The Lightest Pseudo-Goldstone Boson at future e^+e^- Colliders

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DESY - August 21, 2000

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Talk based on hep-ph/9809523

Summary

- The effective description
- The P^0 at LEP, LEP2
- The P^0 at the Tevatron and the LHC
- The P^0 at e^+e^- colliders
- The P^0 at $\gamma - \gamma$ colliders
- Effective low-energy parameters for P^0 from NLC?
- The P^0 at muon colliders
- Conclusions

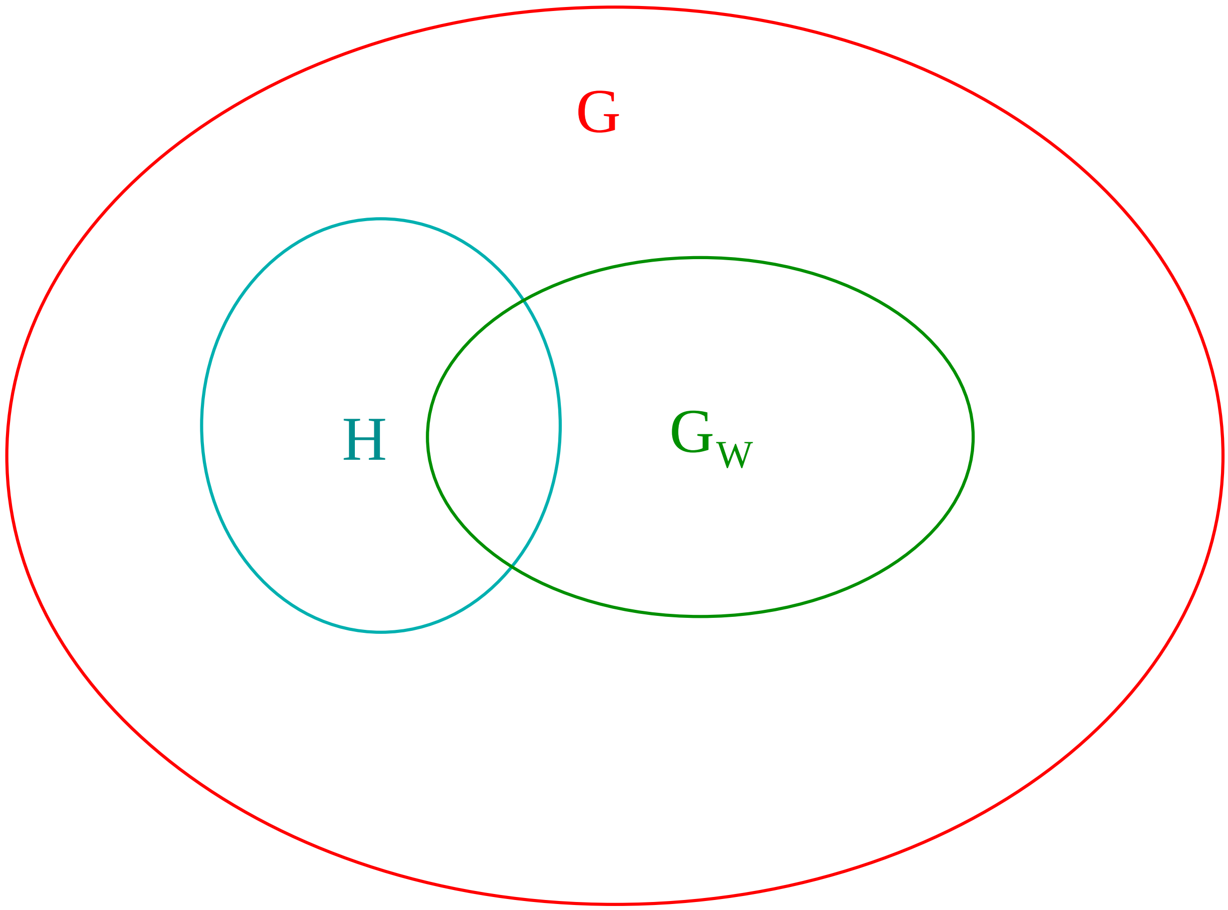
The effective description

EWSB in terms of an effective lagrangian $\mathcal{L}$

- Spontaneous breaking $G \to H$
- Gauge group $G_W = SU(3) \otimes SU(2)_L \otimes U(1)_Y$
  G_W included in G

Strategy:
- Write the most general Yukawa couplings invariant under G_W
- Compute the effective potential $V_{\rm eff}$ including Yukawa and gauge interactions
- Cut off the theory at some scale $\Lambda \approx$ few TeV

This procedure allows for an estimate of the mass spectrum of the PNGB deriving from the SB.


G
H
G_W

We will discuss the specific example of

$$G = SU(8)_L \otimes SU(8)_R \to H = SU(8)_V$$

This is the breaking occurring, for instance, in TC theories with T-fermions $T = (U_i, D_i, N, E)$, $i = 1, 2, 3 \in SU(3)_c$. The quantum numbers of the PNGB's are easily seen from the product $\bar{T} \otimes T$

$$[(\bar{3}, 2) \oplus (1, 2)] \otimes [(3, 2) \oplus (1, 2)] =$$

$$= (1, 1) \oplus (1, 3) \oplus (1, 3) \quad (\#7 \text{ colorless PNGB's})$$

$$\oplus (8, 1) \oplus (8, 3) \quad (\#32 \text{ color octect PNGB's})$$

$$\oplus (3, 1) \oplus (\bar{3}, 1) \oplus (3, 3) \oplus (\bar{3}, 3)$$

(#24 color triplet and antitriplet PNGB′s)

More detailed properties:

Table of PNGB

The 63 Goldstone bosons with their quantum numbers and transformation properties under $SU(2)_L$ and $SU(3)_c$ (here $Y = 2(Q - T^3)$ is the hypercharge):

	$SU(2)_L$	$SU(3)_c$	Q	Y
π^+ $(\tilde{\pi}^+)$ π^- $(\tilde{\pi}^-)$ π^3 $(\tilde{\pi}^3)$	3	1 (1)	1 -1 0	0
π_D	1	1	0	0
π_8^α	1	8	0	0
$\pi_8^{\alpha+}$ $\pi_8^{\alpha-}$ $\pi_8^{\alpha 3}$	3	8	1 -1 0	0
P_3^{0i} $(\bar{P}_3^{0i})$	1	3 $(\bar{3})$	$\frac{2}{3}$ $(-\frac{2}{3})$	$\frac{4}{3}$ $(-\frac{4}{3})$
P_3^{+i} $(\bar{P}_3^{+i})$ P_3^{-i} $(\bar{P}_3^{-i})$ P_3^{3i} $(\bar{P}_3^{3i})$	3	3 $(\bar{3})$	$\frac{5}{3}$ $(-\frac{5}{3})$ $-\frac{1}{3}$ $(\frac{1}{3})$ $\frac{2}{3}$ $(-\frac{2}{3})$	

We will concentrate on color singlet PNGB's:
$(1,1) \oplus (1,3) \oplus (1,3) = \pi_D,\ \pi_a,\ \tilde{\pi}_a,\ a = 1,2,3$
π_a absorbed by W and Z.
Physical colorless PNGB: $\pi_D,\ \tilde{\pi}_3, \tilde{\pi}^\pm$.

The effective Lagrangian

- Gauge Lagrangian

The low energy effective lagrangian will contain interaction terms among the PNGB's and the ordinary gauge bosons, collected in $\mathcal{L}_g$ (Chada and Peskin, 1981):

$$\mathcal{L}_g = \frac{v^2}{16} Tr \left(\mathcal{D}_\mu U^\dagger \mathcal{D}_\mu U \right)$$

where

$$U = \exp\left(\frac{2iT^s \pi^s}{v} \right)$$

with $v = 246\ GeV$. T^s $(s = 1, \ldots, 63)$ are the $SU(8)$ generators.
The covariant derivative $\mathcal{D}_\mu U$ is given by

$$\mathcal{D}_\mu U = \partial_\mu U + \mathcal{A}_\mu U - U \mathcal{B}_\mu$$

where

$$\mathcal{A}_\mu = igT^a W_\mu^a + ig' \frac{T_D}{\sqrt{3}} B_\mu + i \frac{g_s}{\sqrt{2}} T_8^\alpha G_\mu^\alpha$$

$$\mathcal{B}_\mu = ig' T^3 B_\mu + ig' \frac{T_D}{\sqrt{3}} B_\mu + i \frac{g_s}{\sqrt{2}} T_8^\alpha G_\mu^\alpha$$

W_μ^a $(a = 1, 2, 3)$, B_μ and G_μ^α $(\alpha = 1, \cdots, 8)$, are the gauge vector fields related to the gauge group G_W.

- Yukawa Lagrangian (R.C. et al., 1992)

In order to write the Yukawa couplings between the Goldstone bosons and the ordinary fermions we decompose the matrix U according to

$$U = \begin{pmatrix} U^{ij}_{uu} & U^{ij}_{ud} & U^{k}_{u\nu} & U^{k}_{ue} \\ U^{ij}_{du} & U^{ij}_{dd} & U^{k}_{d\nu} & U^{k}_{de} \\ U^{l}_{\nu u} & U^{l}_{\nu d} & U_{\nu\nu} & U_{\nu e} \\ U^{l}_{eu} & U^{l}_{ed} & U_{e\nu} & U_{ee} \end{pmatrix} \qquad i,j,k = 1,2,3$$

The most general Yukawa coupling invariant with respect to G_W for the third family and for the muons (relevant for muon colliders) is given by

$$\begin{aligned}
\mathcal{L}_Y = & - m_1 \left(\bar{t}^i_R U^{\dagger ij}_{uu} t^j_L + \bar{t}^i_R U^{\dagger ij}_{du} b^j_L\right) - m_2 \left(\bar{b}^i_R U^{\dagger ij}_{ud} t^j_L + \bar{b}^i_R U^{\dagger ij}_{dd} b^j_L\right) \\
& - m_4 \left(\bar{\tau}_R U^{\dagger}_{\nu e} \nu_{\tau L} + \bar{\tau}_R U^{\dagger}_{ee} \tau_L\right) - m_4^{(2)} \left(\bar{\mu}_R U^{\dagger}_{\nu e} \nu_{\mu L} + \bar{\mu}_R U^{\dagger}_{ee} \mu_L\right) \\
& - m_5 \left(\bar{t}^i_R U^{\dagger jj}_{uu} t^i_L + \bar{t}^i_R U^{\dagger jj}_{du} b^i_L\right) - m_6 \left(\bar{b}^i_R U^{\dagger jj}_{ud} t^i_L + \bar{b}^i_R U^{\dagger jj}_{dd} b^i_L\right) \\
& - m_7 \left(\bar{t}^i_R U^{\dagger}_{\nu\nu} t^i_L + \bar{t}^i_R U^{\dagger}_{e\nu} b^i_L\right) - m_9 \left(\bar{b}^i_R U^{\dagger}_{\nu e} t^i_L + \bar{b}^i_R U^{\dagger}_{ee} b^i_L\right) \\
& - m_{10} \left(\bar{\tau}_R U^{\dagger jj}_{ud} \nu_{\tau L} + \bar{\tau}_R U^{\dagger jj}_{dd} \tau_L\right) \\
& - m_{10}^{(2)} \left(\bar{\mu}_R U^{\dagger jj}_{ud} \nu_{\mu L} + \bar{\mu}_R U^{\dagger jj}_{dd} \mu_L\right) \\
& - m_{11} \left[\bar{t}_R \lambda^\alpha \begin{pmatrix} U^\dagger_{uu} & U^\dagger_{du} \end{pmatrix} \begin{pmatrix} \lambda^\alpha & 0 \\ 0 & \lambda^\alpha \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}\right] \\
& - m_{12} \left[\bar{b}_R \lambda^\alpha \begin{pmatrix} U^\dagger_{ud} & U^\dagger_{dd} \end{pmatrix} \begin{pmatrix} \lambda^\alpha & 0 \\ 0 & \lambda^\alpha \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}\right] \\
& + \cdots + h.c.
\end{aligned}$$

The magnitudes of the Yukawa couplings are naturally set by the scale of the corresponding fermionic masses.

- In a multiscale technicolor model, the above would be generalized by writing $\mathcal{L}_g$ and $\mathcal{L}_Y$ as a sum of terms with different v_i ($v_i \leq v$).

Expanding U to first order in $1/v$, we get the relevant terms for the colorless neutral fields

$$
\begin{aligned}
U_{uu}^{ij} &\sim \delta^{ij}\left[1+\frac{i}{v}\sqrt{6}\frac{P^{0\,\prime}}{3}\right]+\ldots \\
U_{dd}^{ij} &\sim \delta^{ij}\left[1-\frac{i}{v}\sqrt{6}\frac{P^{0}}{3}\right]+\ldots \\
U_{\nu\nu} &\sim \left[1-\frac{i}{v}\sqrt{6}P^{0\,\prime}\right]+\ldots \\
U_{ee} &\sim \left[1+\frac{i}{v}\sqrt{6}P^{0}\right]+\ldots
\end{aligned}
$$

with

$$
P^0=\frac{\tilde{\pi}_3-\pi_D}{\sqrt{2}}, \qquad P^{0\,\prime}=\frac{\tilde{\pi}_3+\pi_D}{\sqrt{2}}
$$

or (in a TC realization)

$$
P^0=\frac{1}{\sqrt{12}}(3E\overline{E}-D\overline{D}) \qquad P^{0\,\prime}=\frac{1}{\sqrt{12}}(U\overline{U}-3N\overline{N})
$$

which are purely $T_3=-1/2$ and $T_3=+1/2$ weak isospin states.

From the $\mathcal{O}(1)$ terms in the expansion of $\mathcal{L}_Y$ we easily recover the expressions for the fermion masses

$$m_t = m'_1 + m_7 \quad m_b = m'_2 + m_9$$

$$m_\tau = m_4 + 3m_{10} \quad m_\mu = m_4^{(2)} + 3m_{10}^{(2)}$$

$$m_{\nu_\tau} = m_{\nu_\mu} = 0$$

where

$$m'_1 \equiv m_1 + 3m_5 + \frac{16}{3}m_{11} \quad m'_2 \equiv m_2 + 3m_6 + \frac{16}{3}m_{12}$$

Evaluate one-loop effective potential

$$m^2_{P^0} = \frac{4\Lambda^2}{\pi^2 v^2}(2m'_2 m_9 + 2m_4 m_{10} + 2m_4^{(2)} m_{10}^{(2)}) \equiv \frac{4\Lambda^2}{\pi^2 v^2}\rho_8$$

$$m^2_{P^{0\prime}} = \frac{4\Lambda^2}{\pi^2 v^2} 2m'_1 m_7 \equiv \frac{4\Lambda^2}{\pi^2 v^2}\rho_7$$

Λ is the UV cut-off, situated in the TeV region.

The colored PNGB get masses generally higher (from QCD contribution $\approx g_s \Lambda$)

A few comments are in order

- P^0 and $P^{0\prime}$ masses do not receive contributions from gauge interactions

- m_{P^0} gets contribution only from terms involving b, τ and μ

- $m_{P^{0\prime}}$ gets contribution only from terms involving top

- no $P^0 - P^{0\prime}$ mixing

- A very crucial point: P^0 and $P^{0\prime}$ are the mass eigenstates and not the isosinglet π_D and the isotriplet $\tilde{\pi}_3$

$$\Downarrow$$

$$\boxed{m_{P^0} << m_{P^{0\prime}}}$$

The colorless charged states are also heavier than P^0 since $m^2_{P^\pm} = \frac{1}{2}(m^2_{P^0} + m_{P^{0\prime}}{}^2)$

A general argument for understanding the mass matrix in the colorless neutral sector is the following

• In the chiral limit $SU(2)_L \otimes SU(2)_R$ is unbroken

• The mass operator breaking $SU(2)_L \otimes SU(2)_R$ must commute with the charge operator $T_3 = T_{3L} + T_{3R}$

Then

$$m^2 = A + BK_3 + CT_3, \quad K_3 = T_{3L} - T_{3R}$$

In the basis $(\tilde{\pi}_3, \pi_D)$

$$m^2 = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$$

meaning that generically the mass eigenstates are P^0 and $P^{0\,\prime}$.

Notice that for the neutral states

$$T_{3L}^2 = T_{3R}^2 = -T_{3L}T_{3R} = \frac{1}{4}$$

$$0 = (T_{3L} + T_{3R})^2 = \frac{1}{2} + 2T_{3L}T_{3R}$$

- P^0 couplings

The P^0 boson couples to the $T_3 = -1/2$ component of the fermion doublets (while $P^{0\prime}$ couples to the $T_3 = +1/2$ component):

$$\mathcal{L}_Y = -i\lambda_b \bar{b}\gamma_5 b P^0 - i\lambda_\tau \bar{\tau}\gamma_5 \tau P^0 - i\lambda_\mu \bar{\mu}\gamma_5 \mu P^0 + \cdots$$

with

$$\lambda_b = -\frac{\sqrt{6}}{3v}\left(m_2' - 3m_9\right) \quad \lambda_\tau = \frac{\sqrt{6}}{v}\left(m_4 - m_{10}\right)$$

$$\lambda_\mu = \frac{\sqrt{6}}{v}\left(m_4^{(2)} - m_{10}^{(2)}\right)$$

The coefficients depend on the same m_2' combination as the fermion masses

- The phenomenolgy of P^0 depends on 6 parameters (4 neglecting the muon terms)

$$m_2', \; m_4, \; m_9, \; m_{10}, \; m_4^{(2)}, \; m_{10}^{(2)}$$

Since we have 3 fermionic masses (m_b, m_τ, m_μ) and 3 Yukawa couplings involving different parameter combinations we can determine all the $\mathcal{L}_Y$ parameters for the P^0 by measuring masses and widths. This would allow to determine ρ_8

$$m_{P^0}^2 = \frac{4\Lambda^2}{\pi^2 v^2}\rho_8$$

and therefore Λ via m_{P^0}.

Since the parameters m_i are expected of the same order of the corresponding m_f we will make a special choice as representative for this study

$$m_1' = m_7 = \frac{m_t}{2} \quad m_2' = m_9 = \frac{m_b}{2}$$

$$m_{10} = -m_4 = \frac{m_\tau}{2} \quad m_{10}^{(2)} = -m_4^{(2)} = \frac{m_\mu}{2}$$

The corresponding one-loop P^0 and $P^{0\prime}$ masses are

$$m_{P^0}^2 = \frac{2\Lambda^2}{\pi^2 v^2} m_b^2 \quad m_{P^{0\prime}}^2 = \frac{2\Lambda^2}{\pi^2 v^2} m_t^2$$

$$\Rightarrow \quad m_{P^0} \sim 9 \text{ GeV} \times \Lambda(\text{TeV})$$
$$m_{P^{0\prime}} \sim 310 \text{ GeV} \times \Lambda(\text{TeV})$$

and the fermionic couplings

$$\lambda_b = \sqrt{\frac{2}{3}} \frac{m_b}{v}, \quad \lambda_\tau = -\sqrt{6} \frac{m_\tau}{v}, \quad \lambda_\mu = -\sqrt{6} \frac{m_\mu}{v}$$

- The previous choice just as a first assessment about the prospects of determining directly from the experiments all the parameters m_i for P^0, as well as Λ

- Typically $m_{P^{0\prime}}/m_{P^0} \approx m_t/m_b$. Then $m_{P^{0\prime}}$, $m_{P^\pm} \gg m_{P^0}$

- The PNGB couple to pairs of SM gauge bosons via ABJ anomaly. The couplings are model dependent. We will use the ones obtained for the standard TC models

$$g_{PV_1V_2} = \alpha N_{TC} \frac{A_{PV_1V_2}}{\pi v} \epsilon_{\lambda\mu\nu\rho} p_1^\lambda \epsilon_1^\mu p_2^\nu \epsilon_2^\rho$$

where for $P = P^0$ we have:

$$\begin{aligned}
A_{P^0\gamma\gamma} &= -\frac{4}{\sqrt{6}}\left(\frac{4}{3}\right) \approx 2.2 \\
A_{P^0Z\gamma} &= -\frac{4}{2\sqrt{6}}\left(\frac{1-4s_W^2}{4s_W c_W} - \frac{t_W}{3}\right) \approx 0.11 \\
A_{P^0ZZ} &= -\frac{4}{\sqrt{6}}\left(\frac{1-2s_W^2}{2c_W^2} - \frac{t_W^2}{3}\right) \approx -0.41 \\
A_{P^0gg} &= \frac{1}{\sqrt{6}} \approx 0.41
\end{aligned}$$

with $s_W = \sin\theta_W$, *etc.*.

- Notice the dominance of the $\gamma\gamma$ coupling with respect to $Z\gamma$ and to ZZ
- The relevance of the mass eigenstate composition is shown by

$$A^2_{P^0\gamma\gamma} : A^2_{\pi_D\gamma\gamma} : A^2_{\tilde{\pi}_3\gamma\gamma} = 8 : 1 : 9$$
$$A^2_{P^0gg} : A^2_{\pi_Dgg} : A^2_{\tilde{\pi}_3gg} = 1 : 2 : 0$$

• How big is N_{TC}? Difficult to say. In naive TC there is the constraint from EW precision measurements (N_D number of TC-doublets)

$$S \approx 0.28 N_D \frac{N_{TC}}{3}$$

against

$$S_{\text{exp}} = -0.07(-0.09) \pm 0.11$$
$$M_H(GeV) = 100(300)$$

This would imply N_{TC} as low as possible (together with N_D). But this is the result of the QCD-scaled TC which has a lot of other problems, as FCNC, the mass of the top (requiring Λ_{ETC} too close to Λ_{TC}). These problems are currently solved by introducing things as walking TC and color-top assisted models. This makes very difficult to get a realistic estimate for S. For instance, the Weinberg sum rules are violated in walking TC making unrealistic the previous estimate.

- P^0 **partial widths and BR's**

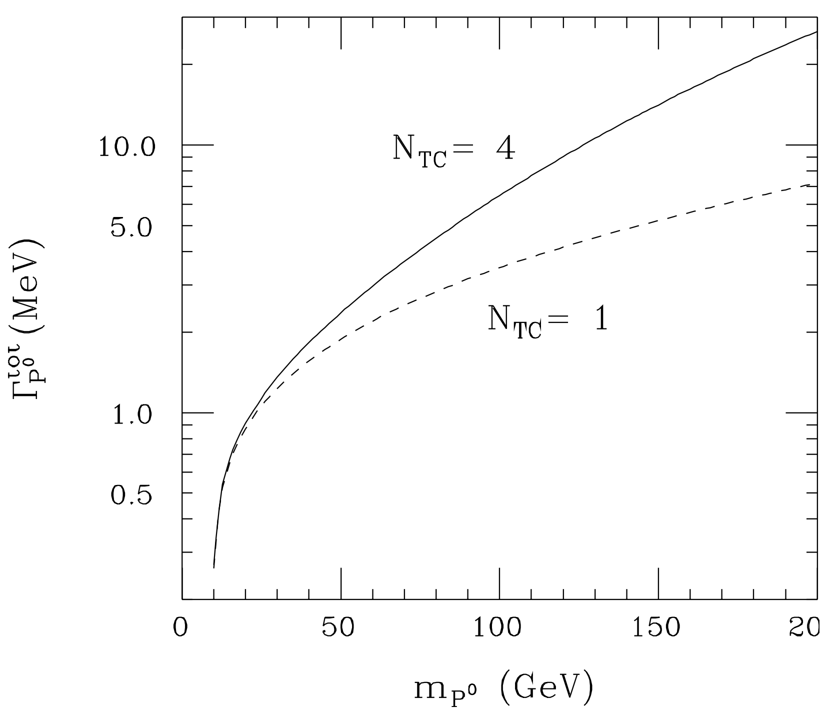
We will need

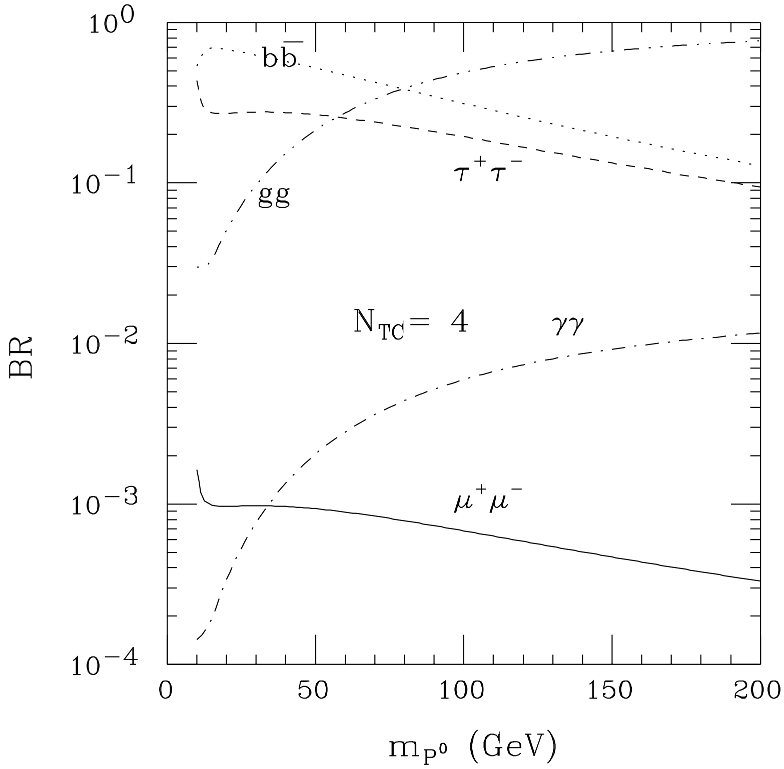
$$\Gamma(P^0 \to f\bar{f}) = C_F \frac{m_{P^0}}{8\pi}\lambda_f^2 \left(1 - \frac{4m_f^2}{m_{P^0}^2}\right)^2$$

$$\Gamma(P^0 \to gg) = \frac{\alpha_s^2}{48\pi^3 v^2} N_{TC}^2 m_{P^0}^3$$

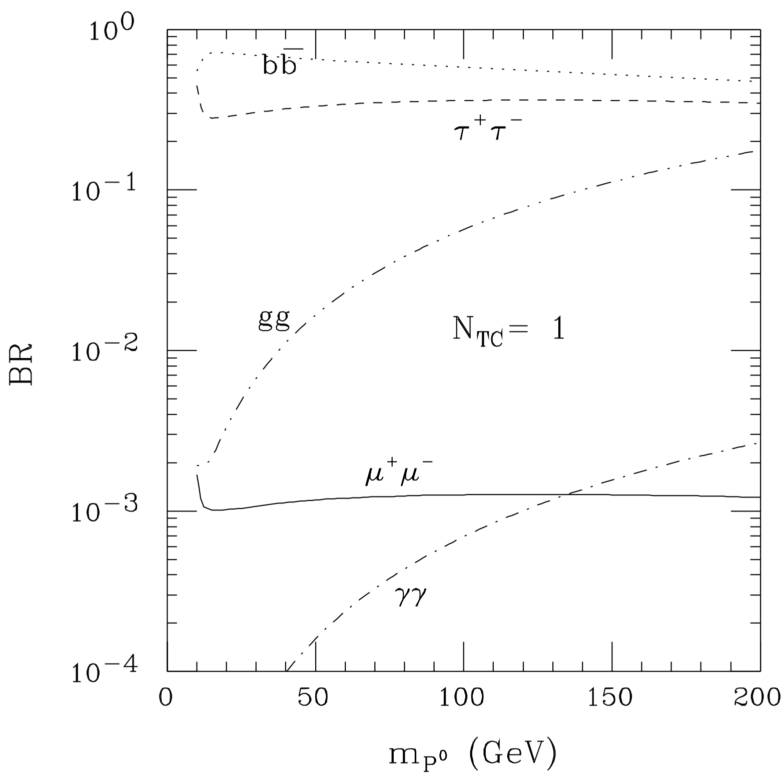
$$\Gamma(P^0 \to \gamma\gamma) = \frac{2\alpha^2}{27\pi^3 v^2} N_{TC}^2 m_{P^0}^3$$

$C_F = 1(3)$ for leptons(down-type quarks).
All results for $N_{TC} = 1, 4$ to show the N_{TC} dependence





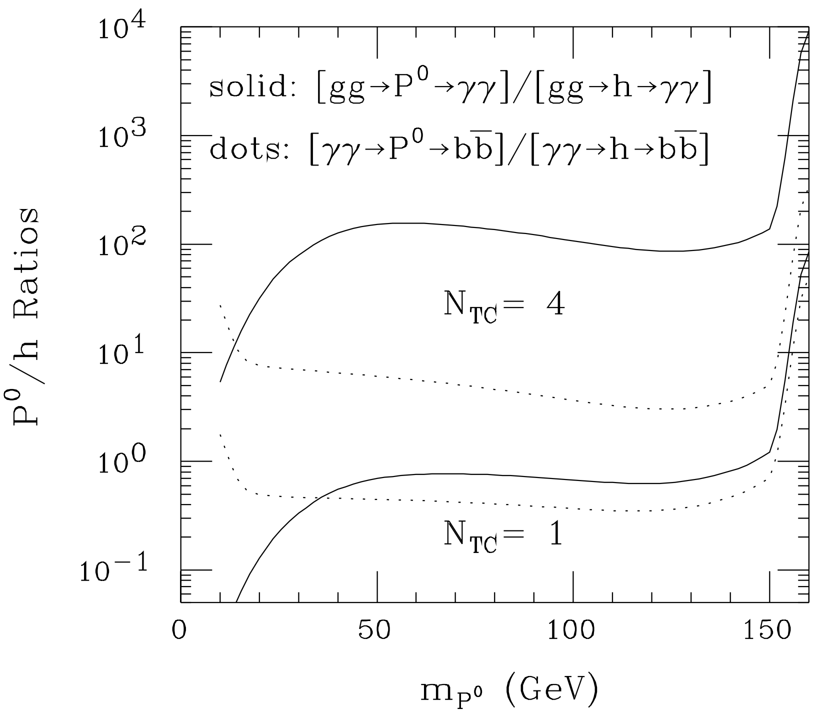
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
BR
$b\bar{b}$
$\tau^+\tau^-$
gg
N_{TC}= 4
$\gamma\gamma$
$\mu^+\mu^-$
0
50
100
150
200
m_{P^0} (GeV)



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
BR
$b\bar{b}$
$\tau^+\tau^-$
gg
N_{TC}= 1
$\mu^+\mu^-$
$\gamma\gamma$
0
50
100
150
200
m_{P^0} (GeV)

- Total width in the few MeV range, similar to Higgs
- Largest BR's: $b\bar{b}$, $\tau^+\tau^-$, gg
- $\Gamma(P^0 \to gg)/\Gamma(H \to gg) \approx 1.5 N_{TC}^2$
- $B(P^0 \to \gamma\gamma)/B(H \to \gamma\gamma) \approx 4$ for $50 \leq m_{P^0}(GeV) \leq 150$ and $N_{TC} = 4$. Much smaller for $N_{TC} = 1$.

$[\Gamma_{P^0 \to gg} B_{P^0 \to \gamma\gamma}]/[\Gamma_{H \to gg} B_{H \to \gamma\gamma}]$



- The P^0 discovery in the $\gamma\gamma$ channel at a hadron collider much easier than for the Higgs, if $N_{TC} > 1$. The same is true for discovering P^0 in $\gamma\gamma$ colliders. For $N_{TC} = 1$ things about as for the Higgs

- This depends again on the precise mass eigenstate composition of P^0. For π_D we get reduction factors 8 and 8/3 for the ratios $gg \to \pi_D \to \gamma\gamma$ and $\gamma\gamma \to \pi_D \to b\bar{b}$ to the Higgs

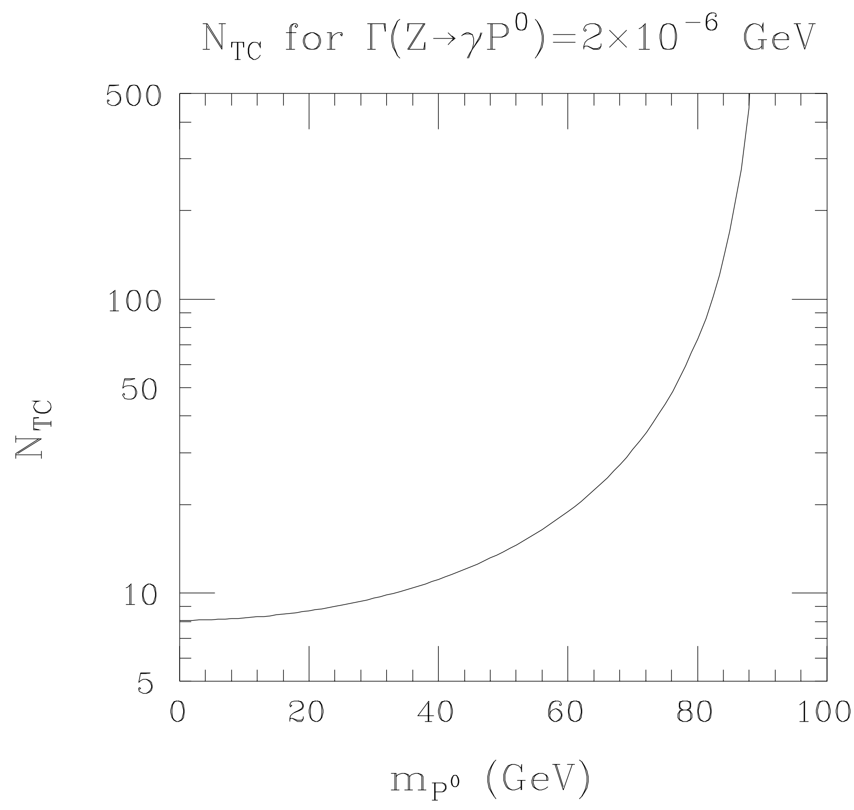
- The possibility of having large P^0/H ratios for the ΓB products together with P^0 being naturally light makes P^0 **a unique particle!!**

P^0 at LEP and LEP2

- At LEP the dominant production mode is $Z \to \gamma P^0$ (Manohar and Randall, 1990)

$$\Gamma(Z \to \gamma P^0) = \frac{\alpha^2 m_Z^3}{96\pi^3 v^2} N_{TC}^2 A_{P^0 Z\gamma}^2 \left(1 - \frac{m_{P^0}^2}{m_Z^2}\right)^3$$

Requiring $Z \to \gamma P^0 > 2 \times 10^{-6}$ GeV to make the P^0 visible in a sample of 10^7 Z bosons, we get the minimum N_{TC} required as a function of m_{P^0}. $N_{TC} \gtrsim 8$ is required at $m_{P^0} = 0$, rising rapidly as m_{P^0} increases.



- LEP2 - $\sigma(e^+e^- \to P^0\gamma)$ is dominated by $e^+e^- \to \gamma \to \gamma P^0$ ($A_{P^0\gamma\gamma}$ larger than $A_{P^0 Z\gamma}$). For $\sqrt{s} = 200\ GeV$ and an angular cut $20^0 \leq \theta_\gamma \leq 160^0$ (to avoid F/B singularities but still 91% efficient)

$$\sigma(e^+e^- \to \gamma P^0) < 1 \text{ fb}, \quad N_{TC} = 4$$

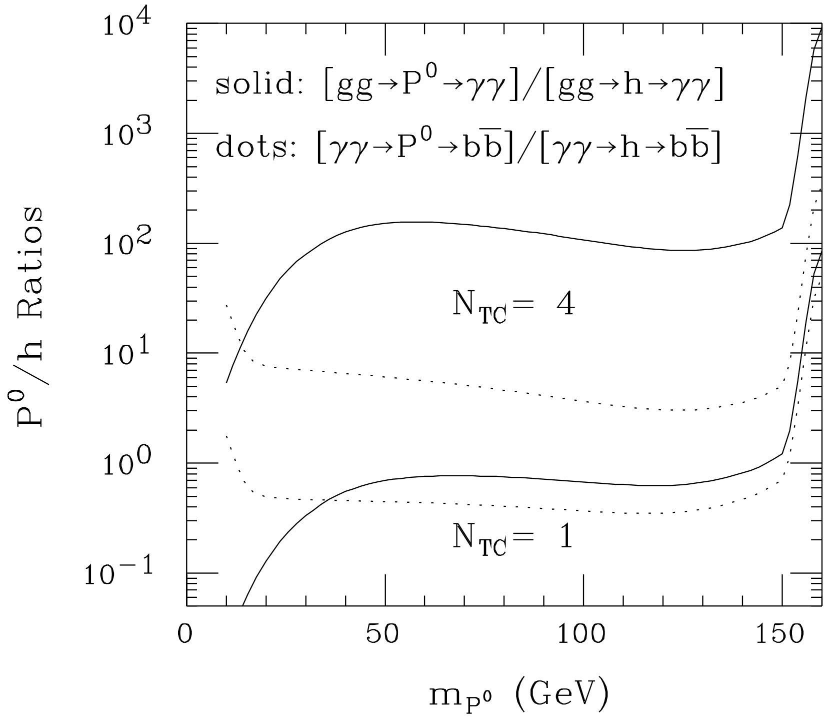
With $L = 0.5$ fb^{-1} P^0 not detectable at LEP2 unless N_{TC} is very large

P^0 at the Tevatron and the LHC

- Most important discovery mode both at the Tevatron and the LHC

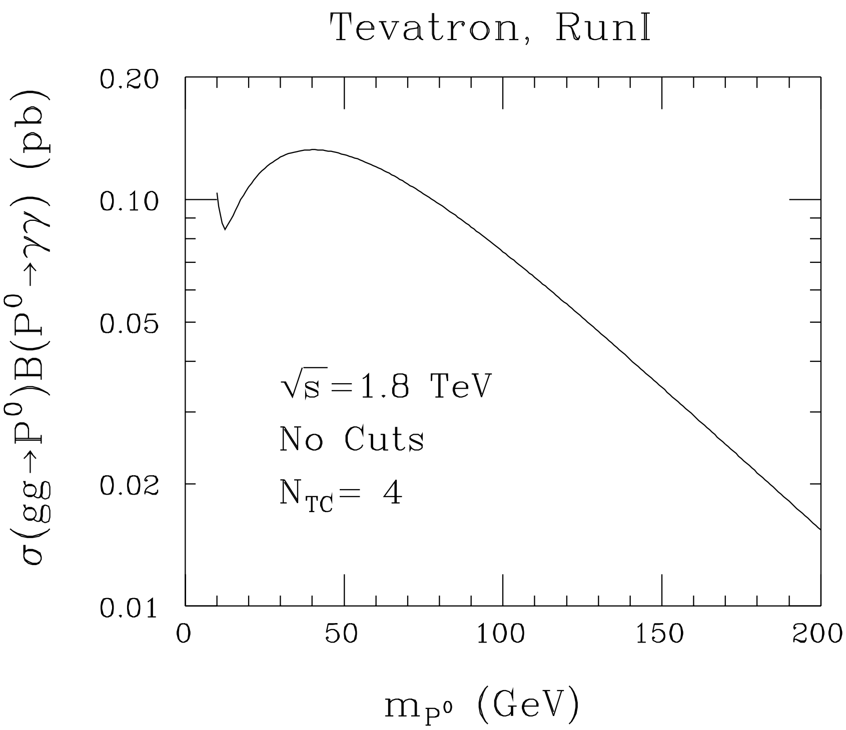
  $$gg \to P^0 \to \gamma\gamma$$

  Very robust, remember ratios to Higgs



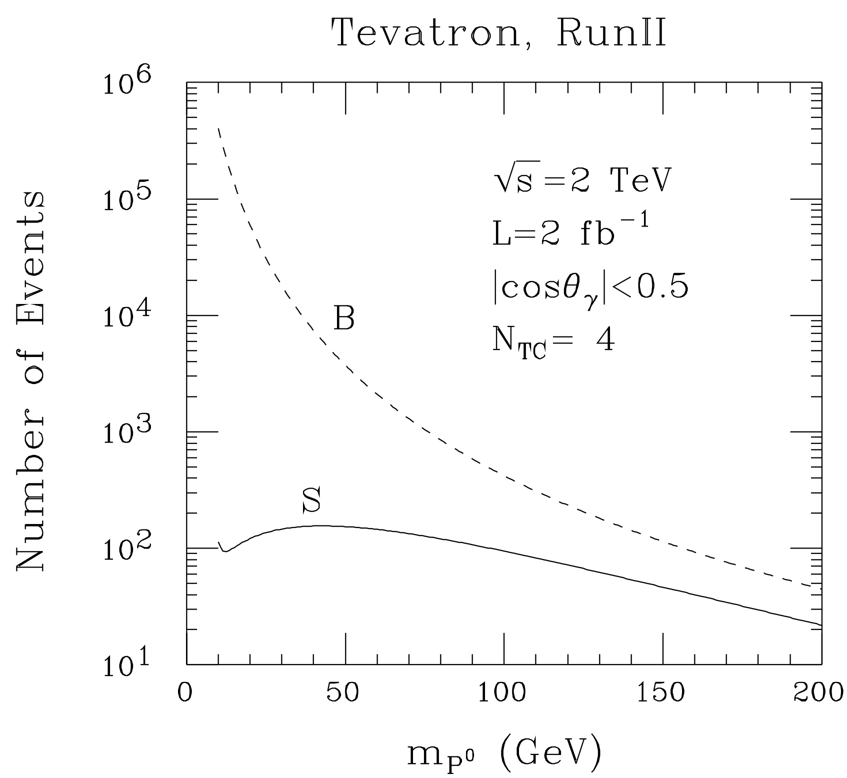
- Rate proportional to $\Gamma(P^0 \to gg)B(P^0 \to \gamma\gamma)$. For $N_{TC} = 4$, factor ≈ 10 w.r. to the Higgs for $m_{P^0} \approx 12\ GeV$ up to ≈ 100 for $m_{P^0} > 100\ GeV$ and increasing rapidly since $B(H \to \gamma\gamma)$ declines for the opening of $H \to WW, ZZ$ channels ($P^0 \to WW$ vanishes and $P^0 \to ZZ$ is negligible). Signal rate decreases only due to phase space

- **Tevatron RunI**

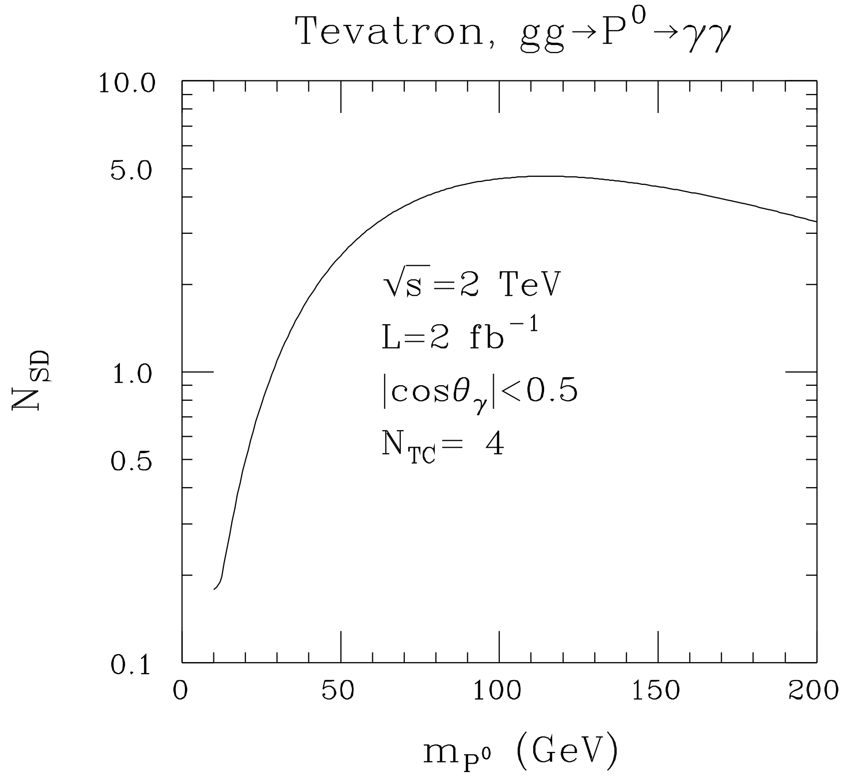


- For L=100 fb^{-1}, # events $\approx$ 10 at $m_{P0} \approx$ 40 GeV down to $\approx$ 2 at $m_{P0} \approx$ 200 GeV. Further reduction from efficiencies and cuts. A more accurate analysis shows that RunI could, at best, exclude the possibility of a light P^0 for $N_{TC} > 12$

- **Tevatron RunII**



- Sizeable rate for L=2 fb^{-1} (but for $N_{TC} = 1$, $S < 1$). The background shown is the irreducible $\gamma\gamma$. Reducible γj and jj requires detailed detector simulation



- The P^0 is detectable for $m_{P0} > 60\ GeV$ ($N_{SD} > 3$). A luminosity of about 30 fb^{-1} would allow a determination of $\Gamma(P^0 \to gg)B(P^0 \to \gamma\gamma)$ of about $5 \div 10\%$. At low masses a serious study of the reducible background needed. Again, if π_D is the mass eigenstate, the process $gg \to \pi_D \to \gamma\gamma$ much less enhanced. Poor prospectives for the Tevatron RunII.

- **LHC**

- Both ATLAS and CMS claim the possibility of discovering the SM Higgs in $gg \to H \to \gamma\gamma$. No problems for P^0 at LHC even for $N_{TC} = 1$. For $N_{TC} \geq 4$ it is safe to assume that the discovery range is $50 \leq m_{P^0}(GeV) \leq 200$. The background studies of the LHC collaborations are not available for $m_H > 200\ GeV$, but we expect that the discovery region can be extended beyond 200 GeV. For $m_{P^0} < 50\ GeV$ limitations come from the irreducible $\gamma\gamma$ BG and in rejecting the reducible γj and jj. We estimate that for $N_{TC} = 4$, $m_{P^0} > 30\ GeV$ is possible, but $m_{P^0} \leq 20\ GeV$ is a problem

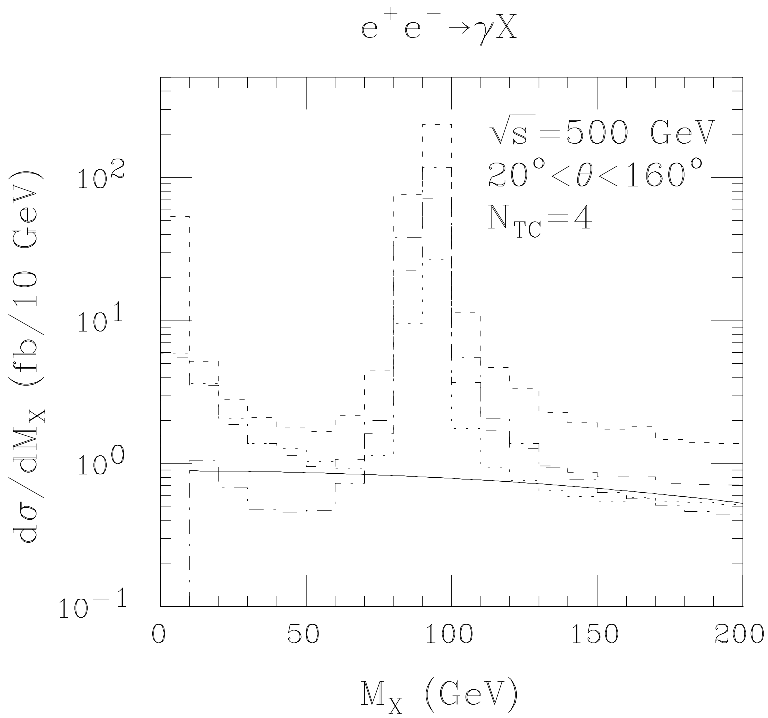
- A detailed study of the case $N_{TC} = 1$ shows that for $70 \leq m_{P^0}(GeV) \leq 200$, the P^0 is detectable at LHC

- The statistical errors for $gg \to P^0 \to \gamma\gamma$ we get, are

$$N_{TC} = 4 \quad \longmapsto \quad \approx 1\%$$
$$N_{TC} = 1 \quad \longmapsto \quad \approx 20\%$$

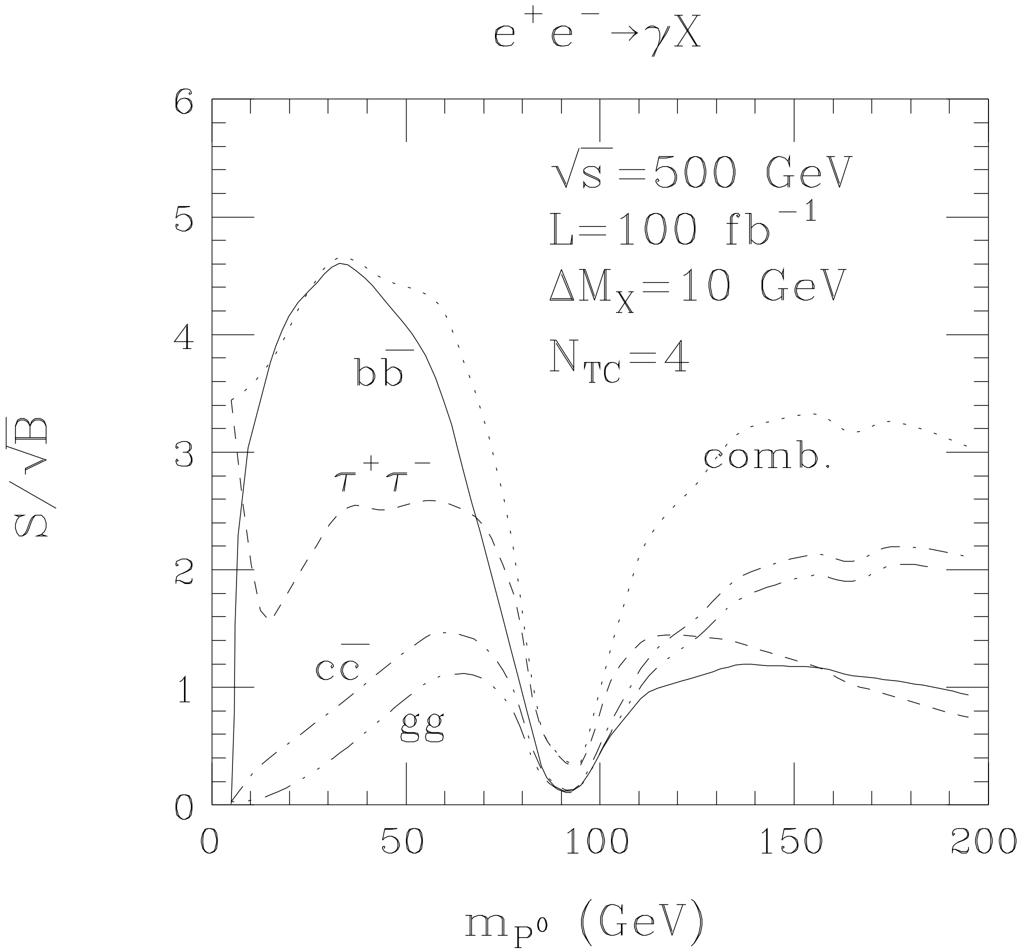
- Another possible production channel is through T-vectors as ρ_T or ω_T in $pp \to V^{\pm}X \to P^{\pm}P^0X$, or $pp \to V^0X \to P^{\pm}P^{\mp}X$. The rates are decent only if the T-vector V is relatively low in mass, as in walking TC.

P^0 at future e^+e^- colliders



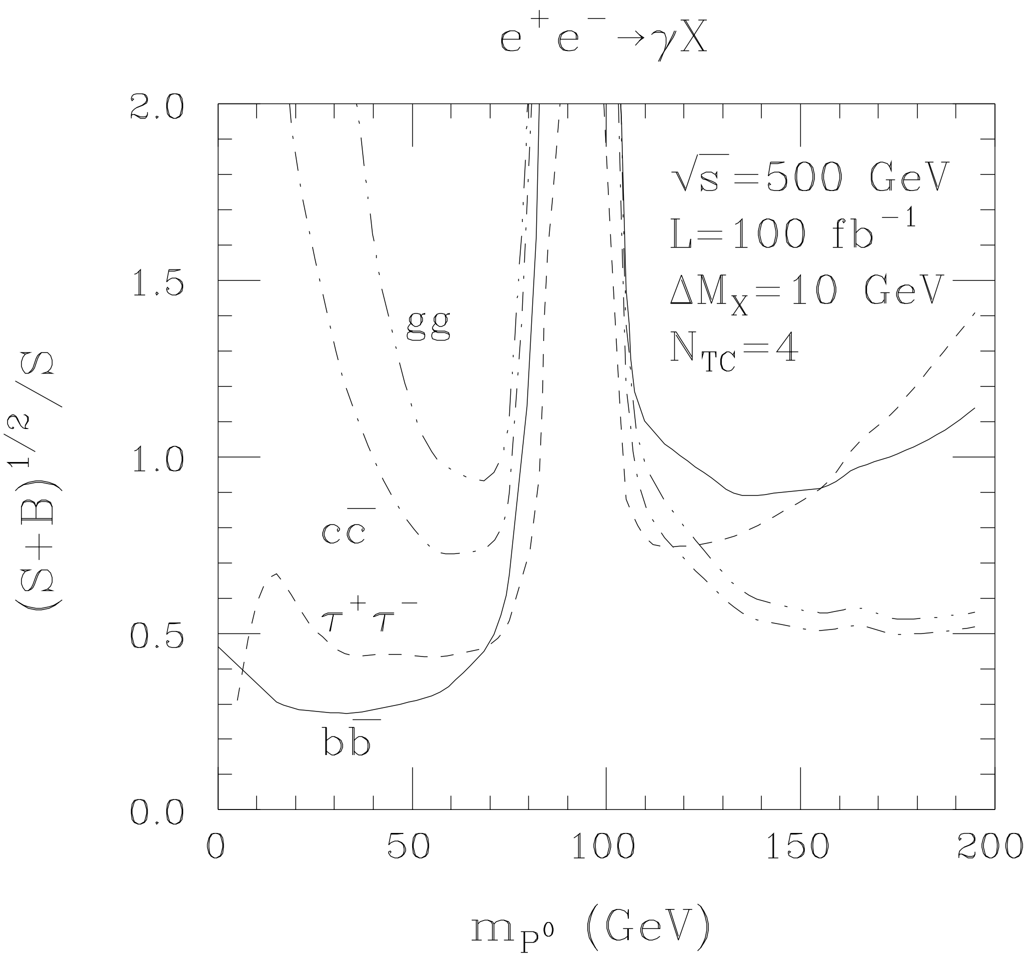
- $d\sigma_{e^+e^-\to\gamma P^0}/dm_{P^0}$ (solid curve), $\gamma b\bar{b}$ (dotdash), $\gamma c\bar{c}$ (dashes), $\gamma q\bar{q}$ (small dashes), $\gamma\tau^+\tau^-$ (dashdoubledots)

- Dominant mode $e^+e^- \to P^0\gamma$. At TESLA, L=500 fb^{-1}, we get about $2500 \div 4500$ events ($e^+e^- \to ZP^0$ is about 1%). For $N_{TC} = 1$ only less than 30 events. For the following discussion $N_{TC} = 4$. The relevant BG's are $\gamma b\bar{b}$, $\gamma c\bar{c}$, $\gamma q\bar{q}$ ($q = u, d, s$)and $\gamma\tau^+\tau^-$. The corresponding σ's are integrated over a bin of $\Delta M_X = 10\ GeV$. Angular cuts have been applied to signal and B's. Tagging and mistagging have been included



- Notice that $S/\sqrt{B}$ is not as good as for the SM Higgs in the ZH mode, since $P^0 \to ZZ$ only through the ABJ anomaly. The discovery regions are ($S/\sqrt{B} \geq 3$) for L = 100 (500) fb^{-1}

$$m_{P^0} \leq 75(80)\ GeV$$
$$m_{P^0} \geq 130(100)\ GeV$$



- After discovery one can extract ratios of BR's via the rates with the fractional accuracy shown in Figure. The only reasonable channel is $b\bar{b}$ with an error $\geq$ 15% for L=500 fb^{-1}.

- A model independent determination of the absolute BR's $B(P^0 \to F)$ (F any final state) would be possible through the ratio of the rate $P^0 \to F$ to the inclusive rate

$$B(P^0 \to F) = \frac{\sigma(e^+e^- \to \gamma P^0) B(P^0 \to F)}{\sigma(e^+e^- \to \gamma P^0)}$$

  Crucial the detection of P^0 inclusively as a peak in the recoil spectrum. Resolution fixed by the E_γ resolution. For

$$\frac{\Delta E_\gamma}{E_\gamma} = \frac{0.12}{E_\gamma(GeV)} \oplus 0.01$$

  we get $= \pm 1\sigma$ mass windows

$$(0, 78), \ (83.5, 114), \ (193, 207)$$

  or, for

$$\frac{\Delta E_\gamma}{E_\gamma} = \frac{0.08}{E_\gamma(GeV)} \oplus 0.005$$

  we get $= \pm 1\sigma$ mass windows

$$(36, 69), \ (91, 108), \ (196, 204)$$

  with $m_{P^0}(GeV) = 55$, 100 and 200 respectively. Knowing m_{P^0} in advance one can choose the appropriate window and estimate the BG's, but the errors will be large.

The conclusion is that at an e^+e^- collider will be very difficult to get more than a very rough determination of the parameters entering in the effective lagrangian, for $N_{TC} = 4$. For $N_{TC} = 1$ one would need more than 500 fb^{-1} even for detecting P^0.

Results at future $\gamma\gamma$

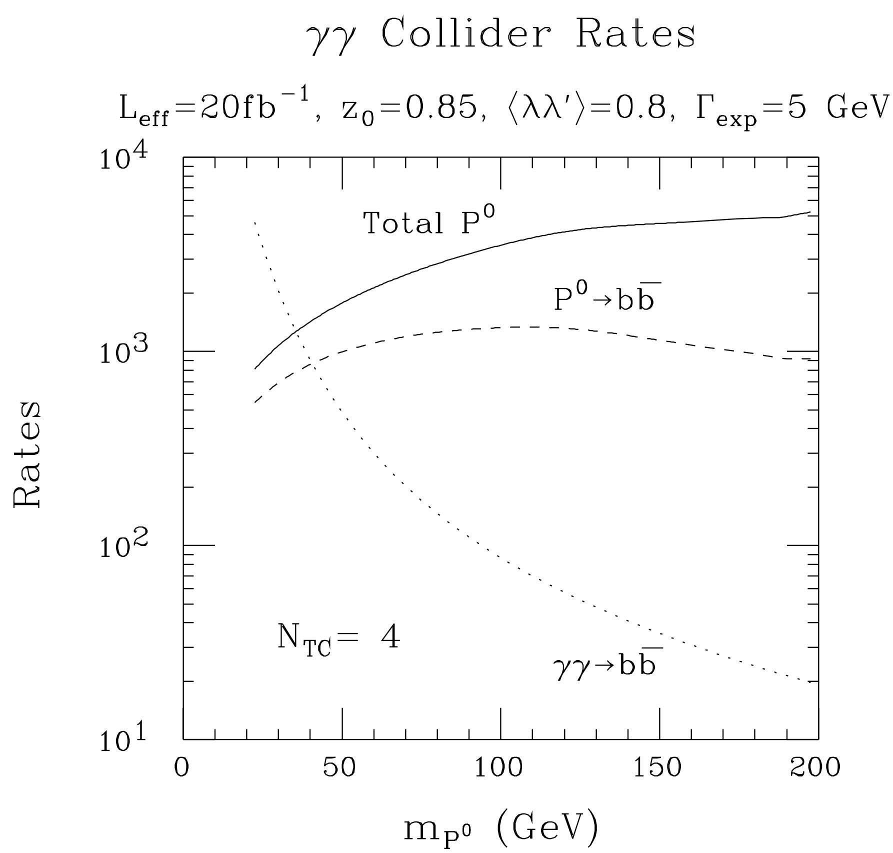
By folding the cross section for the P^0 production at a given energy $E_{\gamma\gamma}$ of a $\gamma\gamma$ collider with the differential luminosity (Gunion and Haber, 1993)

$$N(\gamma\gamma \to P^0 \to X) = \frac{8\pi\Gamma(P^0 \to \gamma\gamma)B(P^0 \to X)}{m_{P^0}^2 E_{e^+e^-}} \times \tan^{-1}\frac{\Gamma_{\mathrm{exp}}}{\Gamma_{P^0}^{\mathrm{tot}}}\left(1 + \langle\lambda\lambda'\rangle\right) G(y_0) L_{e^+e^-}$$

where $y_0 = m_{P^0}/E_{e^+e^-}$, λ and λ' are the helicities of the colliding photons, Γ_{exp} is the mass interval accepted in the final state. The function $G(y)$ is defined by

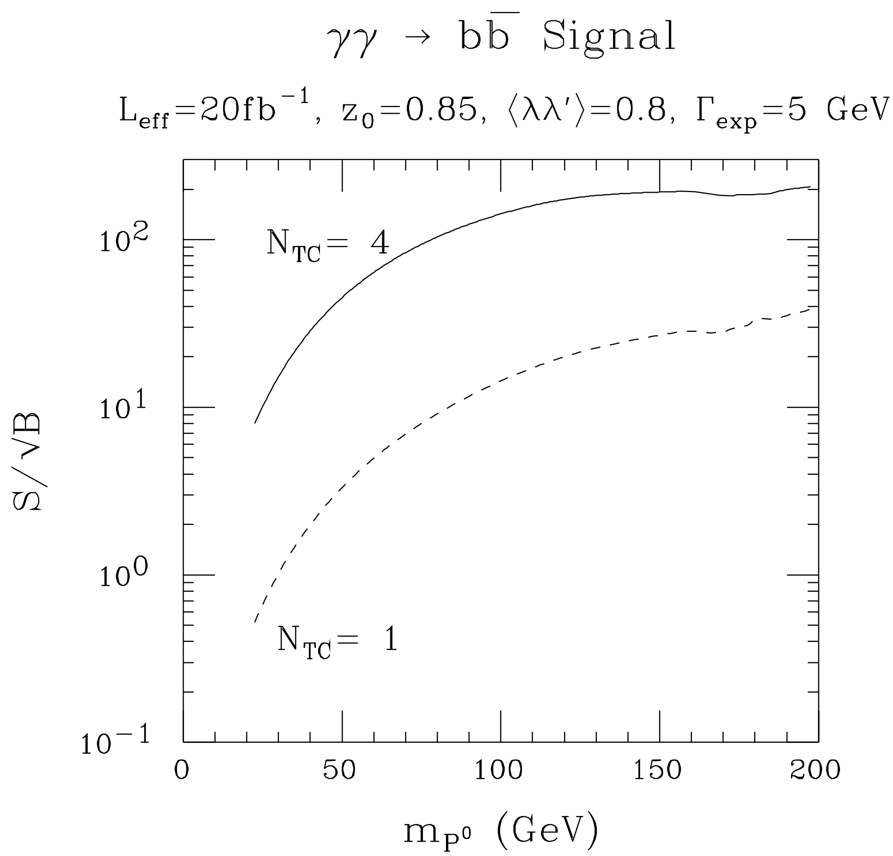
$$\frac{dL_{\gamma\gamma}}{dy} \equiv G(y) L_{e^+e^-}$$

$G(y)$ and $\langle\lambda\lambda'\rangle$ are obtained after convoluting over the possible energies and polarizations of the colliding photons that yeld a fixed value of y. For initial discovery one chooses a setup with initial laser circular polarizations and e^+e^- helicities for a broad spectrum ($2\lambda_e P_c = +1$). One has $G(y_0) \gtrsim 1$ and $\langle\lambda\lambda'\rangle \sim 0.8$.



Angular cut $|\cos\theta| \le z_0$. $L_{eff} \equiv G(y_0) L_{e^+e^-}$. Also shown the irreducible $\gamma\gamma \to b\bar{b}$ BG.

- The large value of $\Gamma(P^0 \to \gamma\gamma) B(P^0 \to b\bar{b})$ compared to the Higgs case implies very good rates. We stress once more the relevance of the composition of the mass eigenstate, for π_D rates down by a factor 8. Good values of $S/\sqrt{B}$, from 8 for $m_{P^0} \approx 20\ GeV$ to 200 for $m_{P^0} \approx 200\ GeV$. Even for $N_{TC} = 1$, the P^0 will be detectable for $m_{P^0} > 60\ GeV$.



- Once discovered it would be possible to measure the rate $\Gamma(P^0 \to \gamma\gamma)B(P^0 \to b\bar{b})$ with a very high statistical accuracy. For $m_{P^0} \approx 100\ GeV$ we get $\approx$ 1.5%. The systematic will dominate this error. For $N_{TC} = 1$ we get $\approx$ 5%. Going down to $m_{P^0} \approx 20\ GeV$ we get $\approx$ 10% for $N_{TC} = 4$

- Other channels? $\gamma\gamma \to P^0 \to \tau^+\tau^-,\ gg$ have large event rates, but
  - BG too large for the gg final state
  - BG not large for $\tau^+\tau^-$ but not a sharp peak

  However, optimization of the set-up knowing already m_{P^0} could make it possible.

- $\gamma\gamma \to P^0 \to \gamma\gamma$ event rate is $\approx$ 20(50) at $m_{P^0} \approx$ 100 GeV(200) with possible improvements through optimization. Irreducible BG probably small except for jets faking photons (detailed study needed). For $N_{TC} =$ 1 things more difficult. The relevance is the possibility of measuring $\Gamma^{\rm tot}_{P^0}$. To do that

  - Rate for $\gamma\gamma \to P^0 \to \gamma\gamma$

    $$\Gamma(P^0 \to \gamma\gamma) B(P^0 \to \gamma\gamma) = \frac{|\Gamma(P^0 \to \gamma\gamma)|^2}{\Gamma^{\rm tot}_{P^0}}$$

  - Rate for $\gamma\gamma \to P^0 \to b\bar{b}$ proportional to $\Gamma(P^0 \to \gamma\gamma) B(P^0 \to b\bar{b})$

  - If $B(P^0 \to b\bar{b})$ known from e^+e^- (but we saw very difficult) then extract $\Gamma(P^0 \to \gamma\gamma)$ from $b\bar{b}$ and then get $\Gamma^{\rm tot}_{P^0}$.

Using NLC to fix the effective low energy parameters for P^0?

The available well-measured quantities from the LHC and the $\gamma\gamma$ colliders would be

- $\Gamma(P^0 \to gg)B(P^0 \to \gamma\gamma)$ from the LHC
- $\Gamma(P^0 \to \gamma\gamma)B(P^0 \to b\bar{b}, \tau^+\tau^-, \gamma\gamma)$ from the $\gamma\gamma$ collider

The dependence of the measured quantities on the parameters is

$$\Gamma(P^0 \to gg)\Gamma(P^0 \to \gamma\gamma) \propto N_{TC}^4/v^4$$
$$\Gamma(P^0 \to \gamma\gamma)\Gamma(P^0 \to b\bar{b}) \propto N_{TC}^2/v^2 \times [4m_2' - 3m_b]^2/v^2$$
$$\Gamma(P^0 \to \gamma\gamma)\Gamma(P^0 \to \tau^+\tau^-) \propto N_{TC}^2/v^2 \times [\tfrac{4}{3}m_4 - \tfrac{1}{3}m_\tau]^2/v^2$$
$$\Gamma(P^0 \to \gamma\gamma)\Gamma(P^0 \to \gamma\gamma) \propto N_{TC}^4/v^4$$

To get the actual rates one has to divide by $\Gamma_{P^0}^{\rm tot}$, depending on the same parameters if gg, $\tau^+\tau^-$ and $b\bar{b}$ final states dominate the P^0 decays. Determine

- N_{TC}/v
- $|4m_2' - 3m_b|/v$
- $|\frac{4}{3}m_4 - \frac{1}{3}m_\tau|/v$

One can check consistency of rates $gg \to \gamma\gamma$ relative to $\gamma\gamma \to \gamma\gamma$ to verify anomalous coupling ratios.
Up to a discrete set of ambiguities it is possible to fix these 3 parameters. Then, use

$$m_b = m_2' + m_9, \quad m_\tau = m_4 + 3m_{10}$$

to fix m_9 and m_{10}.

This allows the determination of the Yukawa couplings of P^0 to b and τ

$$\begin{aligned}\lambda_b &= -\frac{\sqrt{6}}{3v}(m_2' - 3m_9) \\ \lambda_\tau &= \frac{\sqrt{6}}{v}(m_4 - 3m_{10})\end{aligned}$$

If, as likely, (m_2', m_9), (m_4, m_{10}) and $(m_4^{(2)}, m_{10}^{(2)})$ are related respectively to m_b, m_τ and m_μ, we can approximate

$$\begin{aligned}\rho_8 &= 2m_2' m_9 + 2m_4 m_{10} + 2m_4^{(2)} m_{10}^{(2)} \\ &\approx 2m_2' m_9 + 2m_4 m_{10}\end{aligned}$$

and from m_{P^0}

$$m_{P^0}^2 = \frac{4\Lambda^2}{\pi^2 v^2}\rho_8$$

obtain Λ. At the same time from N_{TC}/v we can extract N_{TC}. This assumes $v = 246\ GeV$. In multi-scale TC theories things could be different, in that case one extracts only Λ/v' and N_{TC}/v'. The coupling of P^0 to the μ may eventually be determined at a future muon collider.

$s-$ channel P^0 production at $\mu^+\mu^-$

- The P^0 has a sizeable $\mu^+\mu^-$ coupling (not the $P^{0\prime}$)

- The muon collider has the ability to achieve a very narrow Gaussian spread, $\sigma_{\sqrt{s}} \sim 1$ MeV $\left(\frac{R}{0.003\%}\right)\left(\frac{\sqrt{s}}{50\ \text{GeV}}\right)$ One can achieve $R = 0.003\%$ beam energy resolution with reasonable luminosity (L_{year}(@100 GeV) = 0.1 fb^{-1}).

- Good measurements of rates $\mu^+\mu^- \to P^0 \to b\bar{b}, \tau^+\tau^-, gg$ and $\Gamma^{\text{tot}}_{P^0}$.

In conclusion

- $\Gamma^{\text{tot}}_{P^0}$ from the muon collider

- $\Gamma(P^0 \to gg)B(P^0 \to \gamma\gamma)$ from the LHC

- $\Gamma(P^0 \to \gamma\gamma)B(P^0 \to b\bar{b})$ from the $\gamma\gamma$

- $\Gamma(P^0 \to \mu^+\mu^-)B(P^0 \to F)$ for $F = b\bar{b}, \tau^+\tau^-, gg$ from the muon collider

determine the number of technicolors of the theory and (up to a discrete set of ambiguities) the fundamental parameters of the low-energy effective Lagrangian describing the Yukawa couplings of the P^0, as discussed for the NLC.

Conclusions

Theory

- Getting the eigenstate right is crucial
- The P^0 eigenstate is very special.
  Since its mass $\propto m_b$, it is much lighter than all the other PNGB's; all others bring in m_t into mass formula
- Only a few partial widths of P^0 are important and they are expressed in terms of a relatively small set of parameters of the effective lagrangian

Experiment

- No limits on P^0 from LEP or LEP2 or Tevatron RunI
- Tevatron RunII has quite a chance for finding P^0 ($m_{P^0} > 60\ GeV$)
- LHC will almost certainly discover the P^0, but problems with very low m_{P^0}
- Poor accuracy anticipated for normal $e^+e^- \to \gamma P^0$ measurements at NLC means that $\gamma\gamma$ mode will be crucial. Only clear alternative would be a muon collider.
  In either case we will get mass indications from LHC
- $\gamma\gamma$ mode must measure as many final states as possible. Certainly $b\bar{b}$ and $\tau^+\tau^-$ are required. $\gamma\gamma$ would be very useful

- If $\gamma\gamma \to \gamma\gamma$ measurement can be done with good accuracy it will allow a stand-alone analysis of the low-energy parameters of the P^0

- The LHC $gg \to \gamma\gamma$ measurements can provide a substitute or a cross check

- With the above measurements, many interesting parameters of the theory can be determined
  - Unambiguous determination of N_{TC}/v and Λ/v
  - Effective Yukawa Lagrangian parameters (in units of v) in down-quark/lepton sector, barring a discrete set of ambiguities