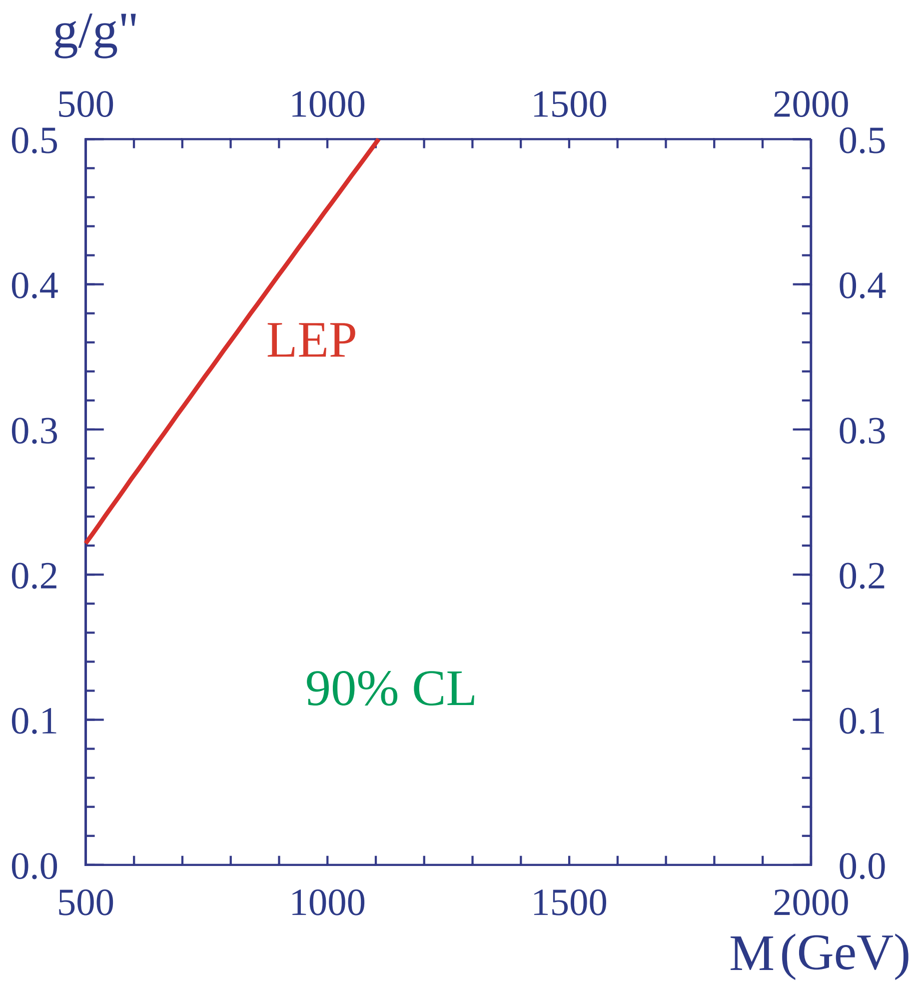


- Since at the NLC the energy resolution, R, should be greater than 1%, we see that almost nowhere in the allowed region one will be able to measure Γ with an error lower than 20%.

- At the μC one should be able to have R of order 1% or smaller (paying something in luminosity, since to improve R of a factor 2 one looses 33% in luminosity). A clear improvement with respect to NLC would be obtained.

Degenerate BESS model

In this model the resonances decouple, and the restrictions on the parameter space from precision experiments are rather mild (see figure).

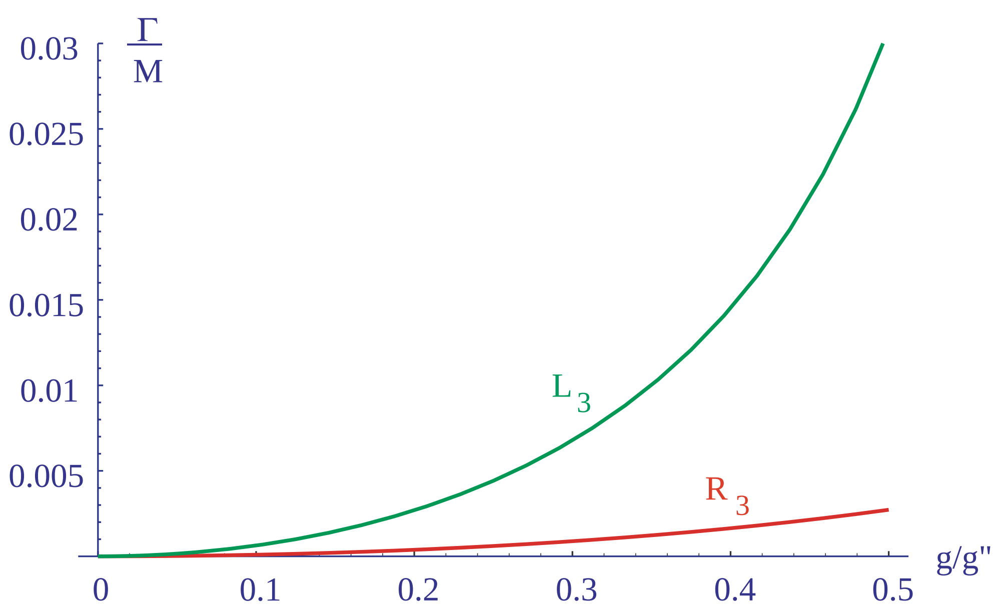


The main decay modes of $\vec{L}$ and $\vec{R}$ are the fermionic ones, since the WW mode is depressed. The Br's are sizeable and almost parameters independent

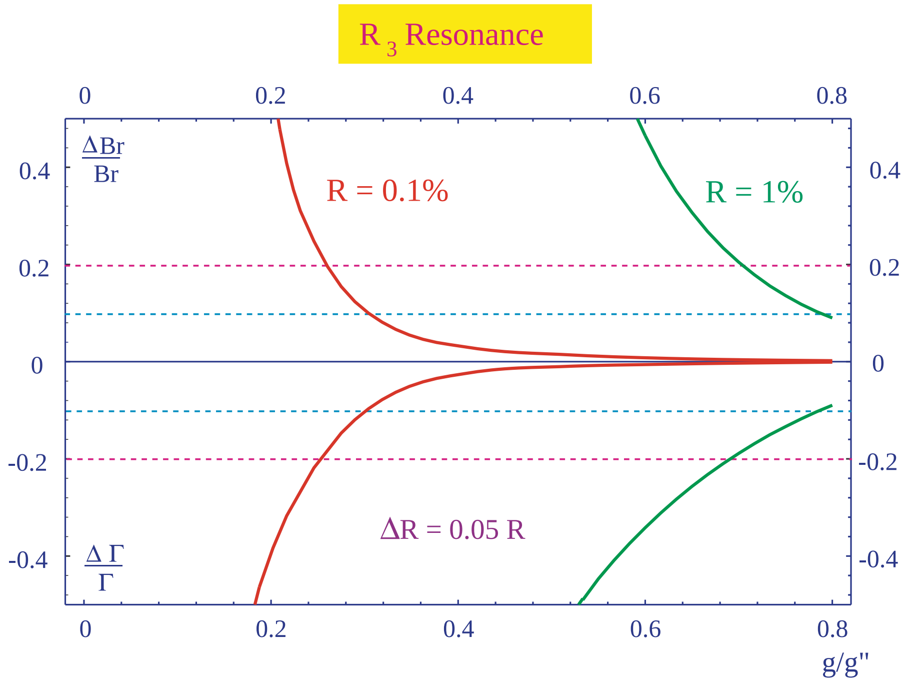
$$Br(L_3 \to \ell^+\ell^-) \approx 4\%, \quad Br(R_3 \to \ell^+\ell^-) \approx 12\%$$

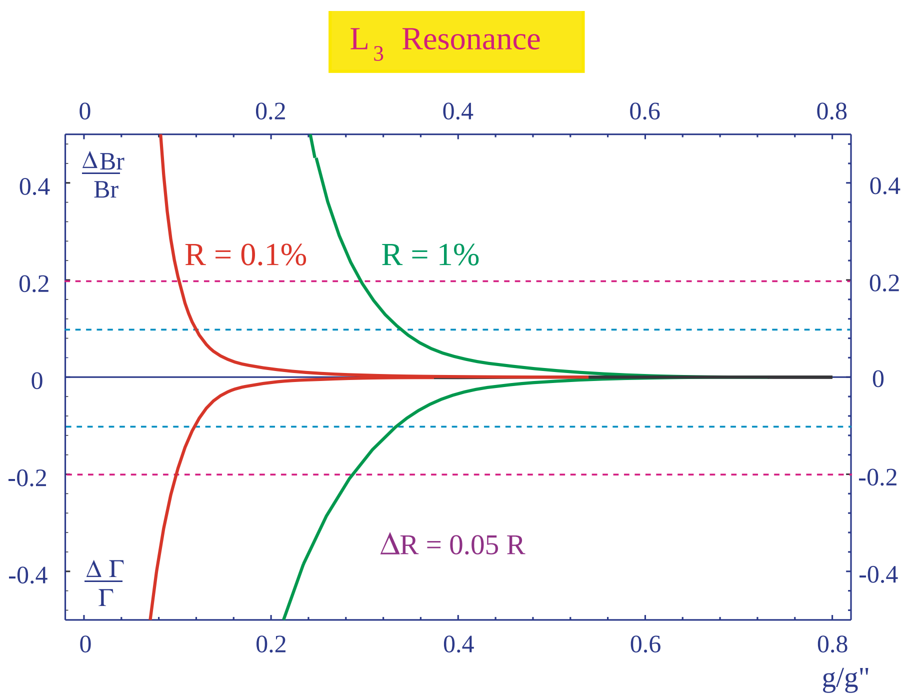
The degeneracy between the resonances is broken by weak corrections. It follows that the widths and the mass splitting $\mathcal{O}_i = (\Gamma_i, \Delta M)$ are given by

$$\mathcal{O}_i = M\, h_i(g/g'')$$



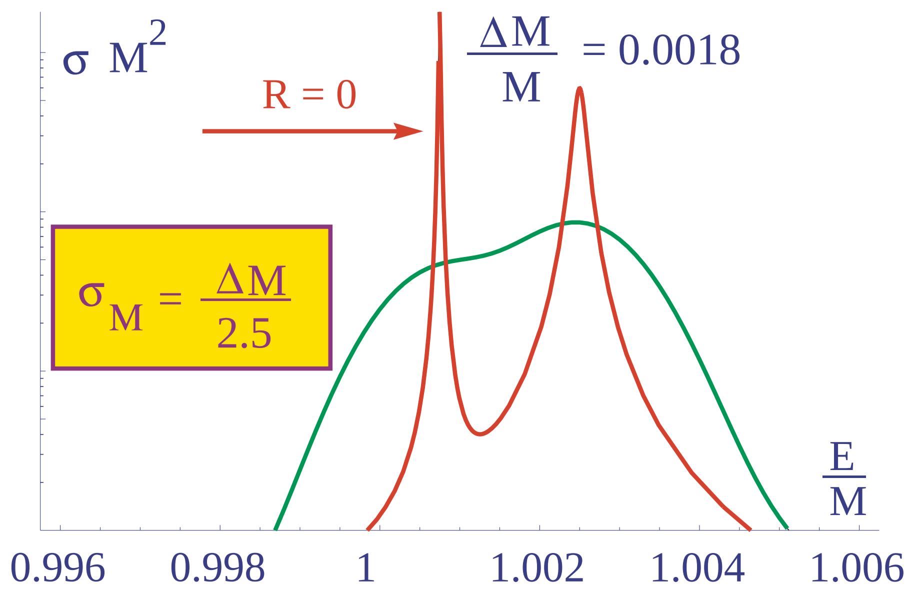
The bounds on Γ/σ_M depend only on g/g'' and R





We have also some restrictions from the mass splitting. In fact, if we require that a machine is able to resolve the two resonances one has to require, for instance, that one starts to detect the two peaks structure from the presence of two maxima in the convolution. This implies, approximately

$$\sigma_M \leq \frac{\Delta M}{2.5}$$



In this way one gets the following bounds on g/g''

$\left(\frac{g}{g''}\right)_{\Delta M}$	$\left(\frac{g}{g''}\right)_{\Gamma_{L_3}}$	$\left(\frac{g}{g''}\right)_{\Gamma_{R_3}}$	$R(\%)$
0.15	0.29	0.7	1
0.05	0.10	0.26	0.1

Since all the functions $h_i(g/g'') = \mathcal{O}_i/M$ for $g/g'' \ll 1$ have the behaviour

$$h_i(g/g'') \approx a_i \left(\frac{g}{g''}\right)^2$$

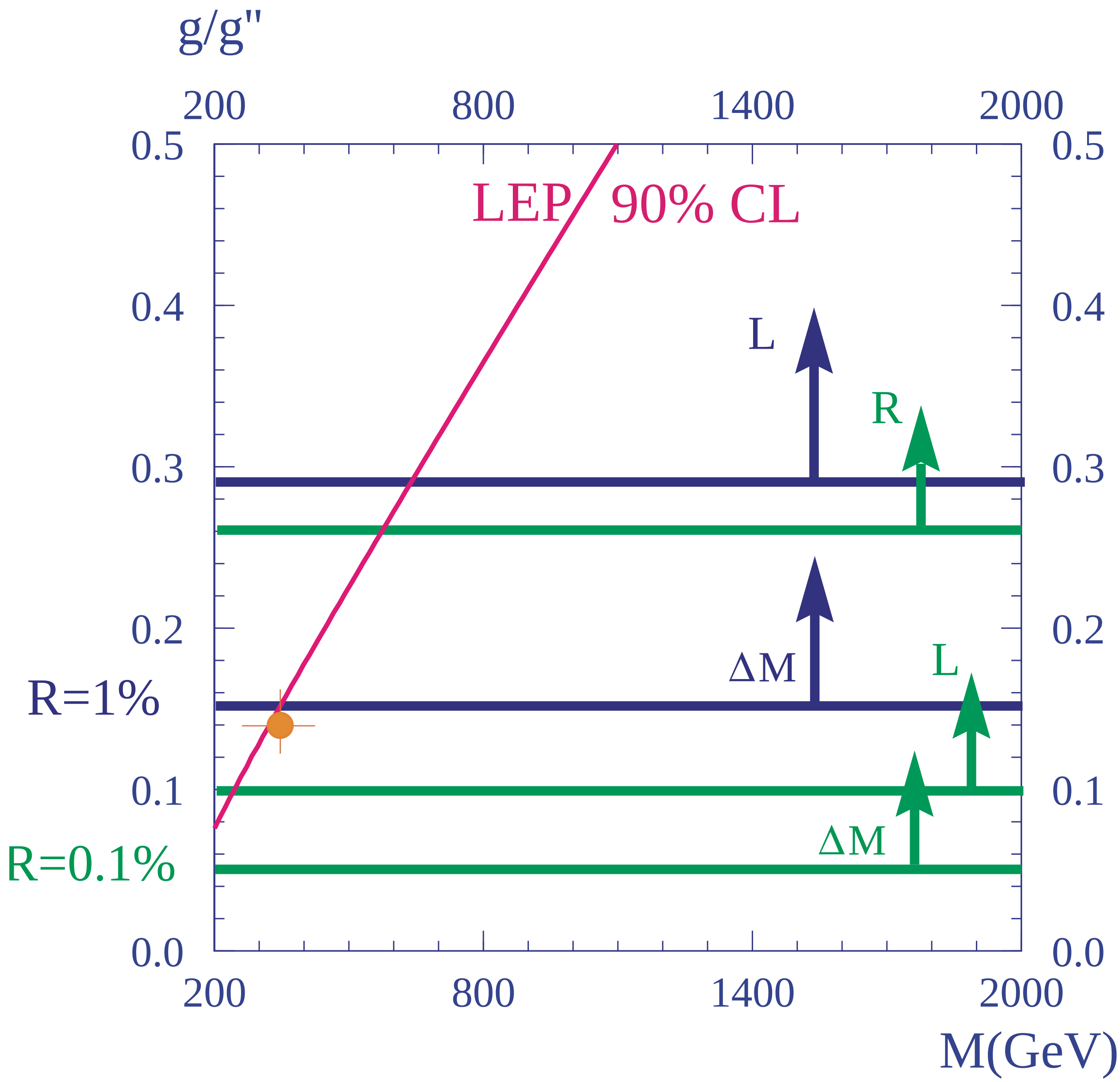
it follows that the bounds scale roughly with $\sqrt{R}$.

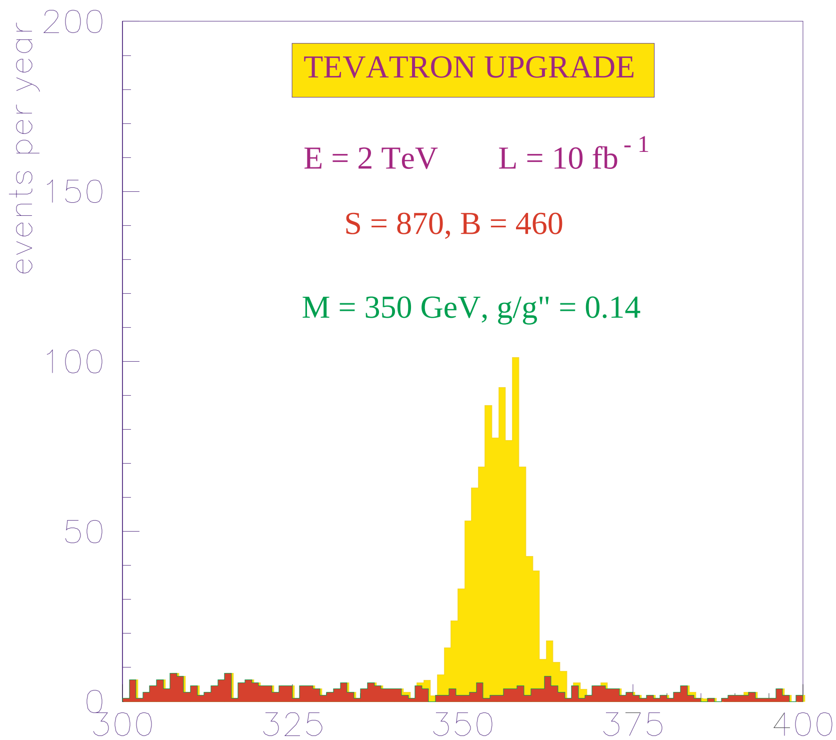
From the precision data we can convert the lower bounds on g/g'' in energy bounds, such that for $E < M_i$ the observable $\mathcal{O}_i$ cannot be resolved at the required level of accuracy. This can be done by using the following approximate relation from the precision data at 90%CL

$$\frac{M}{1000\ GeV} \geq 2.17 \left(\frac{g}{g''} + 0.01 \right)$$

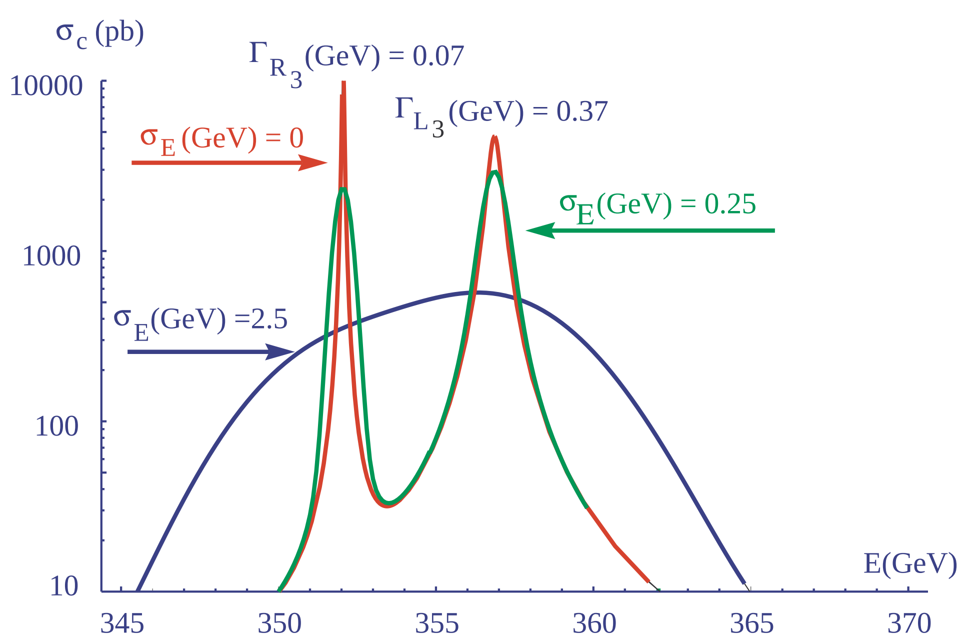
$M_{\Delta M}(GeV)$	$M_{\Gamma_{L_3}}(GeV)$	$M_{\Gamma_{R_3}}(GeV)$	$R(\%)$
350	630	1520	1
110	220	560	0.1

The two resonances can be seen at the Tevatron upgrade but they cannot be resolved


g/g"
200
800
1400
2000
0.5
0.4
0.3
0.2
0.1
0.0
LEP 90% CL
L
R
ΔM
L
R=1%
R=0.1%
ΔM
M(GeV)



M(GeV) = 350, g/g'' = 0.14, ΔM(GeV) = 5.3



Conclusions

We have shown that althought the narrow and low mass resonances arising from SEWS models as BESS and degenerate BESS will be detectable at Tevatron upgrade (and, of course, LHC) and at NLC, **a more detailed study of their properties will require a machine with higher energy resolution such as a μC**.